Consider the case of a pair of identical infinite line currents (current $i$), parallel to the $z$ axis, one piercing the $xy$ plane at $(a, 0)$ going into the page and the other at $(-a, 0)$ coming out of the page. It's easy to directly calculate the force on a length $\ell$ of the lhs line current: The rhs line current produces a clockwise magnetic field:

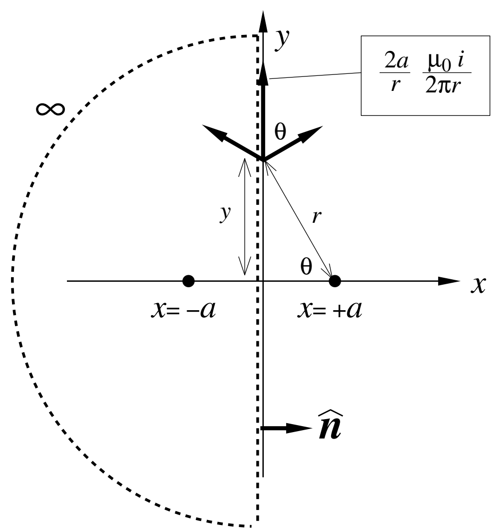
$$B = \frac{\mu_0 i}{2\pi r}$$

So at the location of the lhs line current, the magnetic field is:

$$\mathbf{B} = \hat{\mathbf{j}}\,\frac{\mu_0 i}{2\pi 2a}$$

The magnetic force per length is $\mathbf{I} \times \mathbf{B}$, so length $\ell$ of the lhs line current has total force:

$$\mathbf{F} = -\hat{\mathbf{i}}\,\frac{\mu_0 i^2 \ell}{2\pi 2a}$$




Now the hard way: Imagine surrounding a length $\ell$ of the lhs wire with the section of the $yz$ plane from $z = 0$ to $z = \ell$. Note that for these infinite wires, the fields do not depend on $z$. Infinitely far (so the magnetic field is effectively zero) we 'close' the surface with a half cylinder (that, for example, goes through $x = -\infty$). If we integrate the stress tensor over this closed surface, we should get the net magnetic force on all the enclosed current (which is just the $\ell$ section of the lhs wire). The total magnetic field on the $yz$ plane is the sum of the magnetic fields of both line currents. As shown in the diagram, the $x$ components of the fields cancel and the $y$ componets add. To get the $y$ components we need $\cos\theta$, which is just $a/r$. The result is just in the $y$ direction:

$$\mathbf{B} = \hat{\mathbf{j}}\,\frac{2a}{r}\,\frac{\mu_0 i}{2\pi r}$$

where $r^2 = y^2 + a^2$.

Since $B_x = B_z = 0$, the stress tensor is diagonal:

$$\mathcal{T} = \frac{1}{\mu_0}\begin{bmatrix} -\frac{1}{2}B_y^2 & 0 & 0 \\ 0 & \frac{1}{2}B_y^2 & 0 \\ 0 & 0 & -\frac{1}{2}B_y^2 \end{bmatrix}$$

So for a segment of the $yz$ plane:

$$d\mathbf{F} = \mathcal{T} \cdot \hat{\mathbf{n}} da = -\hat{\mathbf{i}}\ \frac{1}{2\mu_0}\ B_y^2\ \ell dy = -\hat{\mathbf{i}}\ \frac{1}{2\mu_0}\ \left[\frac{2a}{r}\ \frac{\mu_0 i}{2\pi r}\right]^2 \ell dy$$

and the total force is:

$$\mathbf{F} = -\hat{\mathbf{i}}\ \frac{\mu_0 a^2 i^2 \ell}{2\pi^2}\ \int_{-\infty}^{+\infty} \frac{1}{(y^2+a^2)^2}\ dy$$

which when evaluated, agrees with the above.

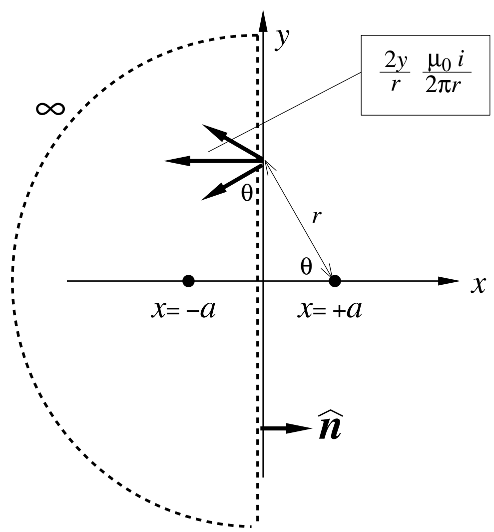
If we reverse the sign of the current in the rhs wire the force between the two is equal to the above but opposite (attractive). Lets see how that works with the stress tensor.

The total magnetic field on the $yz$ plane is the sum of the magnetic fields of both line currents; the sum of these two counter-clockwise vectors points in the $-\hat{\mathbf{i}}$ direction:

$$\mathbf{B} = -\hat{\mathbf{i}}\ \frac{2y}{r}\ \frac{\mu_0 i}{2\pi r}$$

where $r^2 = y^2 + a^2$.
Since $B_y = B_z = 0$, the stress tensor is diagonal:

$$\mathcal{T} = \frac{1}{\mu_0} \begin{bmatrix} \frac{1}{2}B_x^2 & 0 & 0 \\ 0 & -\frac{1}{2}B_x^2 & 0 \\ 0 & 0 & -\frac{1}{2}B_x^2 \end{bmatrix}$$




So for a segment of the $yz$ plane:

$$d\mathbf{F} = \mathcal{T} \cdot \hat{\mathbf{n}} da = \hat{\mathbf{i}}\ \frac{1}{2\mu_0}\ B_x^2\ \ell dy = \hat{\mathbf{i}}\ \frac{1}{2\mu_0}\ \left[\frac{2y}{r}\ \frac{\mu_0 i}{2\pi r}\right]^2 \ell dy$$

and the total force is:

$$\mathbf{F} = \hat{\mathbf{i}}\ \frac{\mu_0 i^2 \ell}{2\pi^2}\ \int_{-\infty}^{+\infty} \frac{y^2}{(y^2+a^2)^2}\ dy$$

which when evaluated, agrees with the expected result.