While there is some dispute about the 'best' values for the cosmological parameters I adopt those from the Particle Data Handbook which are largely those of Planck (2018)

$$H_0 = 67.9 \text{ km/s/Mpc} \tag{1}$$

$$\Omega_m = \frac{8\pi G\rho_0}{3H_0^2} = .306 \tag{2}$$

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2} = .694 \tag{3}$$

Our differential equation for the scale factor $R = 1/(1+z)$ (valid for times when the mass density overwhelms light density) is

$$\dot{R}^2 = H_0^2\left(\frac{\Omega_m}{R} + \Omega_\Lambda R^2\right) \tag{4}$$

Surprisingly Mathematica can find an exact, analytic solution

```
DSolve[{D[R[t],t]^2==H0^2 (OmegaM/R[t]+OmegaL R[t]^2),R[0]==0},R[t],t]
```

we get a list of solutions. If we pull out the positive, real option (`%[[4]]` for me), and apply `FullSimplify` telling Mathematica that everything is positive

```
FullSimplify[%[[4]],{H0>0,OmegaL>0,OmegaM>0,t>0}]
```

we get:

```
                     OmegaM 1/3       3 H0 Sqrt[OmegaL] t 2/3
 Out[3]= {R[t] ->   (------)     Sinh[-------------------]    }
                     OmegaL                     2
```

Applying that result we have the function we seek

```
R[t_]= R[t] /. %
```

Expressed in normal units $H_0$ has the units of inverse time. Report the value of $1/H_0$ in billions of years. Expressed in normal units $\frac{3}{2}H_0\sqrt{\Omega_\Lambda}$ has units of inverse time. Report the value of $\frac{2}{3}(H_0\sqrt{\Omega_\Lambda})^{-1}$ in billions of years.

Using time units of billions of years and the above values of the cosmological parameters define `r[t]`. Plot $r(t)$ for 0 through 25 billion years. Estimate the time when $r(t) = 1$ (that is the time of NOW starting from the Big Bang). Using Mathematica (e.g., `FindRoot`) find an accurate value for the time since the Big Bang.

It is estimated that the cosmological microwave background was emitted about .00038 billion years after the Big Bang (i.e., 380,000 years). Find $r(t)$, $z$, and $T$ (see Eq. 11.3-8 but use 2.725 instead of 2.8) at that time. From the above values (particularly $\Omega_m$), find $\rho_0$ and express the result in terms of H-atoms per m$^3$ (but of course in fact its mostly dark matter).