Physics 366: Relativity EXAM 1 2005 September 22

Complete the last problem and 4 additional problems.

The S' frame moves with a velocity βc down the positive x axis of the S frame. The relationship between coordinates in the two frames is given by:

Boost: $$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \text{and} \quad \begin{matrix} y' = y \\ z' = z \end{matrix}$$

or: $\mathbb{X}' = O \cdot \mathbb{X}$ where: $\mathbb{X} = (\mathbf{r}, ict)$

and O is the orthogonal matrix: $$\begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

4-vectors: $\mathbb{U} = \mathbb{\beta} c = \gamma(\mathbf{v}, ic)$ $\quad \mathbb{P} = m_0\mathbb{U} = (\mathbf{p}, iE/c) = m_0\gamma(\mathbf{v}, ic)$

1. Two photons travel along the x-axis of S, with a constant distance L between them. Prove in S' the distance between these photons is $L(1+\beta)^{\frac{1}{2}}/(1-\beta)^{\frac{1}{2}}$

2. In the S' frame an electron moves straight down the the y'-axis, with $\beta_e = .95$. Find the 4-velocity $\mathbb{U}'$ of this electron in the S' frame. As usual, the S' frame moves with velocity $\beta = .99$ down the x-axis of frame S. Find the 4-velocity $\mathbb{U}$ of this electron in the S frame. Sketch the electron's trajectory as seen in the S frame. Calculate the direction of motion and speed as seen in the S frame.

3. Consider the 4-momentum $\mathbb{P} = a(3, -2, 1, i4)$ where a is a constant. Calculate $\mathbb{P}^2$. What is the rest mass of this particle? What is γ for this particle? What is the velocity of this particle? What is the speed of the particle?

   In the rest frame of this particle, what is the value of $\mathbb{P}'_4$?

4. Consider the tensor $T_{\mu\nu}$ that in the S frame has the following values:

$$T_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a \end{pmatrix}$$

   Find $T_{\mu'\nu'}$, i.e., $T_{\mu\nu}$ in the S' frame. Report an invariant that can be formed from $T_{\mu\nu}$ and its value.

5. In the lab frame (S) projectile particle A (mass: m_A) collides with the stationary target particle B (mass: m_B) to produce a new particle C (mass: m_C)…that is:

$$A + B \rightarrow C$$

   Show (derive) that A must have lab-frame energy:

$$E_A = \frac{m_C^2 - m_A^2 - m_B^2}{2m_B}\, c^2$$

   for this reaction to occur.

6. Assume that $\mathbb{D}$ is a 4-vector field with components

$$\mathbb{D} = (x, y, z, 0)$$

Find $\mathbb{D}'(x', ct')$, i.e., the vector field $\mathbb{D}$ as seen in the S' frame and expressed in terms of the S' coordinates. Calculate $\partial_\mu \mathbb{D}_\mu$ and $\partial'_\mu \mathbb{D}'_\mu$. Is the result invariant?

7. Consider the attached Minkowski diagram. The unit of length is light-years; the unit of time is years. Quartet, the home planet of the Quartons, is motionless in the S frame, three light years to the left of the origin. Three years ago a spaceship left Quartet (i.e., the event $(x, ct) = (-3, -3)$). According to observers in S, the spaceship traveled for 3 years toward the origin at a speed of $\frac{1}{3}c$. It then stopped and sent a radio signal back to Quartet asking if it should continue. As soon as it received the signal, Quartet replied: "yes, continue on to the origin". On the supplied diagram accurately sketch the world line of spaceship and the radio signals.

   According to the S' frame, at approximately what times (t') and locations (x') did the spaceship start and stop according to the S' frame. Does the traveling spaceship have a positive or negative velocity in the S' frame?

   Folks in the S frame measure the length of rod that is at rest in S': one end at $x' = 0$ the other at $x' = 1$. Label with **A**s the two events associated with this measurement. Show that the result is less than one light year. Folks in the S' frame measure the length of rod that is at rest in S: one end at $x = 0$ the other at $x = 1$. Label with **B**s the two events associated with this measurement. Show that the result is less than one light year.

   Folks in S measure the time it takes for a calendar at $x' = 2$ to click off one year (i.e, $t' = 0 \rightarrow t' = 1$) Label with **C**s the two events of this measurement. Show that the result is longer than one year.