Physics 364 FINAL EXAM Spring 1999

Answer 7 of the following 9 questions

1. Prove the Virial Theorem. Use it to explain why the Sun burns stably.

2. According to Allen's *Astrophysical Quantities* if $V = 0$ the F_λ at the center of the V band is: 3.8×10^{-12} W $\cdot$ cm$^{-2} \cdot \mu$m^{-1}. (Allen reports that the V band is centered on $\lambda = .55$ μm with a band width of $\Delta\lambda = .09$ μm.) Using the above data, consider a $V = 6$ star which is just barely visible to the naked eye. (The eye has an aperture of 6 mm.) Find the F_λ of the star. Find the approximate number/sec of V-band photons that could be captured by your eye. ($h = 6.6261 \times 10^{-34}$ J·s, $c = 3 \times 10^8$ m/s)

3. What is $B - V$? I claim that whereas both B and V depend on the distance to the star, $B - V$ is an intrinsic property of a star. Explain! *How* is $B - V$ related to more commonplace quantities? (False example: if $B - V$ increases density increases.) My above claim is not exactly correct. $B - V$ *can* depend on the distance from the star. What would cause this?

4. Consider a binary eclipsing star system with stars S (the small-radius star) and L (the large-radius star). When L totally eclipses S, the combined light drops 0.2 magnitudes (i.e., the magnitude increases by 0.2). Find the luminosity ratio L_S/L_L. When S moves in front of L, the combined light drops 1 magnitude. What fraction of L is covered by S?

5. Consider a uniform gas with mass density ρ and temperature T (i.e., typical molecule KE $\sim kT$). Using dimensional analysis come up with a characteristic *length* (called the Jean's length) that involves just G (units: N $\cdot$ m^2/kg^2), kT and ρ. (It turns our the the Jean's length is an estimate of the minimum size of a molecule cloud able to condense into a star.)

6. Most distance measurements in astronomy are based on the distance modulus: $m - M$. Generally m is easy to determine. Describe two ways that M can be determined.

7. Using the Hubble Space Telescope a picture of a distant galaxy is obtained without the blurring effects of the Earth's atmosphere. The galaxy has a diameter of 10 pixels (at 0.1 arc-seconds per pixel) so detailed structures (like spiral arms) can not be resolved. (A) What additional observations or further analysis might help resolve whether the galaxy is early-type or late-type? (B) If this galaxy is as large as our Galaxy, how far away is it?

8. The attached paper is from a recent edition of Astrophysical Journal. Quickly read this paper and find five concepts/terms that you learned in this course. Provide descriptions/definitions of those terms.

9. We derived in class the equation of motion for "Newtonian cosmology:"

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = E$$

where M is the mass enclosed by a sphere of radius R and E is the energy per mass. If we add a cosmological constant Λ we would get:

$$\left(\frac{dR}{dt}\right)^2 - \frac{2GM}{R} - \frac{\Lambda R^2}{3} = 2E$$

We can scale R to remove the constant E, but we are left with the sign of E: $k = -1, 0, 1$ depending on if $E < 0$, $E = 0$ or $E > 0$. (In general relativity, k turns out to be minus the sign of curvature of space.)

$$\left(\frac{dR}{dt}\right)^2 - \frac{2GM}{R} - \frac{\Lambda R^2}{3} = k$$

(a) For small R and constant M show: $R \propto t^{2/3}$. Say what defines "small R".

(b) For large R and constant M show: $R \propto \exp(\sqrt{\Lambda/3}\; t)$. Say what defines "large R".

(c) If R is small and M is in the form of a photon gas with temperature inversely proportional to R, show: $R \propto t^{1/2}$.

(d) Show that the Hubble "constant" $H = \dot{R}/R$ and so:

$$\begin{aligned} H^2 &= k/R^2 + \frac{8\pi}{3}\, G\rho + \frac{\Lambda}{3} \\ 1 - k/(HR)^2 &= \frac{8\pi G\rho}{3H^2} + \frac{\Lambda}{3H^2} \end{aligned}$$

Thus if the rhs is > 1, $= 1$ or < 1 determines the sign of the curvature of space. Remark: currently popular cosmological models use the $k = 0$ case with (currently) $\Lambda/3H^2 \sim 10 \times 8\pi G\rho/3H^2$, i.e., the cosmological constant dominates the mass density. Clearly in the past the mass density would dominate the cosmological constant.