Complete 6 of these 7 problems.

The S' frame moves with a velocity βc down the positive x axis of the S frame. The relationship between coordinates in the two frames is given by:

$$\textbf{Boost:} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \text{and} \quad \begin{matrix} y' = y \\ z' = z \end{matrix}$$

$$\textbf{or:} \quad \mathbb{X}' = O \cdot \mathbb{X} \qquad \text{where: } \mathbb{X} = (\mathbf{r}, ict)$$

$$\text{and } O \text{ is the orthogonal matrix: } \begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

$$\textbf{4-vectors:} \quad \mathbb{U} = \mathbb{B}c = \gamma(\mathbf{v}, ic) \qquad \mathbb{P} = m_0\mathbb{U} = (\mathbf{p}, iE/c) = m_0\gamma(\mathbf{v}, ic)$$

$$A^{\mu'} = \left(\frac{\partial x^{\mu'}}{\partial x^\alpha}\right) A^\alpha \qquad B_{\mu'} = \left(\frac{\partial x^\alpha}{\partial x^{\mu'}}\right) B_\alpha$$

$$\Gamma^\alpha_{\mu\nu} \equiv \tfrac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}\right)$$

$$0 = \frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} \qquad \frac{d^2 x_\alpha}{d\lambda^2} = \Gamma^\mu_{\alpha\nu} \frac{dx_\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = \tfrac{1}{2} g_{\mu\nu,\alpha} \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$$

$$\delta A^\mu = -\Gamma^\mu_{\alpha\beta} A^\alpha \delta x^\beta \qquad \delta B_\mu = \Gamma^\alpha_{\mu\beta} B_\alpha \delta x^\beta$$

$$A^j{}_{;i} \equiv \partial_i A_j + \Gamma^j_{i\alpha} A^\alpha \qquad B_{j;i} \equiv \partial_i B_j - \Gamma^\alpha_{ij} B_\alpha$$

$$R^d{}_{abc} = \Gamma^d_{ac,b} - \Gamma^d_{ab,c} + \Gamma^\alpha_{ac}\Gamma^d_{b\alpha} - \Gamma^\alpha_{ab}\Gamma^d_{c\alpha} \qquad R_{ab} = R^d{}_{abd} \qquad R = R^a{}_a$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -8\pi G_N/c^2 \, T_{\mu\nu} + \Lambda g_{\mu\nu}$$

The below problem requires lots individual elements to be calculated. For an in-class exam, you should be able to do any single calculation. The file `366f18P.m` sets mathematica do the zillions of calculations and you can use those results to check your by-hand calculations.

1. Consider the coordinate system $dx^a = (d\chi, d\theta, d\phi)$ with invariant distance:

$$ds^2 = d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Note that for small χ this is like spherical coordinates where:

$$ds^2 = dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Find the non-zero Christoffel symbols.

I claim that with this metric:

$$R_{abcd} = K \left(g_{ac} g_{bd} - g_{ad} g_{bc}\right) = K g_{a[c} g_{d]b}$$

for some scaler constant K (note that all indices are covariant in this expression). Calculate <u>one</u> non-zero R_{abcd}, and then use it in the above expression to determine K. Once you've found K, calculate the Ricci tensor and the scaler curvature.