

# The $\Lambda$ CDM Universe

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**Modern Cosmology**  
**Bremen @ Oct-26-D10**

**Goods Field**  
**HST 2009**

# Summary

The metric of the expanding Universe can be expressed in one of the 3 following ways:

$$ds^2 = -c^2 dt^2 + a^2(t) [d\chi^2 + \chi^2 d\Omega^2] \quad k=0 \text{ Infinite flat Universe}$$

$$ds^2 = -c^2 dt^2 + a^2(t) [d\chi^2 + \sin^2 \chi d\Omega^2] \quad k=1 \text{ Finite closed Universe}$$

$$ds^2 = -c^2 dt^2 + a^2(t) [d\chi^2 + \sinh^2 \chi d\Omega^2] \quad k=-1 \text{ Infinite, open Universe}$$

# Hubble Function in terms of $\Omega$ 's

- The generalized Friedmann equation governing evolution of  $R(t)$  is written in terms of the present  $\Omega$ 's (density parameter terms) as **→ Hubble Func**

$$H^2(z) = H_0^2 \left[ \Omega_R(1+z)^4 + \Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda \right].$$

- The only terms in this equation that vary with time are the scale factor  $R$  and its rate of change  $\dot{R}$
- Once the constants  $H_0$ ,  $\Omega_M$ ,  $\Omega_\Lambda$ ,  $\Omega_k$  are measured empirically (using observations), then the whole past and future of the Universe is determined by solving this equation!
- Solutions, however, are more complicated than for  $\Lambda=0$ .

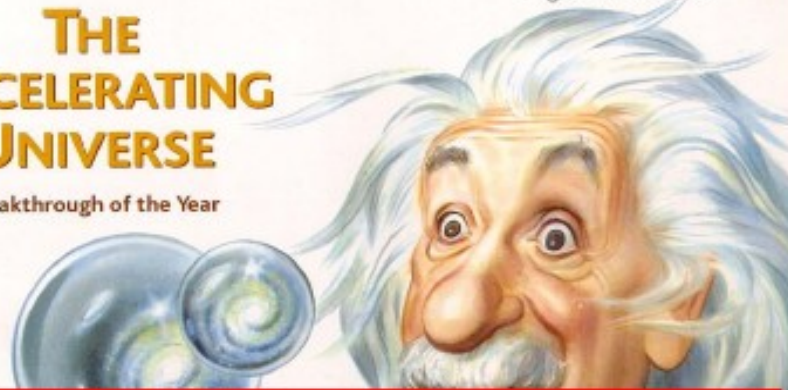
# Science

18 December 1998

Vol. 282 No. 5397  
Pages 2141-2336 \$7

## THE ACCELERATING UNIVERSE

Breakthrough of the Year



... and the  
winner is  
 $\Lambda$ CDM

 AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

# Topics

- The  $\Lambda$ CDM Universe – analytic solution for matter and Dark Energy;
- Transition from deceleration to acceleration;
- The **Fundamental Plane of Cosmology**;
- **Age of the Universe**, Age as a function of redshift, Look-back Time.
- **Luminosity distances** for particular models  
→ how to measure the Expansion with SN Ia.
- Apparent angular diameters in the expanding Universe.
- The Universe of **Dark Energy** – how to probe **Dark Energy**?
- **WMAP7 Results** → **Concordance Model**.

# The $\Lambda$ CDM Universe

$$\Omega_k = 0, \quad \Omega_M + \Omega_\Lambda = 1$$

Friedmann-Equation and its Solution  $\rightarrow$  Limiting cases

$$\dot{a}^2 = H_0^2 \left( \frac{\Omega_M}{a} + \Omega_\Lambda a^2 + \cancel{\Omega_M - \Omega_\Lambda} \right).$$

$$a(t) = \left( \frac{\Omega_M}{\Omega_\Lambda} \right)^{1/3} \sinh^{2/3} \left( 3\sqrt{\Omega_\Lambda} H_0 t / 2 \right)$$

# The $\Lambda$ CDM Exact Solution

$$a(t) = \left( \frac{\Omega_M}{\Omega_\Lambda} \right)^{1/3} \sinh^{2/3} \left( 3\sqrt{\Omega_\Lambda} H_0 t / 2 \right)$$

**Applicable for 99.99% of the age of our Universe**

**Small  $t$ ,  $t \ll t_0$ :  $\rightarrow$  Einstein-deSitter**

$$a(t) \propto t^{2/3}, \quad \ddot{a}(t) \propto -t^{-4/3}, \quad H(t) \propto t^{-1}, \quad D_H(t) \propto t$$

**Large  $t$ :  $\rightarrow$  deSitter**

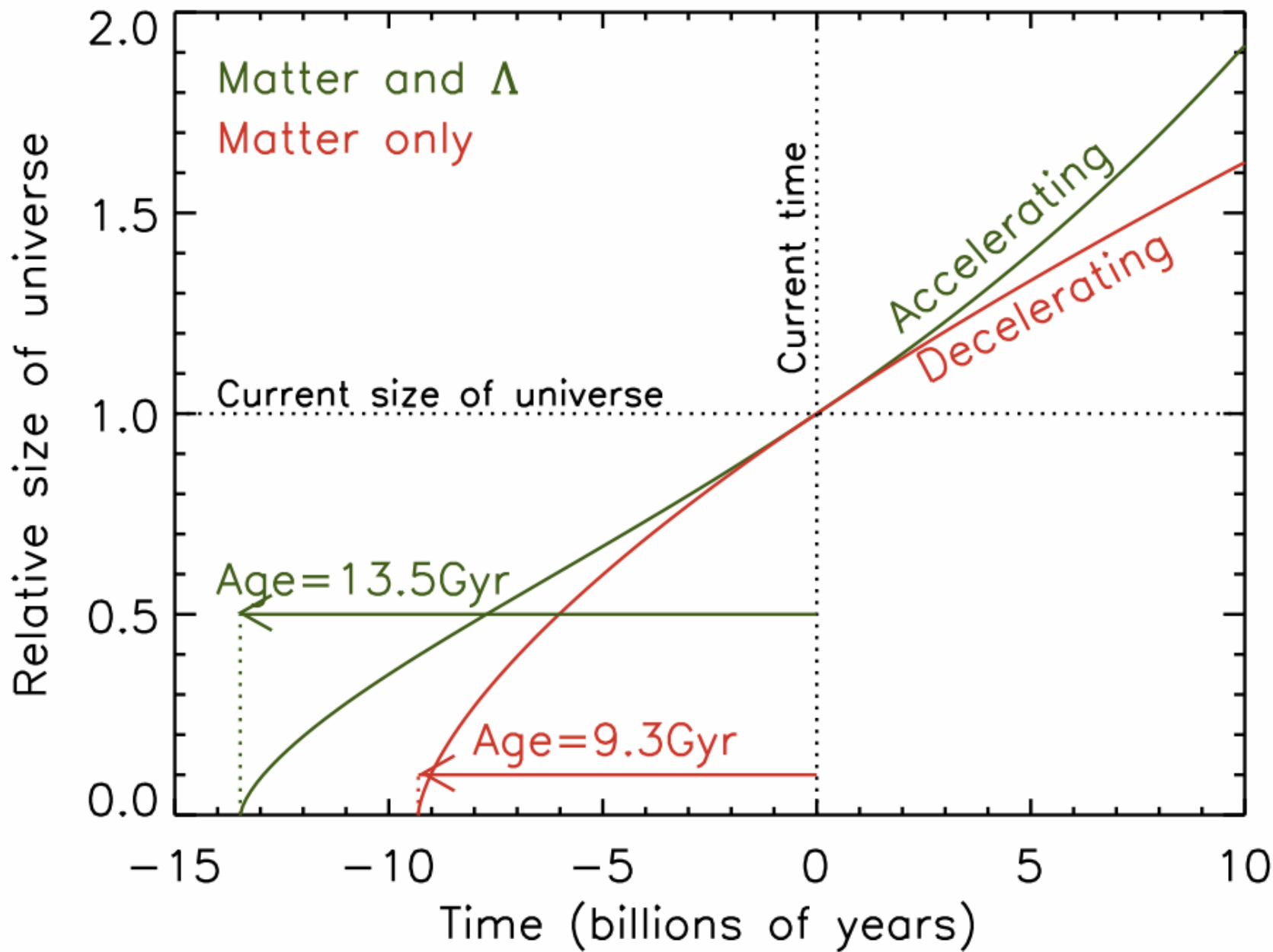
$$\ddot{a}(t) \propto a(t) \propto \exp(H_\infty t)$$

$$H_\infty = H(t = \infty) = H_0 \sqrt{\Omega_\Lambda}$$

# The $\Lambda$ CDM Turning Point

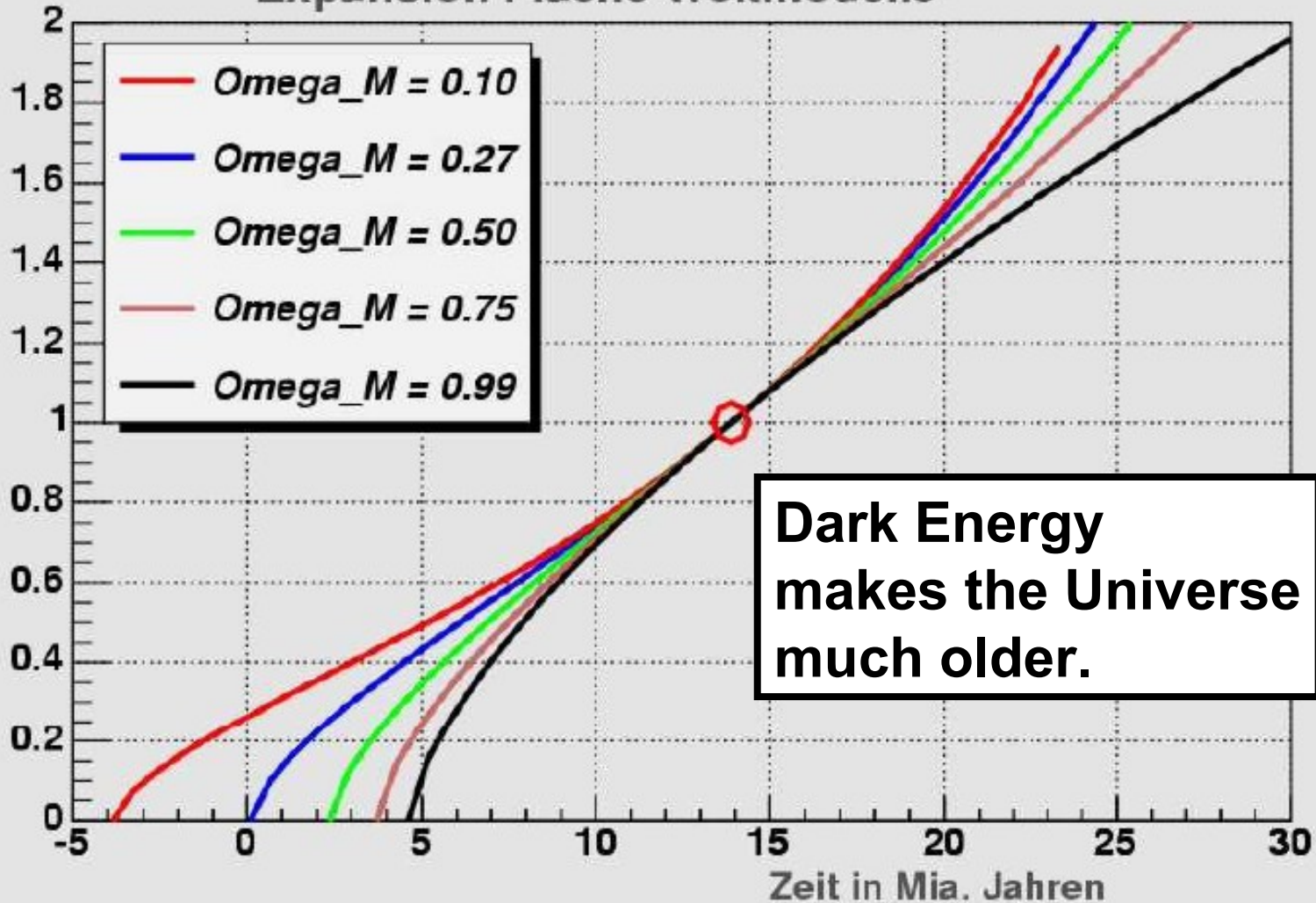
- There will a turning point **from deceleration to acceleration**, following from the 2<sup>nd</sup> Friedmann equation:  $\ddot{a}_T = 0$
- $\ddot{a} / a = - 4\pi G(\rho + 3P/c^2)/3 + c^2\Lambda/3 = 0$
- ✂  $\rightarrow - 1/2 \Omega_M/a^3_T + \Omega_\Lambda = 0$
- ✂  $\rightarrow 1 + z_T = (2\Omega_\Lambda/\Omega_M)^{1/3}$
- ✂  $\rightarrow z_T = 0.785$  for the standard model.





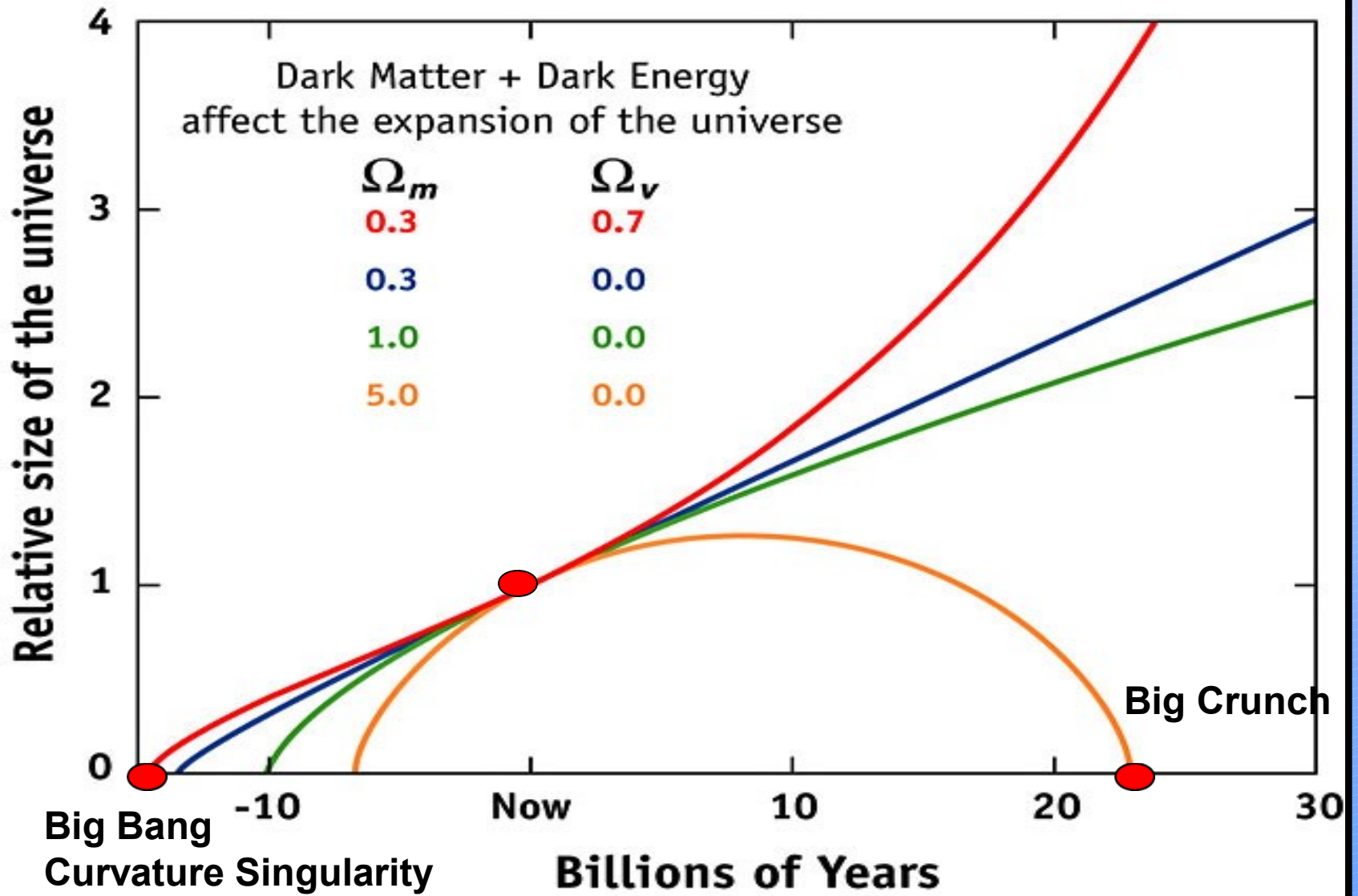
# The $\Lambda$ CDM Universe

Expansion Flache Weltmodelle



**Dark Energy  
makes the Universe  
much older.**

# EXPANSION OF THE UNIVERSE



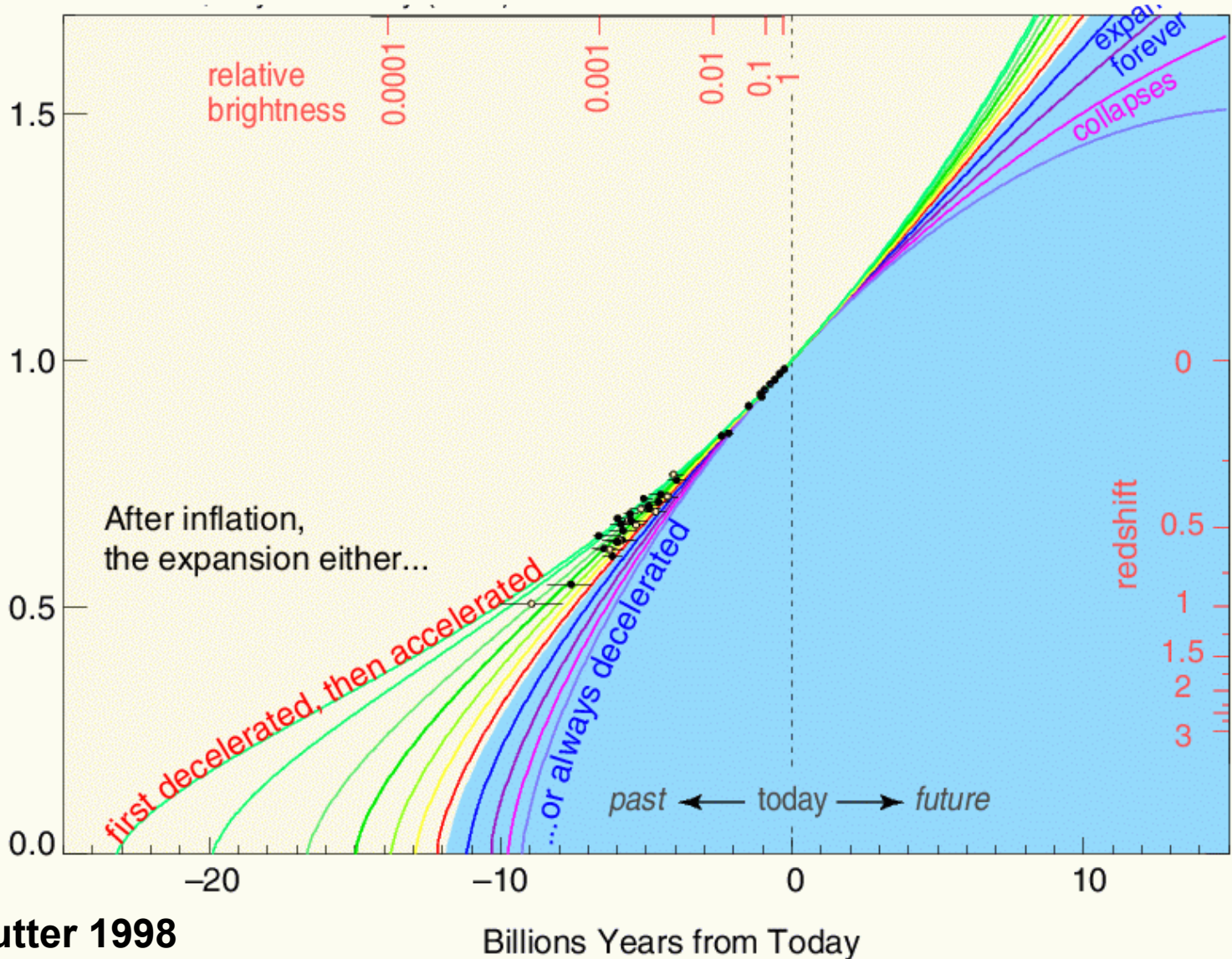
# The $\Lambda$ CDM Universe - 1998

- Supernovae-Observations (Riess et al. 1997, ...) →

The Expansion is  
accelerated!

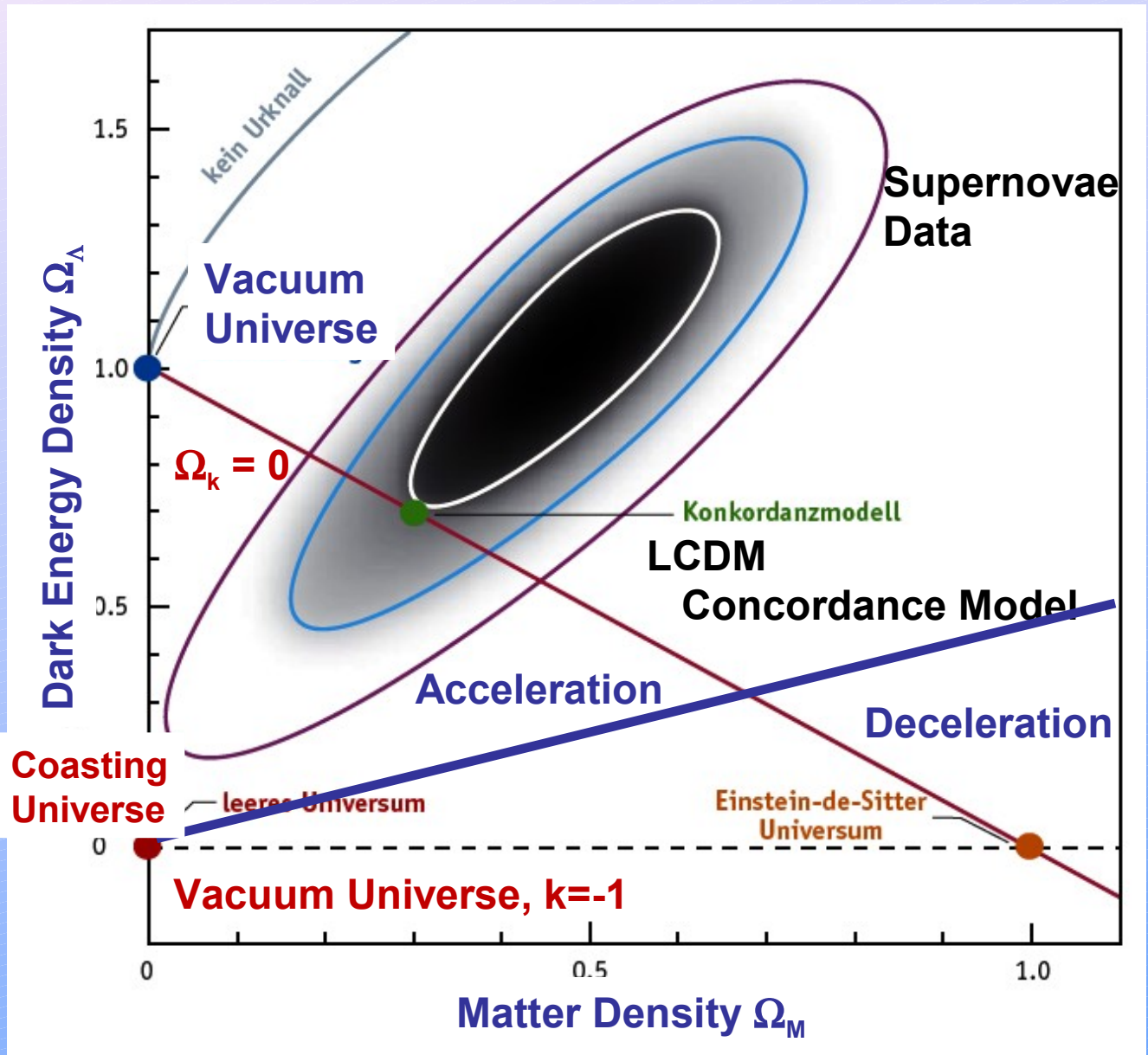
We need therefore a  
negative pressure like  $\Lambda$  !

# 1998 Discovery → Acceleration



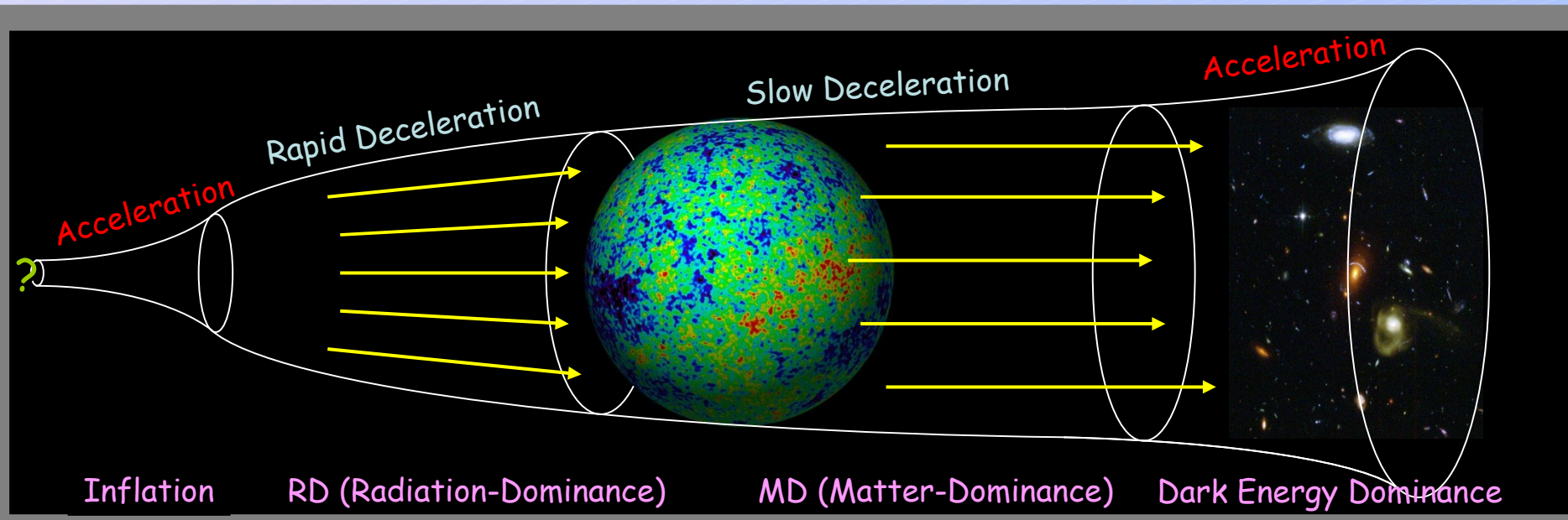
Saul Perlmutter 1998

# The Fundamental Plane of Cosmology



# Phases of Cosmic Expansion

$$\frac{\ddot{R}}{R} = -\frac{4\pi G(\rho c^2 + 3P)}{3c^2} + \frac{c^2 \Lambda}{3}$$



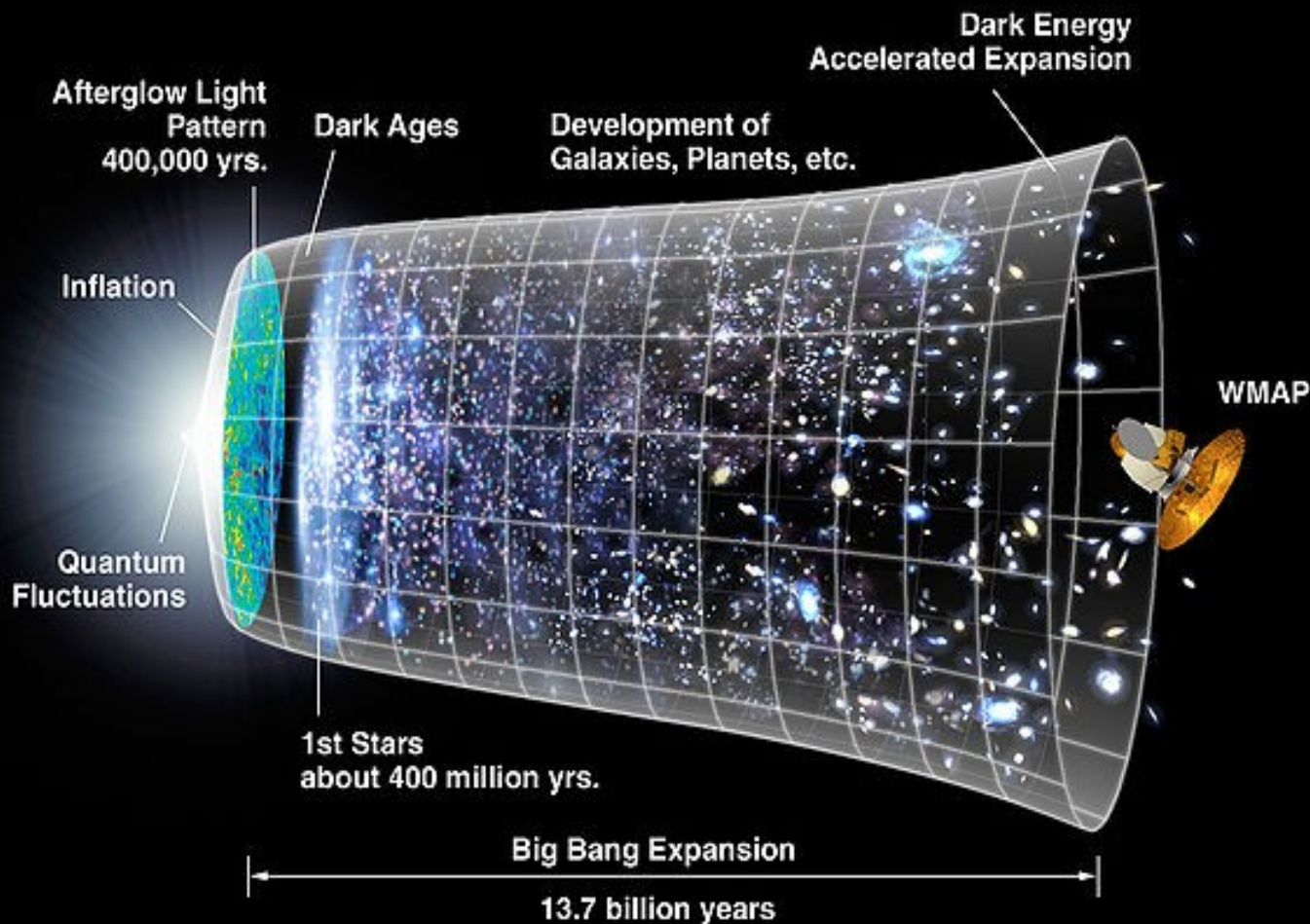
$$a(t) \sim e^{Ht}$$

$$a(t) \sim t^{1/2}$$

$$a(t) \sim t^{2/3}$$

→ Time increases logarithmically

# Phases of Cosmic Expansion





# The Age of the Universe

$$H^2 = H_0^2 \left( \Omega_m a^{-3} + \Omega_w a^{-3(1+w)} \right)$$

(without radiation,  $\Omega_k = 0$ )

- Integrate  $dt = da / (da/dt) = da / [a H(a)]$

$$t = \int_0^1 \frac{da}{aH(a)} = \int_0^1 \frac{da}{H_0 (\Omega_m a^{-3} + \Omega_w a^{-3(1+w)})^{1/2}}$$

- For  $w=-1$  (d.h.  $\Omega_{DE} = \Omega_\Lambda$ ):

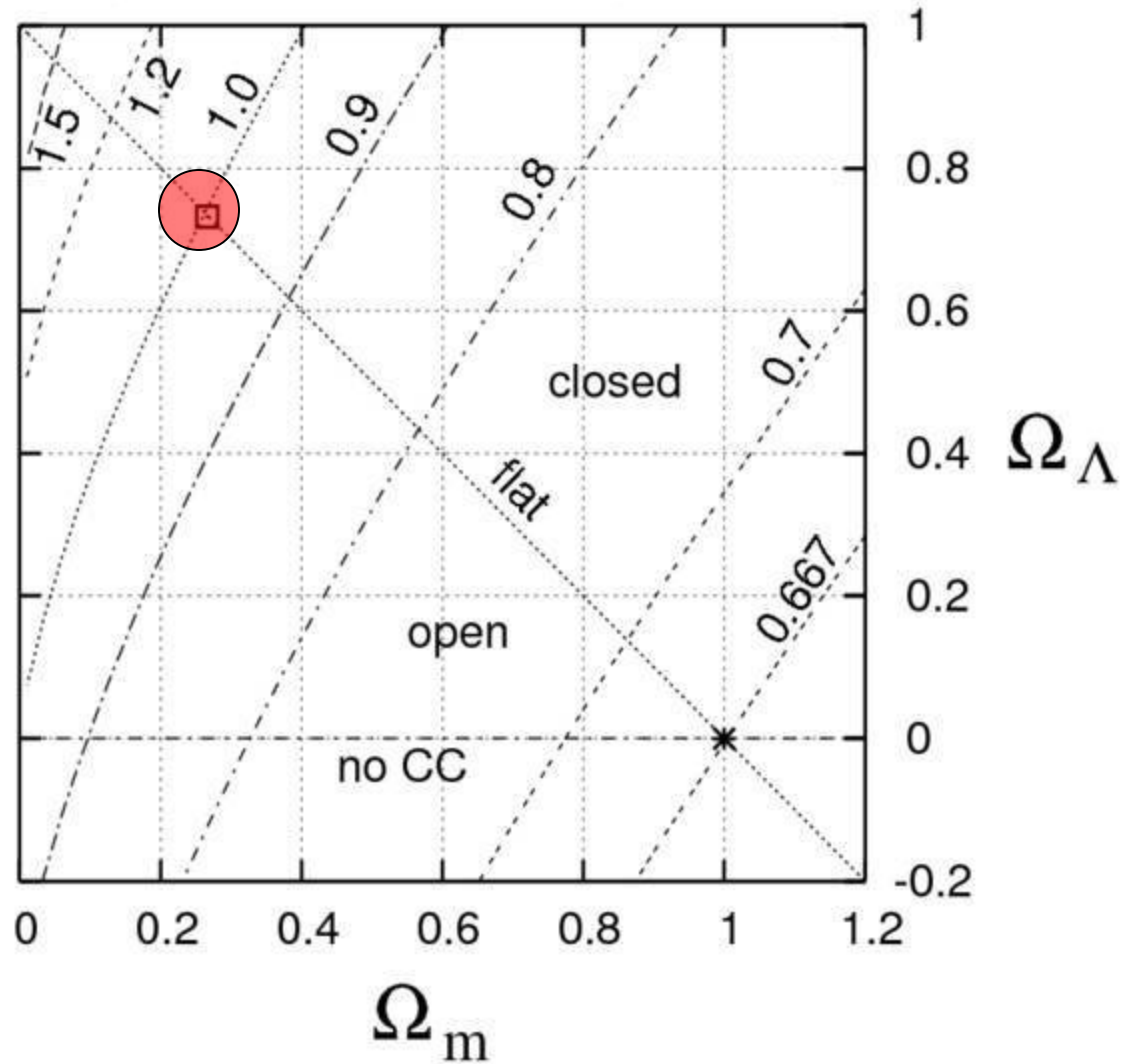
$$t_0 = \frac{2}{3H_0} \frac{\tanh^{-1}(\sqrt{\Omega_\Lambda})}{\sqrt{\Omega_\Lambda}}$$

- Hubble Time:  $1/H_0 = 13,7$  Gyears.

# Fundamental Plane - Age of the Universe -

$$t_0 = \frac{1}{H_0} F(\Omega_m, \Omega_\Lambda, \dots)$$

Age times current Hubble parameter

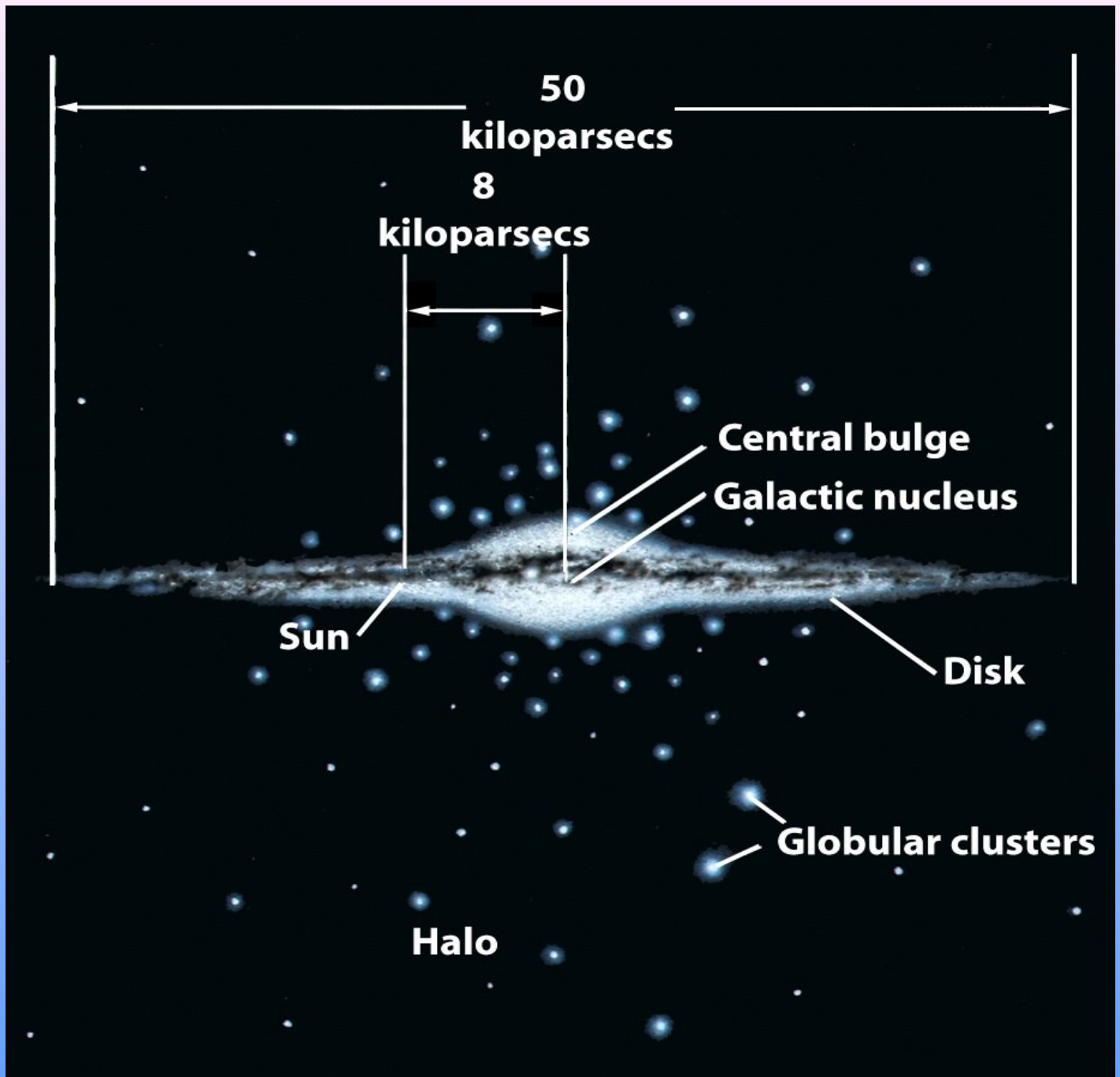


# How to measure the Age

- Method of Globular Clusters (position of knee and horizontal branch in HR Diagramm).
- Cooling of White Dwarfs in the Galactic Disk →  $T > 3000 \text{ K}$  → GAIA. Oldest WDs have cooled down to a surface temperature  $\sim 3000 \text{ K}$ .
- Age of chemical elements in the Galaxy (somewhat inaccurate method).

# Globular Clusters

~ 150 – oldest Objects

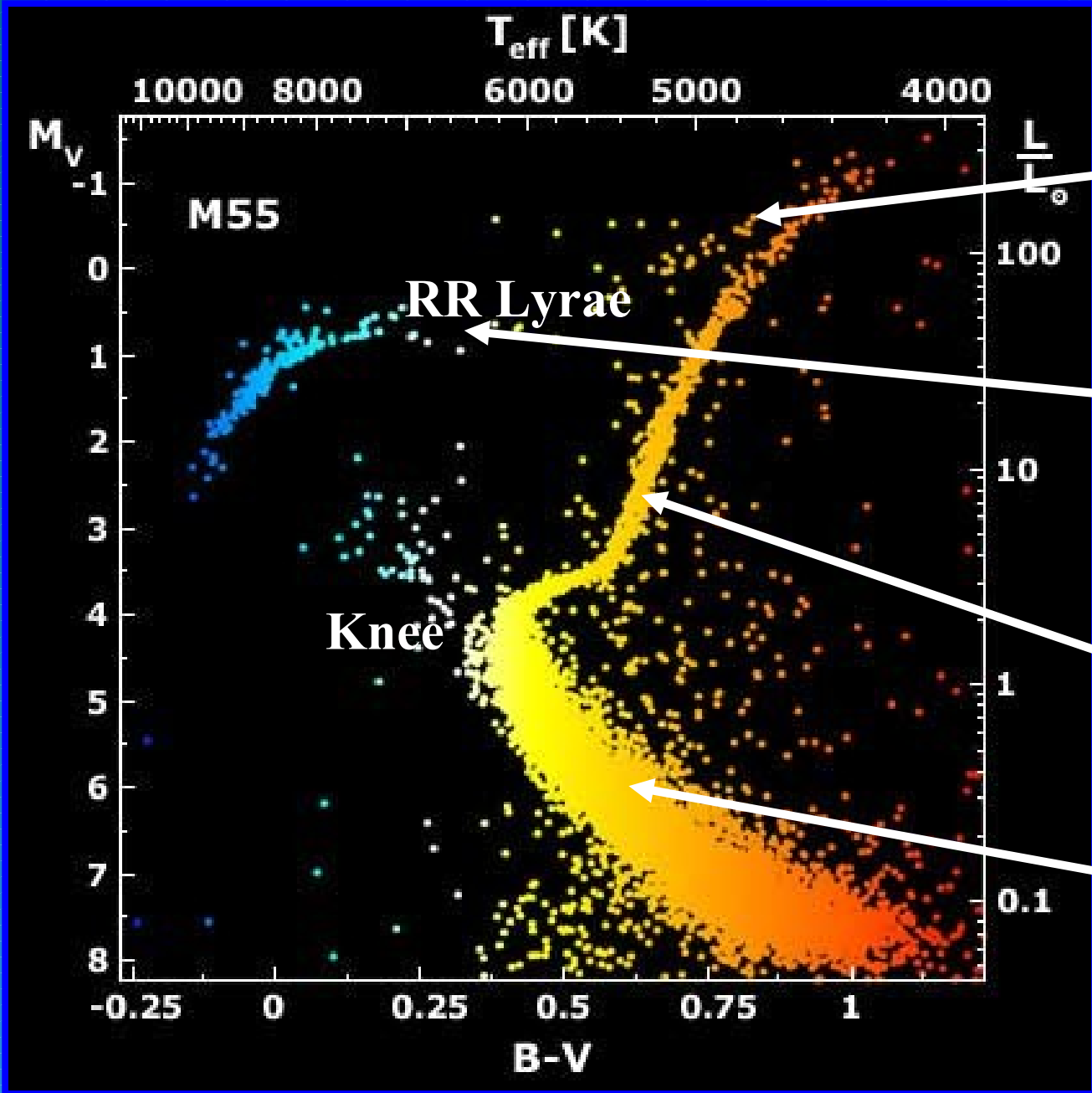


# Globular Clusters

< 1 Mio Stars



# CM-Diagram



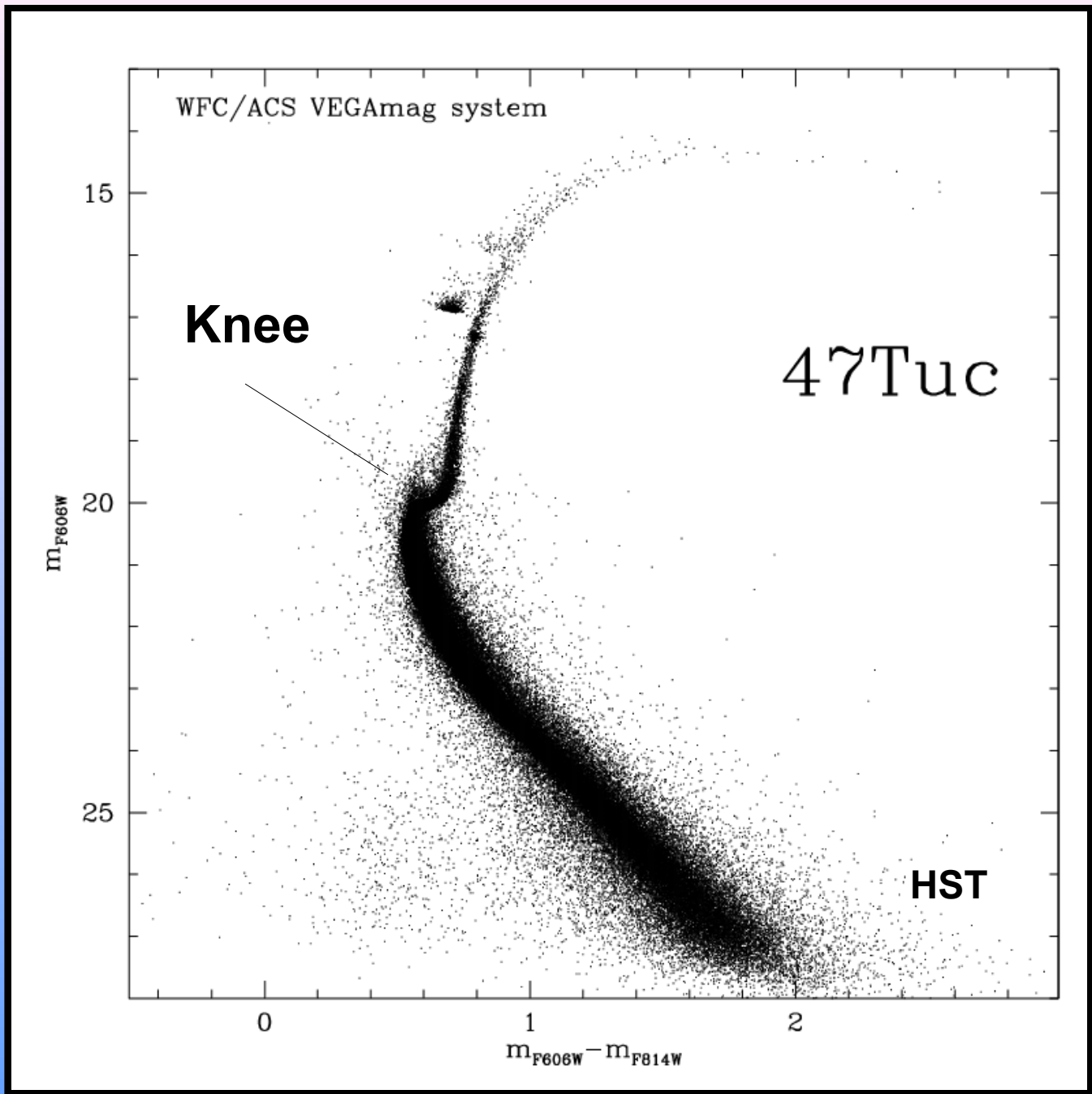
Asympt.  
Giant Branch  
AGB

Horizontal  
Branch  
(He Fusion)

Giant Branch  
(Shell  
Burning)

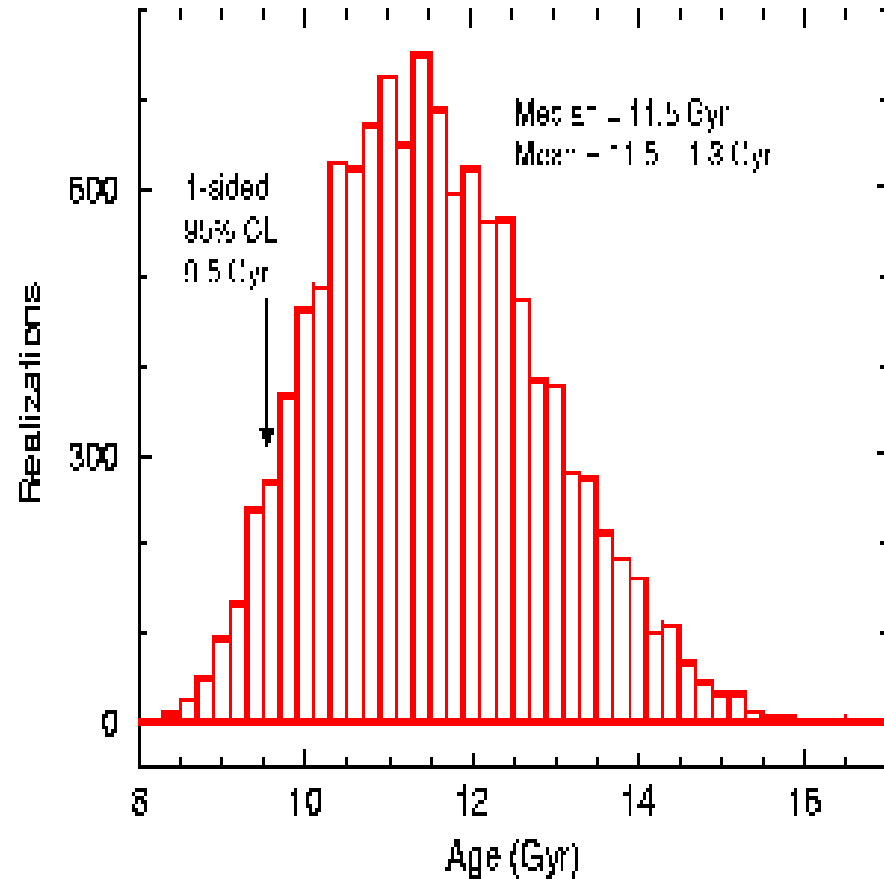
Main Sequ.  
(H Fusion)

# Globular Clusters Age Determination



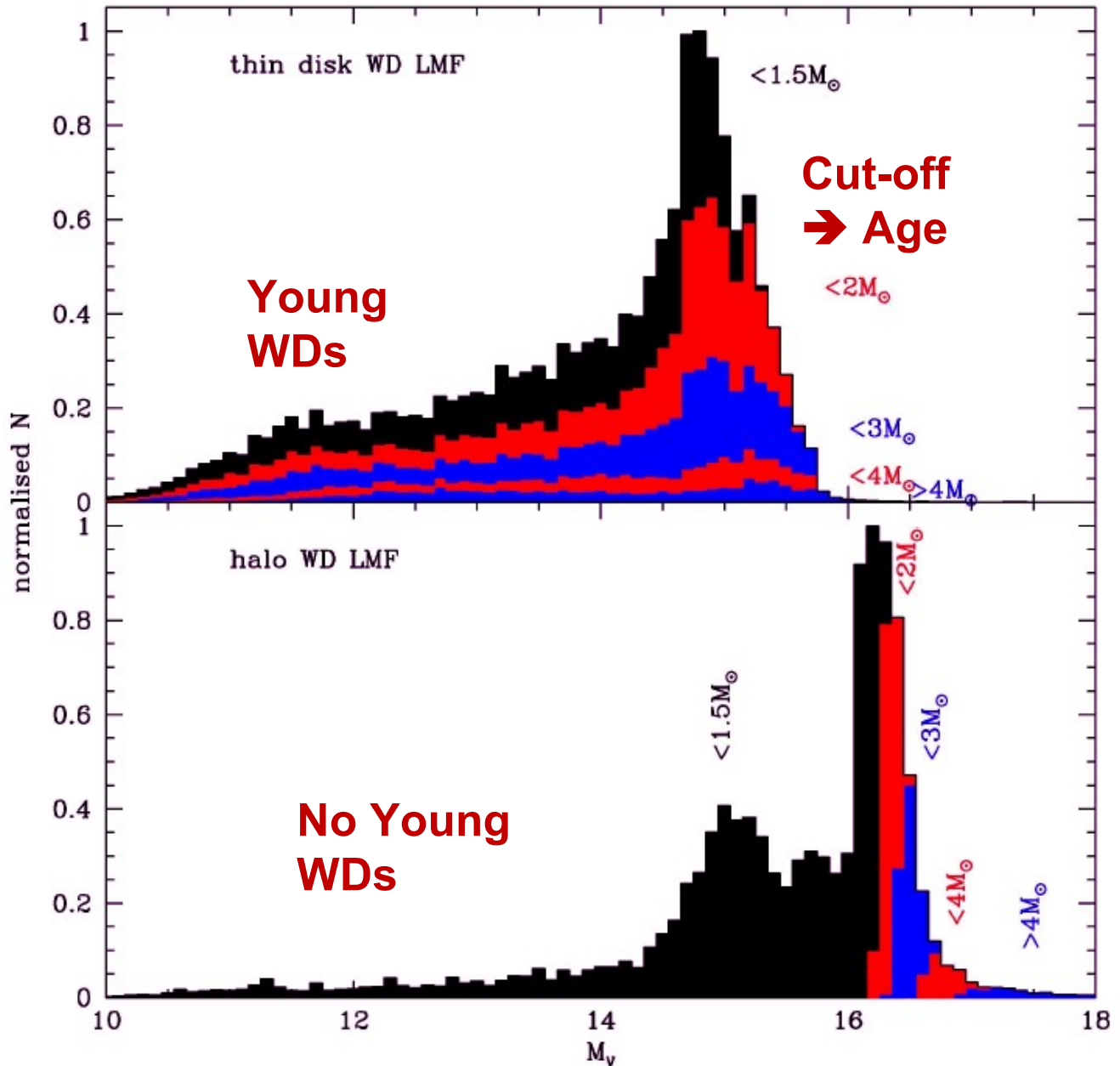
# Age Distribution of Globular Clusters

- Krauss + Chaboyer
  - stars age = 12.4 Gyr
  - estimate  $\sim 1$  Gyrs min for formation
  - $t_0 > 10.2$  Gyr 95 per cent 1-tailed
- CMB + Flatness  $\rightarrow$   
 $t_0 \sim 13.7$  Gyr

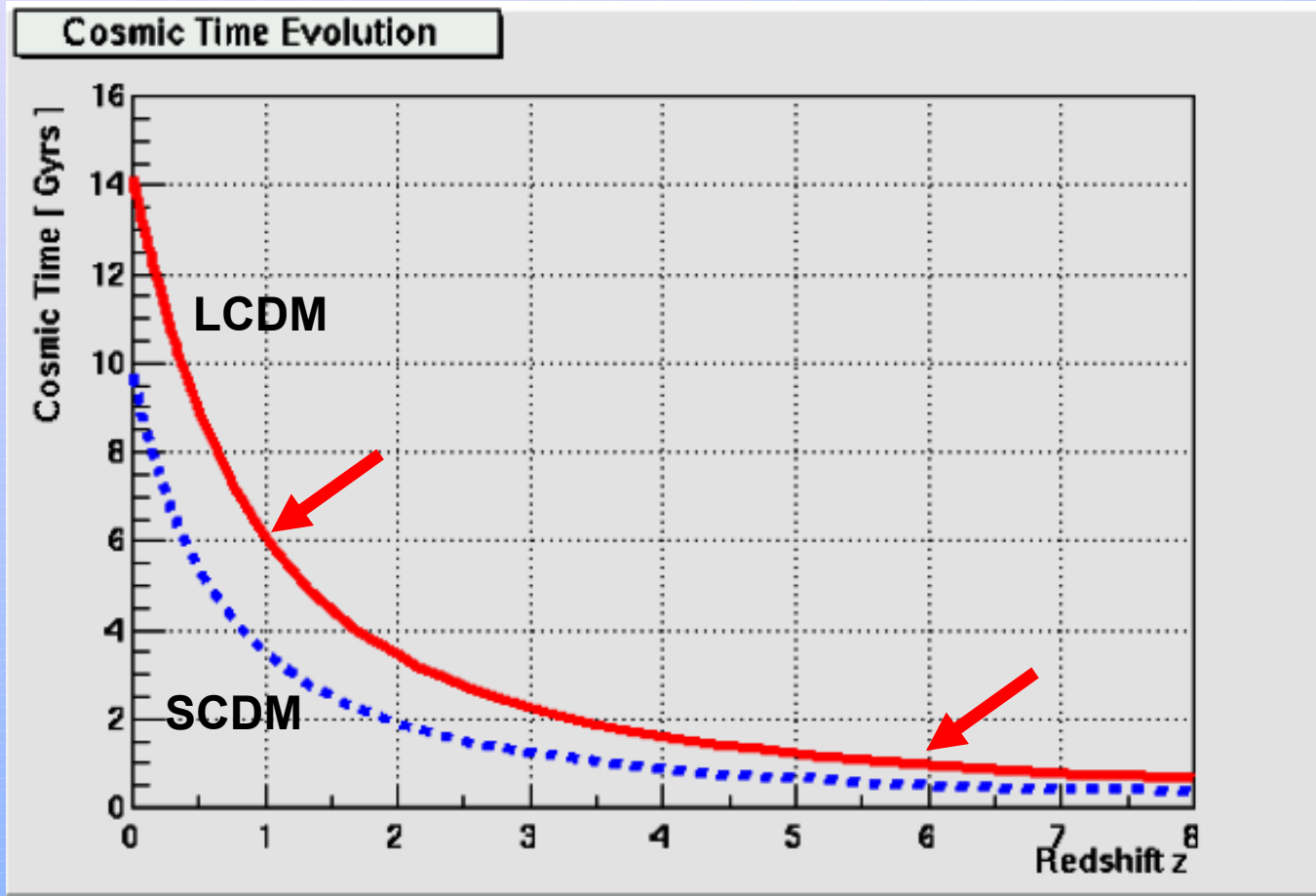




# White Dwarf Luminosity Function



# Age as Function Redshift



# How to Observe in the Friedmann Universe

**Consider the propagation of photons  
under the Expansion of the Universe.**

- SNe Ia Hubble-Diagrams, based on  
the Luminosity distance**
- Angular extensions of Galaxies**
- [ Number counts of galaxy clusters ]**
- [Weak Lensing by Dark Matter]**

# The Comoving Distance

$$\text{✂} \rightarrow dx = c dt = a dr \quad (k=0)$$

$$D_c = a_0 dr = a_0 \int \frac{c dt}{a} = a_0 \int \frac{da}{a \dot{a}} = a_0 \int \frac{da}{a^2 H(a)}$$

$$D_C(z) = \int_0^z \frac{dz'}{H(z')} = \int_0^z \frac{dz'}{H_0 (\Omega_m (1+z')^3 + \Omega_w (1+z')^{3(1+w)})^{1/2}}$$

$$\frac{a}{a_0} = \frac{1}{1+z} \quad \frac{da}{a^2} = \frac{dz}{a_0} \quad \Omega_T = 1$$

$$\frac{H^2(z)}{H_0^2} = (\Omega_m (1+z)^3 + \Omega_w (1+z)^{3(1+w)}) \quad \text{Hubble-Function}$$

# Luminosity Distances

**DeSitter Model:**

$$d_L(z) = \frac{c}{H} z(1+z).$$

**Mattig Formula ( $\Omega_\Lambda = 0$ , 1968):**

$$d_L = r_1 R_0(1+z) = \frac{c}{H_0} \frac{1}{q_0^2} \left( q_0 z + (q_0 - 1) [\sqrt{1 + 2q_0 z} - 1] \right).$$

**FLRW → no closed form:**

$$\begin{aligned} S(x) &= x, & k &= 0 \\ S(x) &= \sin(x), & k &= +1 \\ S(x) &= \sinh(x), & k &= -1 \end{aligned}$$

$$D_L = \frac{(1+z)c}{H_0 \sqrt{|\Omega_K|}} S \left\{ \sqrt{|\Omega_K|} \int_0^z \left[ \Omega_K (1+z')^2 + \Omega_M (1+z')^3 + \Omega_\Lambda \right]^{-1/2} dz' \right\}$$

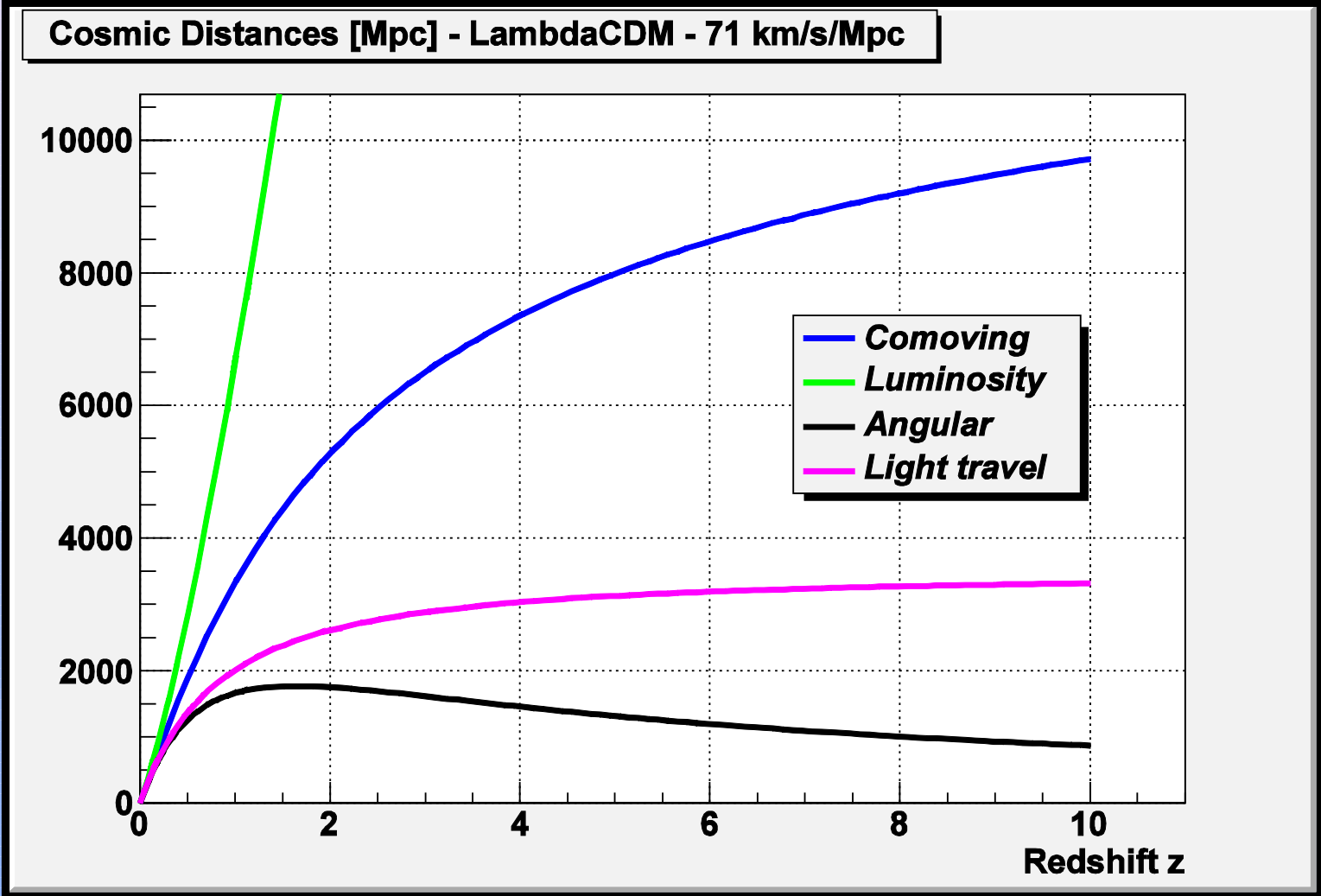
# $\Lambda$ CDM: Ue-Li Pen Approximation for Luminosity Distance

$$d_L(z) = \frac{c}{H_0} (1+z) \left[ \eta(0, \Omega_M) - \eta(z, \Omega_M) \right]$$
$$\eta(z, \Omega_M) = 2\sqrt{s^3 + 1} \left[ (1+z)^4 - 0,1540s(1+z)^3 + 0,4304s^2(1+z)^2 \right. \\ \left. + 0,19097s^3(1+z) + 0,066941s^4 \right]^{-1/8} .$$

$$s^3 = (1 - \Omega_m) / \Omega_m$$

$$d_A = \frac{d_L}{(1+z)^2} = \frac{D_C(z)}{(1+z)}$$

# Distances in LCDM



see Web Portal iCosmos

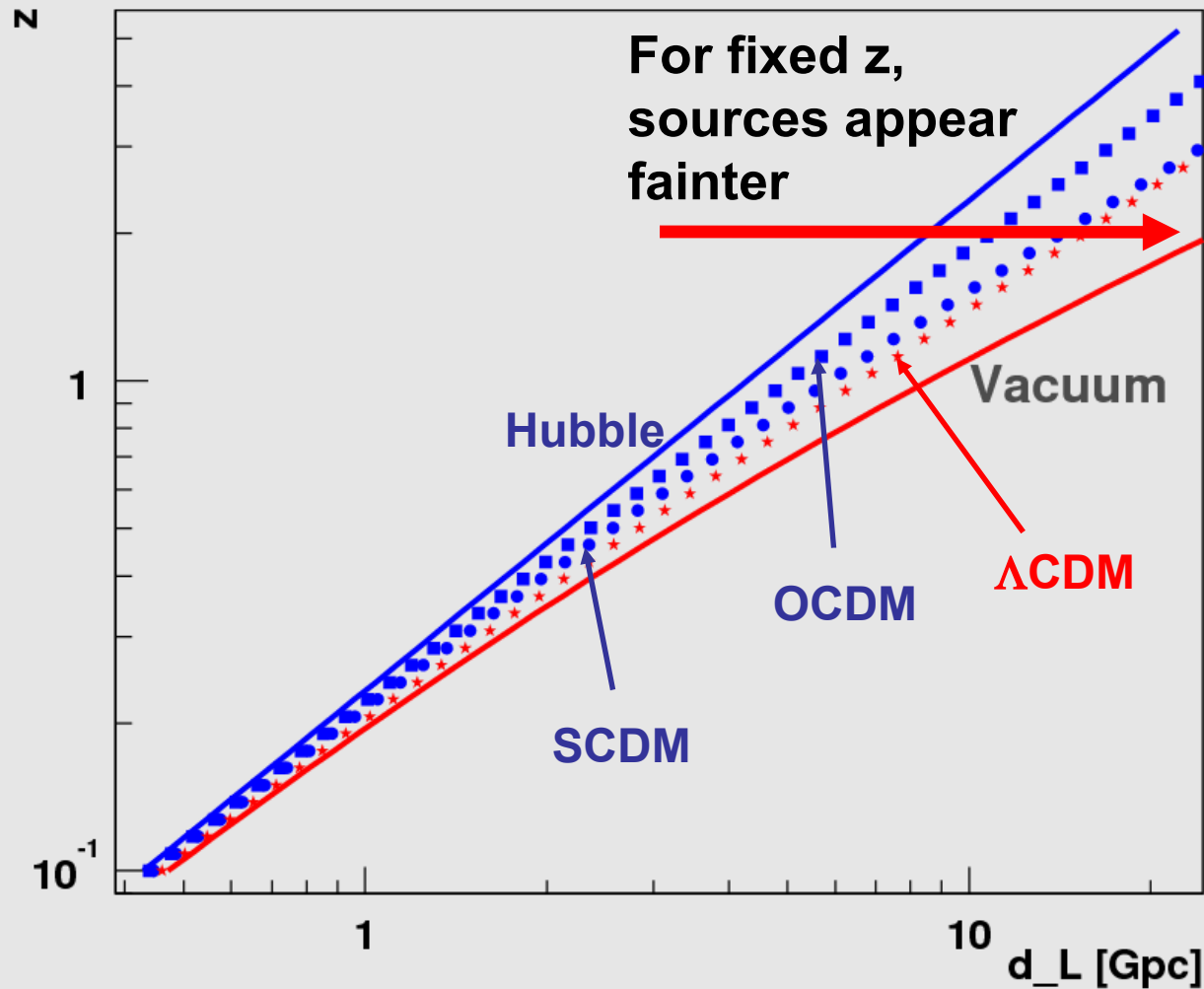
# Distances in LCDM

Quantity	$z = 1$	$z = 6$	$z = 10$
age at $z$	5.94 Gyr	0.952 Gyr	0.484 Gyr
lookback time	7.73 Gyr	12.7185 Gyr	13.18 Gyr
Luminosity dist	6635 Mpc	58,972 Mpc	106,308 Mpc
Angular dist	1659 Mpc	1203 Mpc	878.6 Mpc
Comoving dist	3317 Mpc	8425 Mpc	9664 Mpc
Comoving vol to $z$	153 Gpc <sup>3</sup>	2505 Gpc <sup>3</sup>	3781 Gpc <sup>3</sup>
1 arcsec scale	8.04 kpc	5.83 kpc	4.26 kpc
1 kpc at $z$	0.124 arcsec	0.17 arcsec	0.23 arcsec

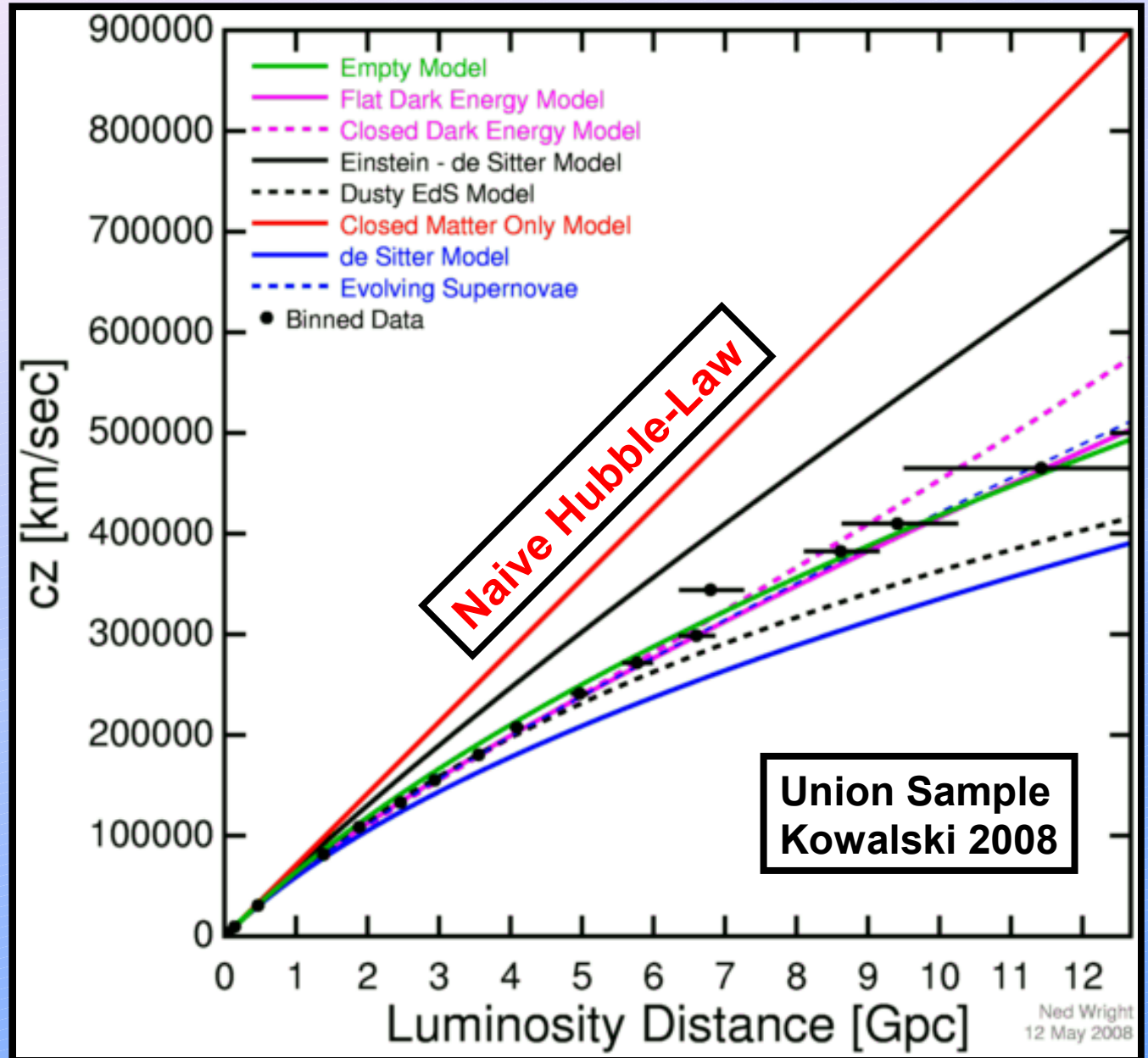


# Distance Comparison

Luminosity Distance Inflationary Universe



# Observed Supernovae Determine World Models



**$m - M \propto \log d_L$  (Luminosity-Dist.)**

$$f = \frac{L}{4\pi d_L^2}$$

$$d_L = \frac{c}{H_0} z$$

for small  $z$

$$\Omega_M = \frac{\rho}{\rho_{crit}}$$
$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$
$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

$\Omega_\Lambda > 0$  ?

0

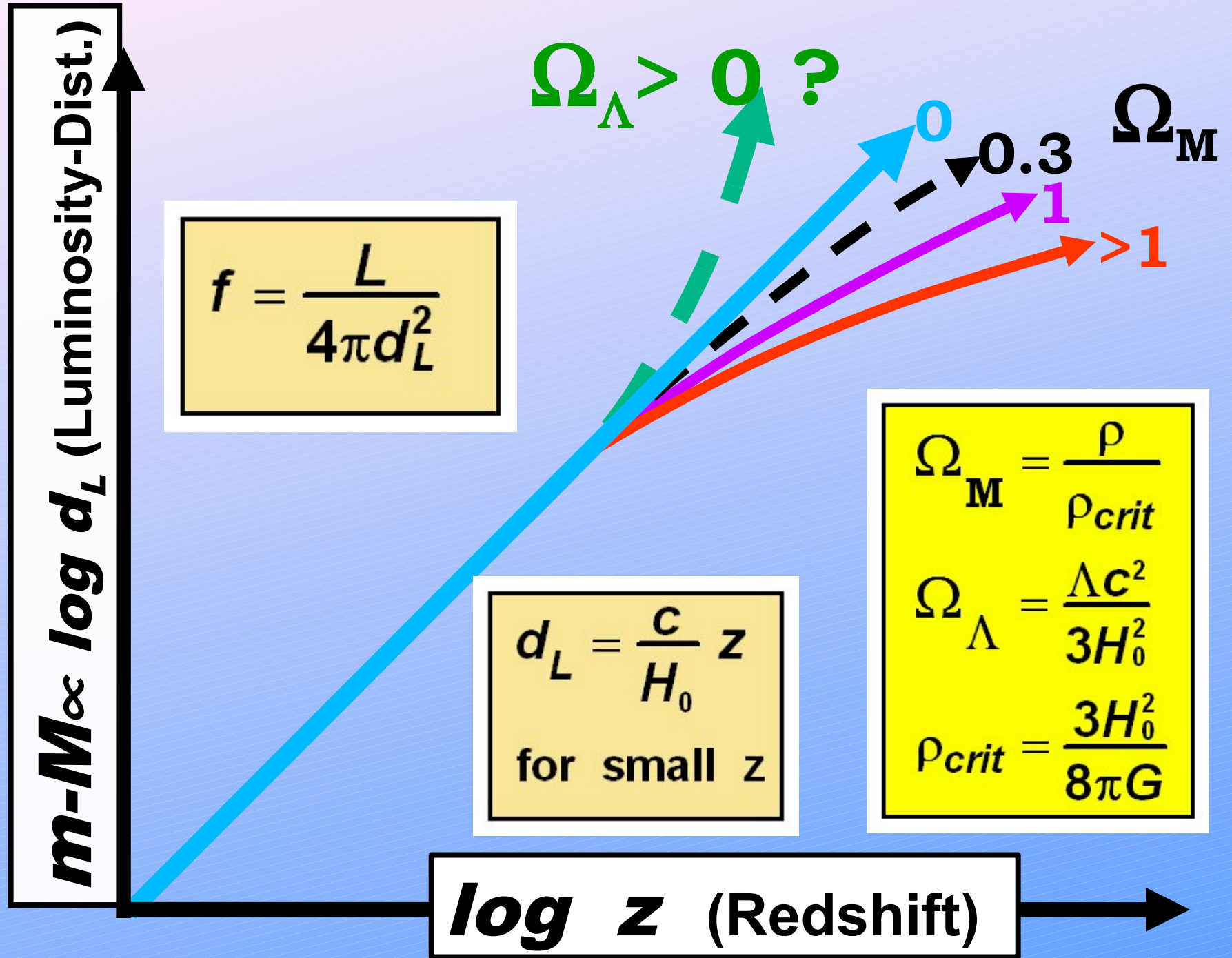
0.3

$\Omega_M$

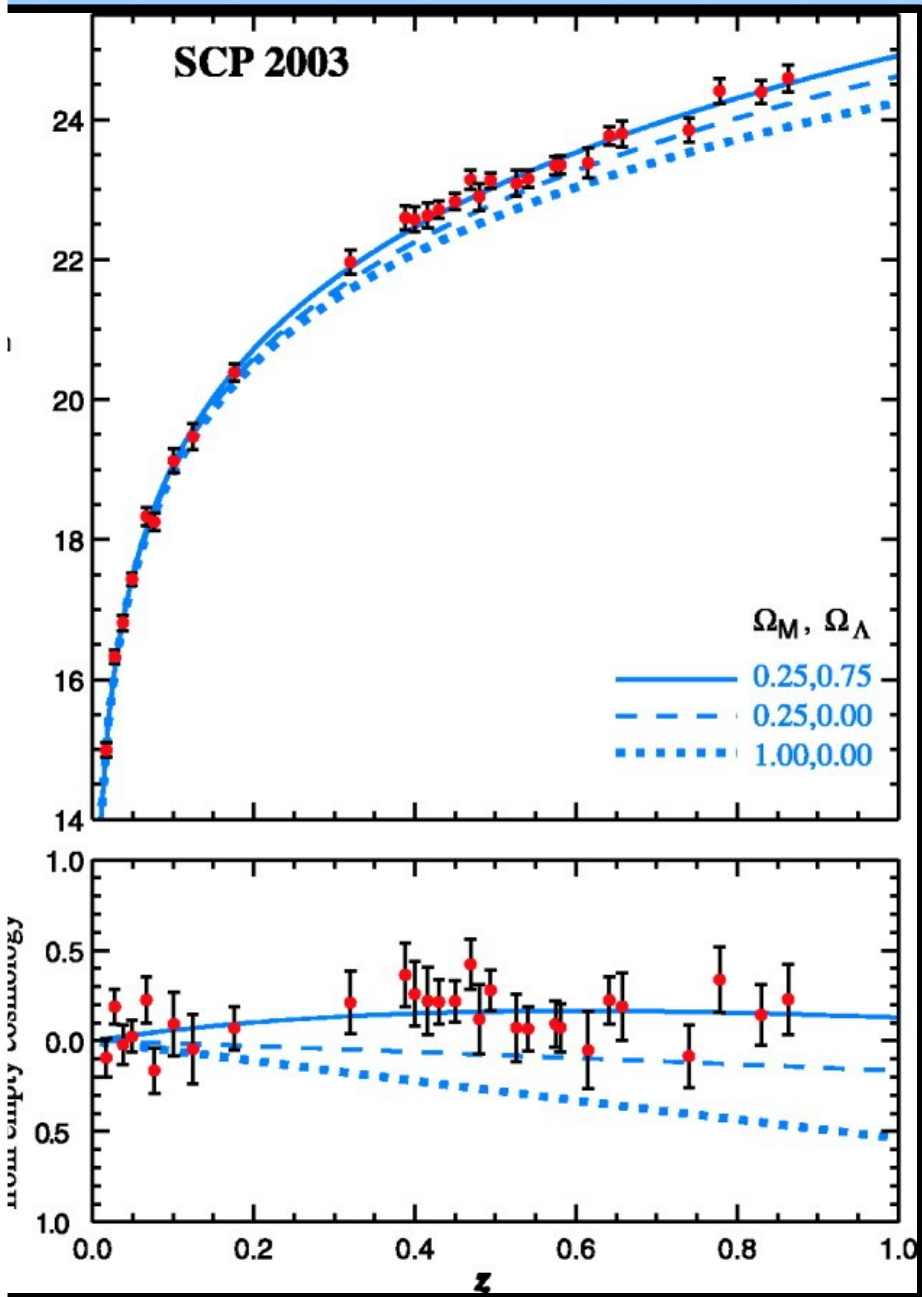
1

> 1

**$\log z$  (Redshift)**



How faint each object appears (when viewed from Earth)



# Type Ia Supernovae (Knop et al. 2003)



Distance  
 $\approx$



$\approx$  Redshift

Redshift (amount universe "stretched" since the object's light was emitted)

# Apparent Angular Diameter

– Defined as

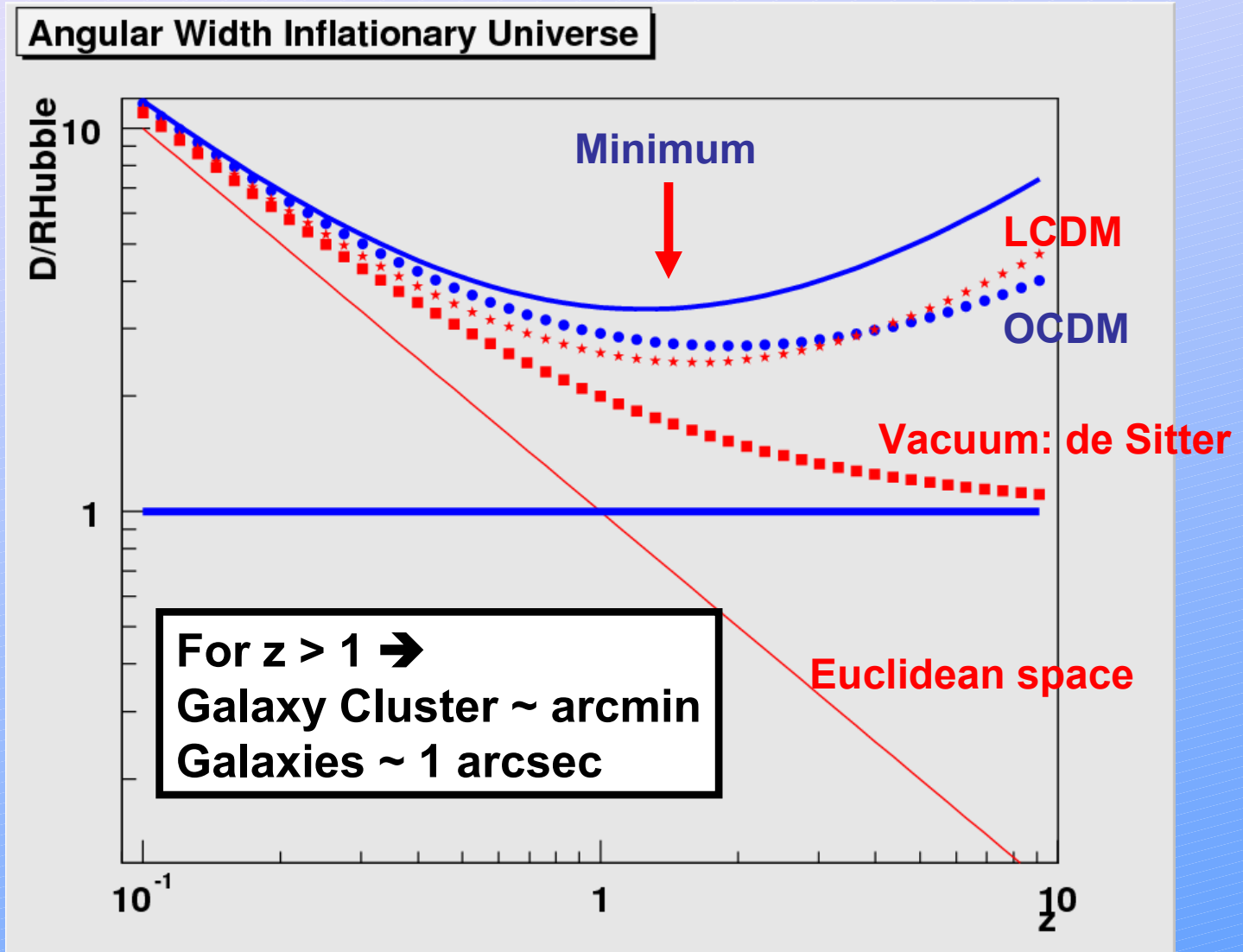
$$\theta = D / d_A = D (1+z)^2 / d_L$$

- $D$  = physical Extension of the Object (Galaxy)
- $\theta$  = measured apparent angular diameter on Sky

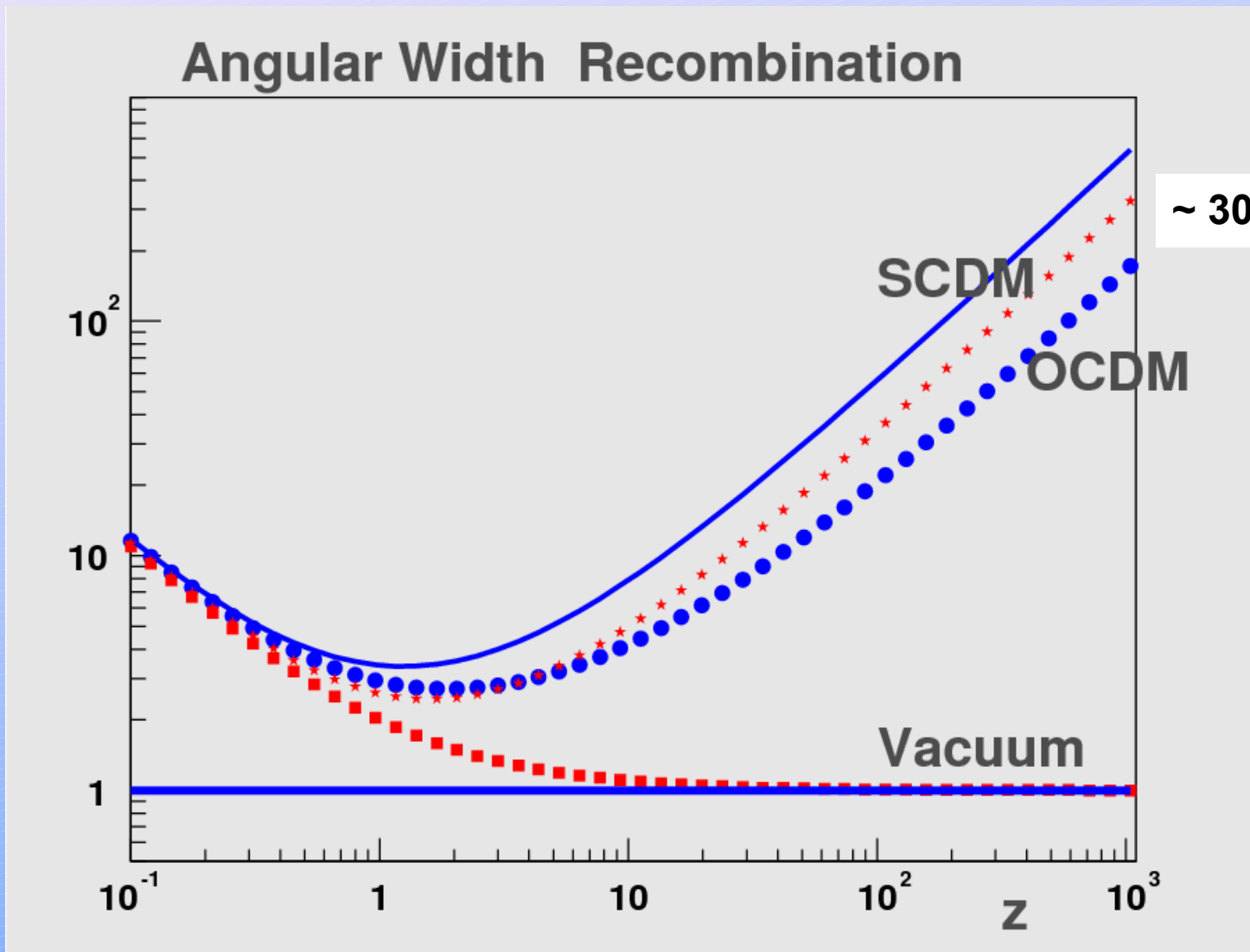
$$d_A = \frac{d_L}{(1+z)^2} = \frac{D_C(z)}{(1+z)}$$

Since  $d_L(z)$  increases in general less steeply than  $\sim z^2$   
→ Angular Diameter reaches a Minimum @  $z \sim 1,4$  !!!!

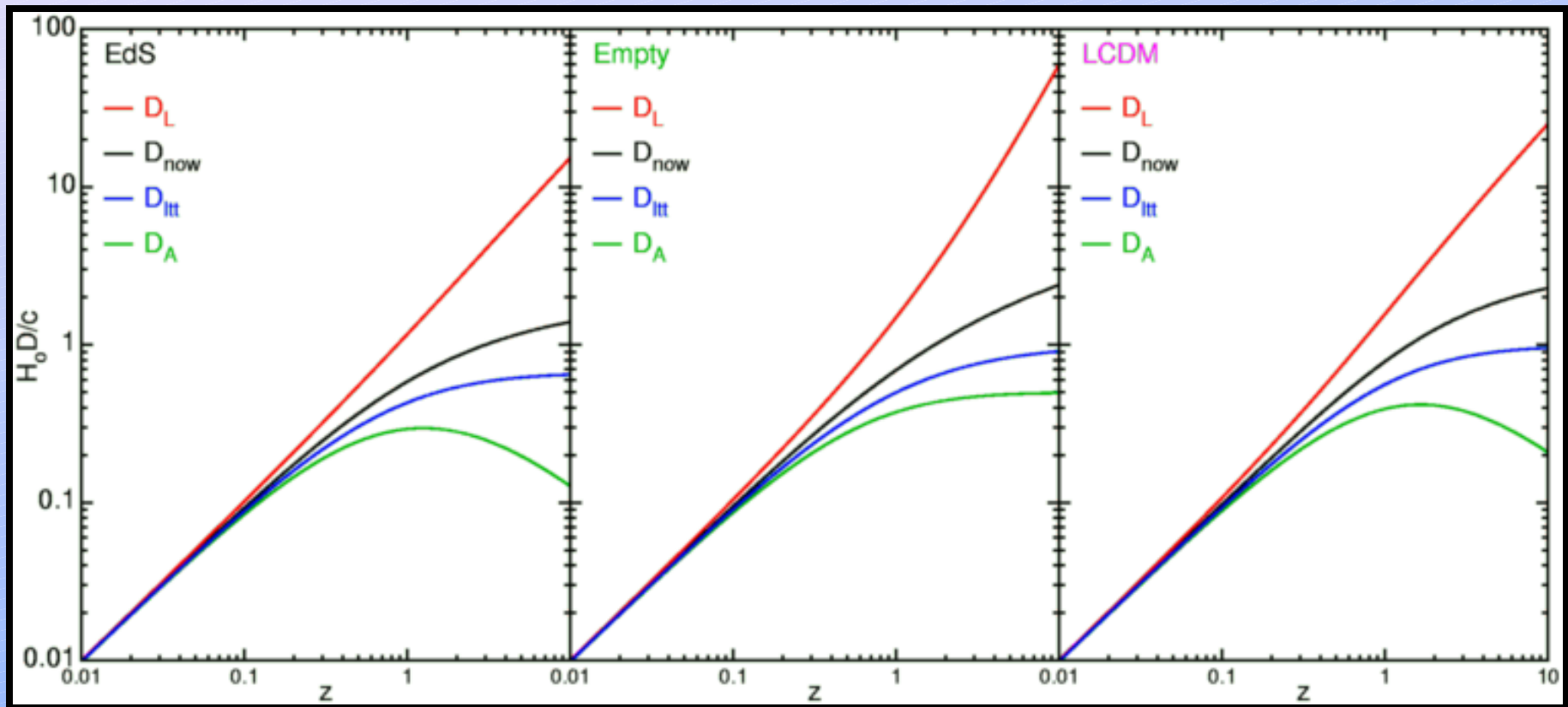
# Angular Diameter in Models



# Angular Diameter @ Recombination



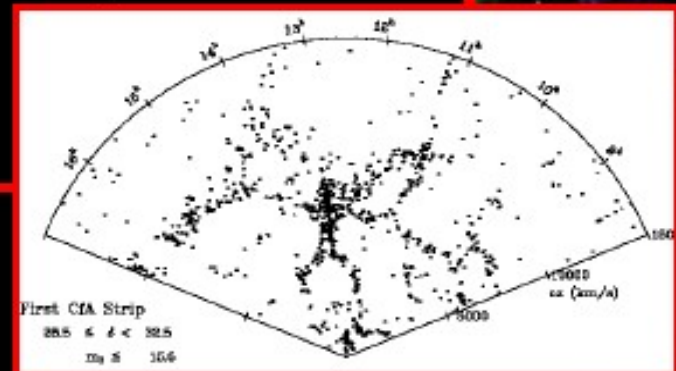
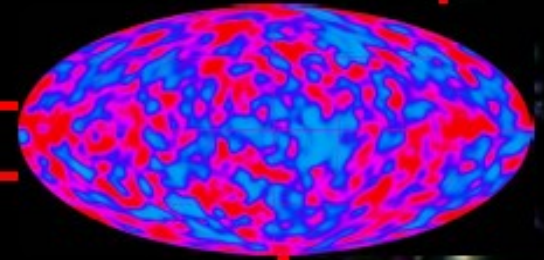
# Summary on Distances in Expanding Universe





# 1990s: Beginning of Data-driven Cosmology

- COBE! and CMB experiments
- Redshift surveys (CfA, IRAS, 2dF, SDSS)
- Large-scale velocity field measurements
- Gravitational lensing
- Big telescopes (Keck, ...) with big CCD cameras
- X-ray, gamma-ray, IR, ...



E.L. Wright: Constraints ...  
2007: arXiv:0701.584

# Physical Observables Probing Dark Energy

1. Luminosity distance vs. redshift:  $d_L(z)$   $m(z)$

Standard candles: SNe Ia

[2. Angular diameter distance vs.  $z$ :  $d_A(z)$

Alcock-Paczynski test: Ly-alpha forest; redshift correlations]

3. Number counts vs. redshift:  $N(M,z)$

probes:

\*Comoving Volume element  $dV/dz d\Omega$

\*Growth rate of density perturbations  $\delta(z)$

Counts of galaxy halos and of clusters; QSO lensing

# Use Luminosity Distance

**Basic assumption:  
homogeneous and isotropic Universe**



**Null geodesics in Friedmann-Robertson-Walker**

**metric:**

$$D_L = \frac{(1+z)c}{H_0 \sqrt{|\Omega_k|}} S \left\{ \sqrt{|\Omega_k|} \int_0^z \left[ \Omega_k (1+z')^2 + \Omega_M (1+z')^3 + \Omega_\Lambda \right]^{-1/2} dz' \right\}$$

$$\Omega_M = \frac{8\pi G}{3H_0^2} \rho_M \quad \Omega_k = -\frac{kc^2}{R^2 H_0^2} \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

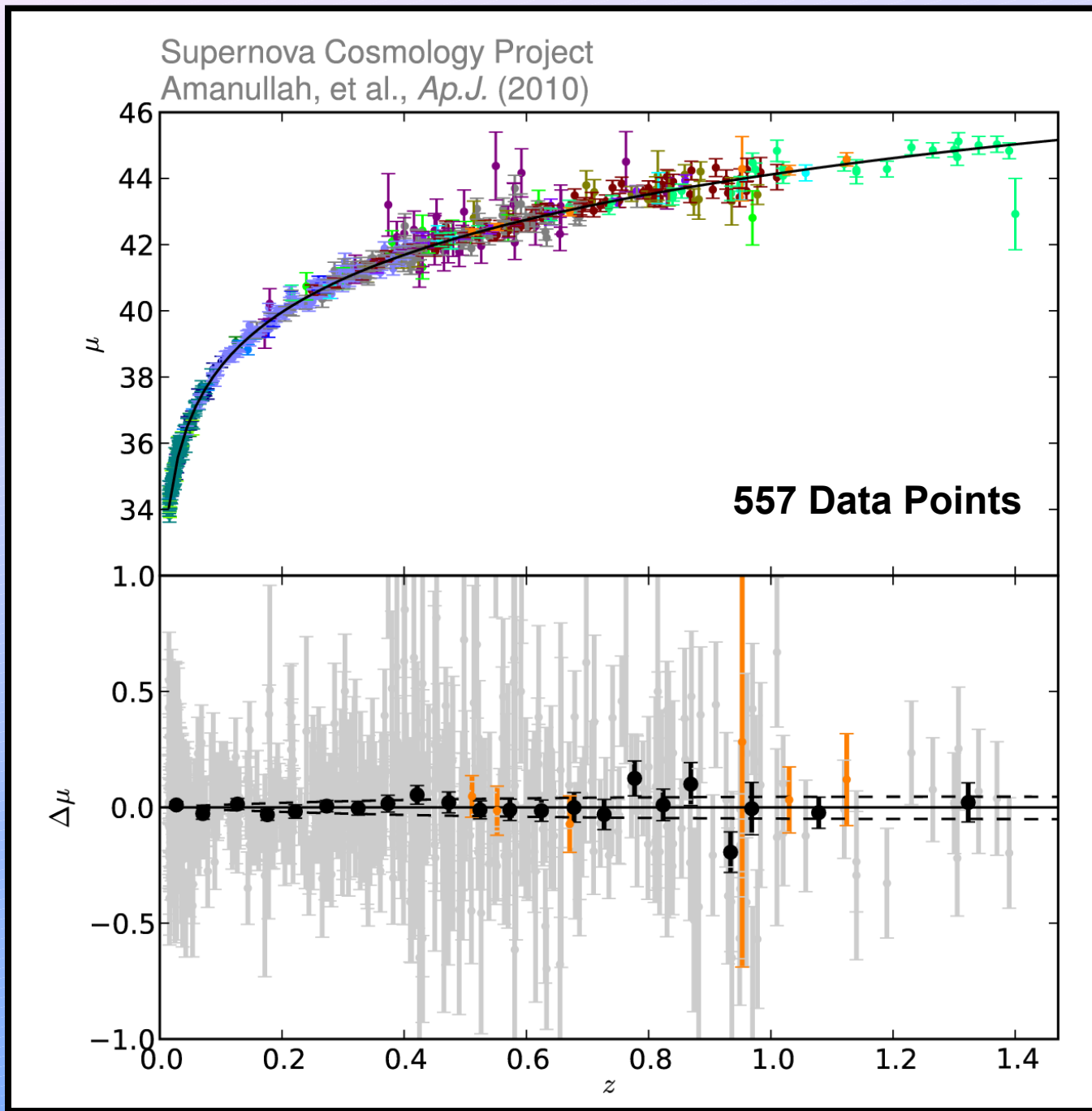
$\Omega_M$ : Matter Density

$\Omega_k$ : Curvature

$\Omega_\Lambda$ : Cosmological  
Constant

# Hubble Diagram SNe Ia

$\mu = 5 \log(D_L/\text{Mpc}) + 25$

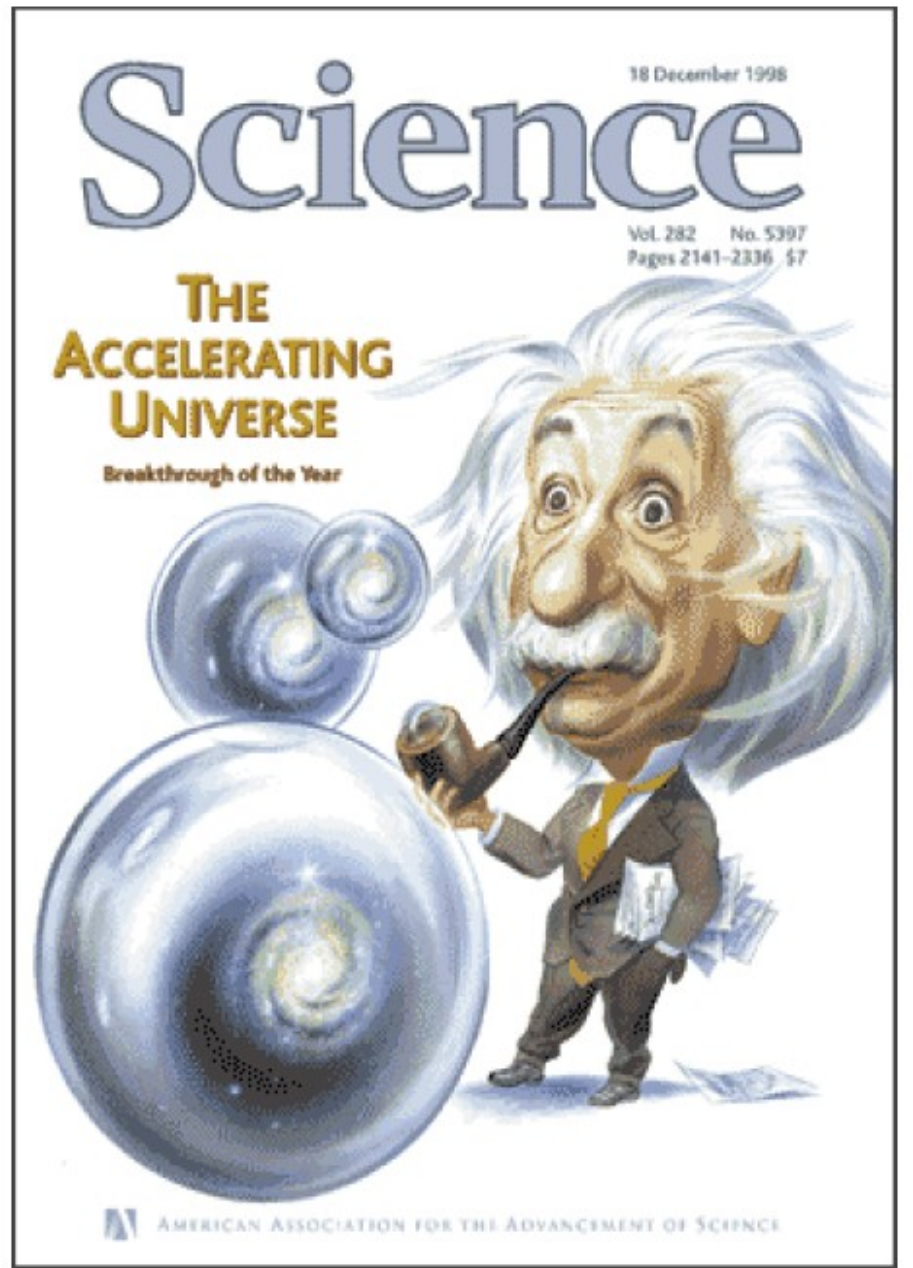


# Accelerating Universe: 1998

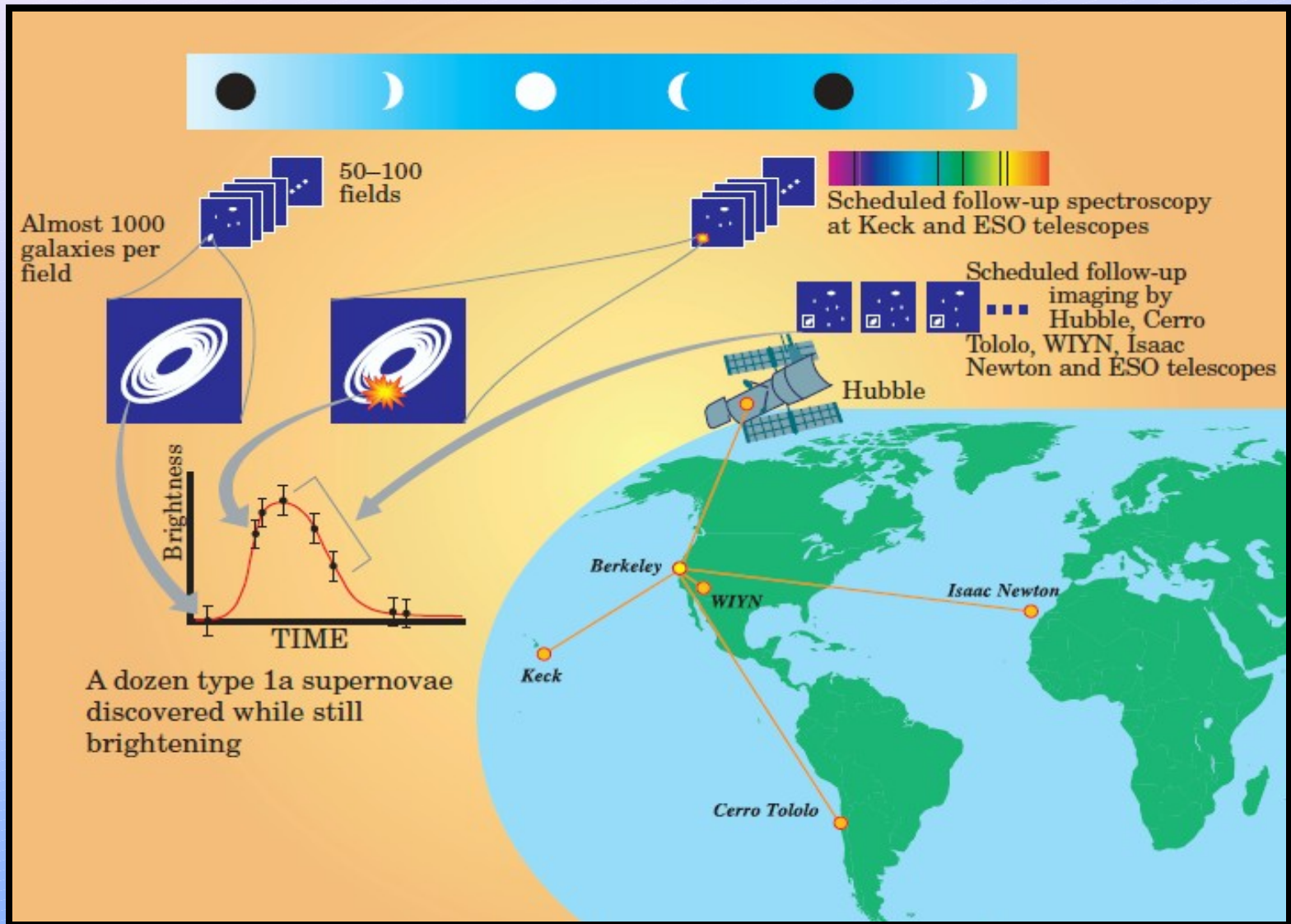
Distant (high  $z$ ) supernovae fainter than expected.

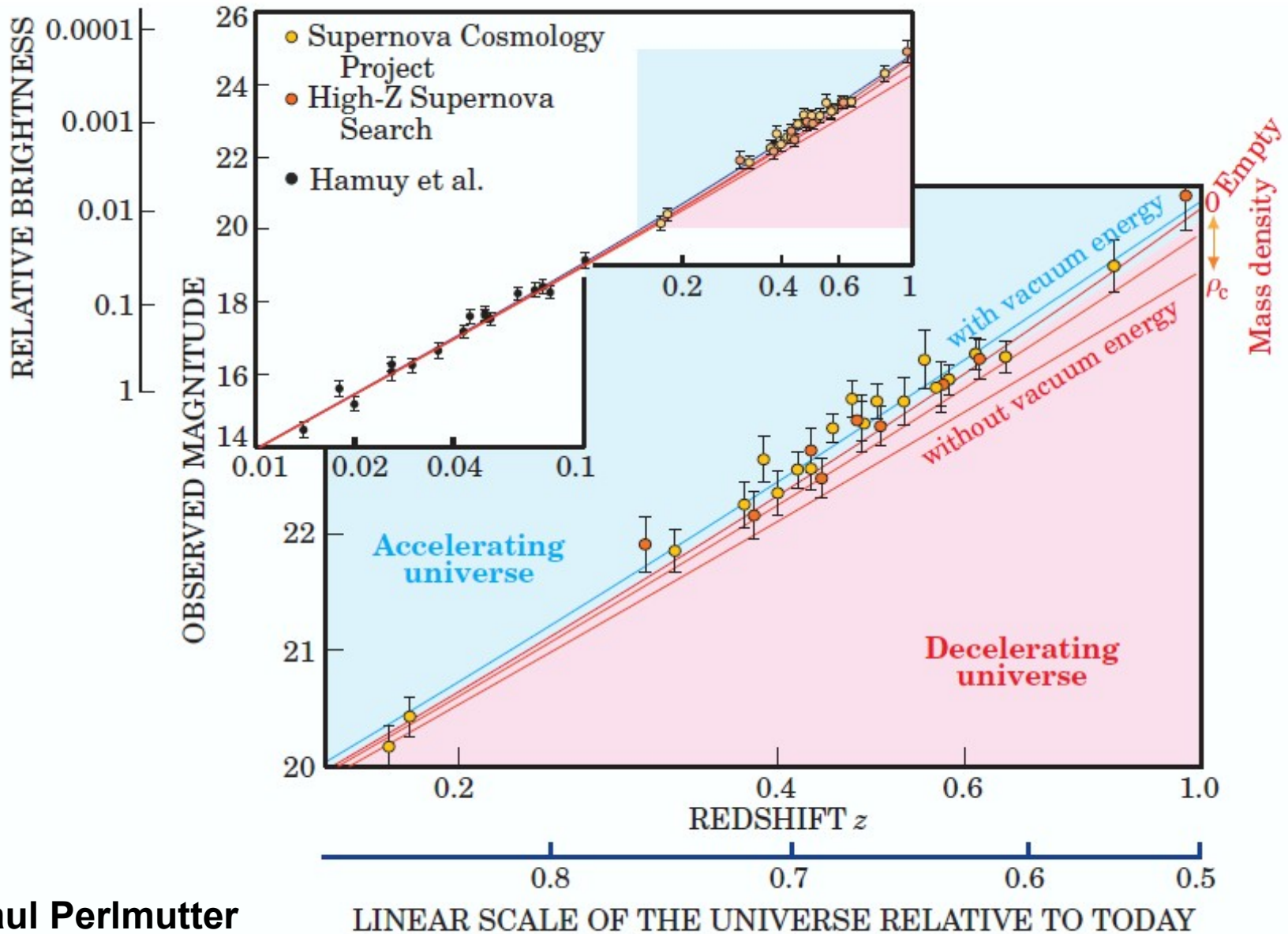
This was the AAAS discovery of the year in 1998.

$\Lambda$  causes acceleration!



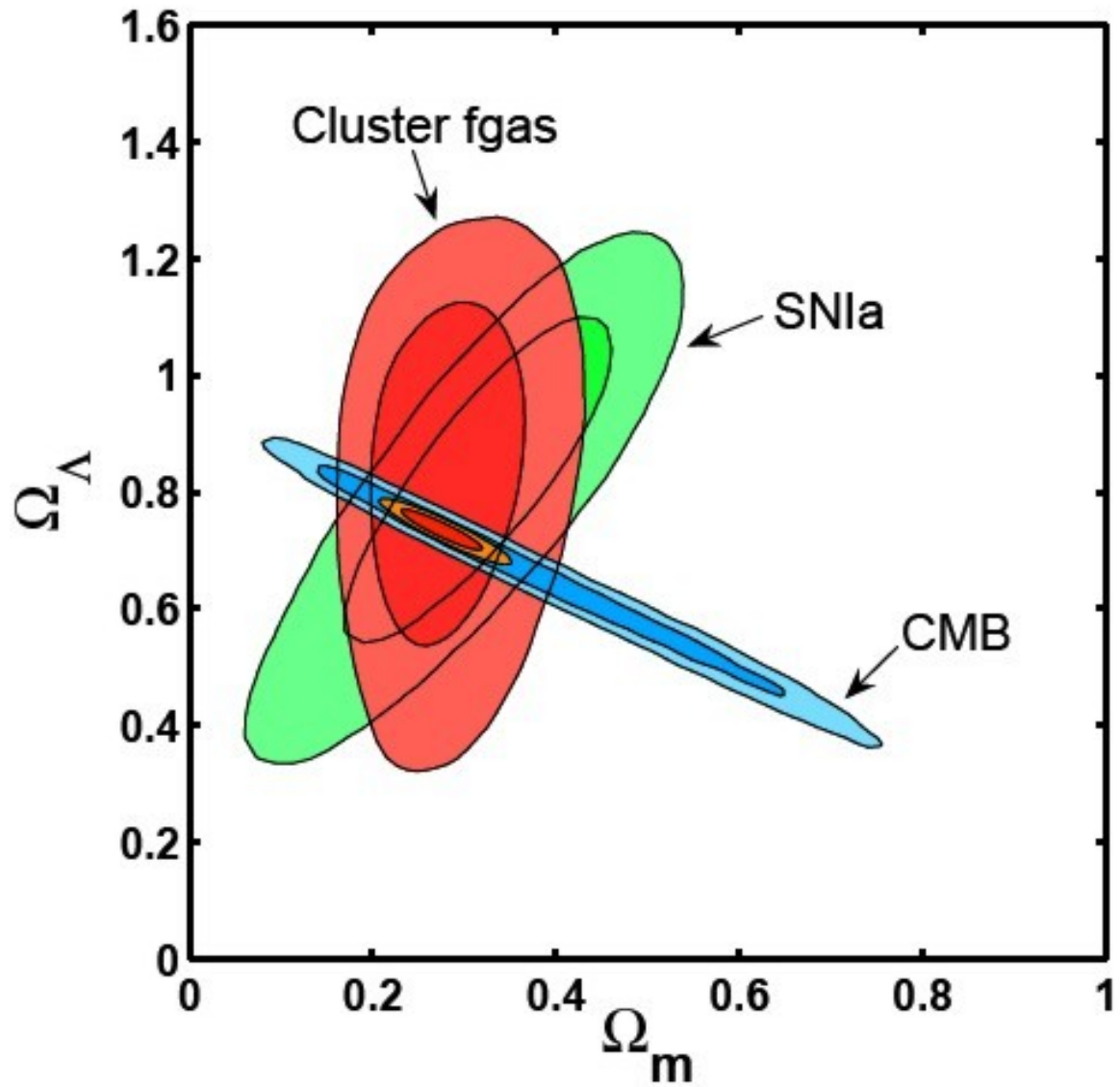
# Observing Strategy 1998





**Saul Perlmutter**  
**Physics Today 1998**

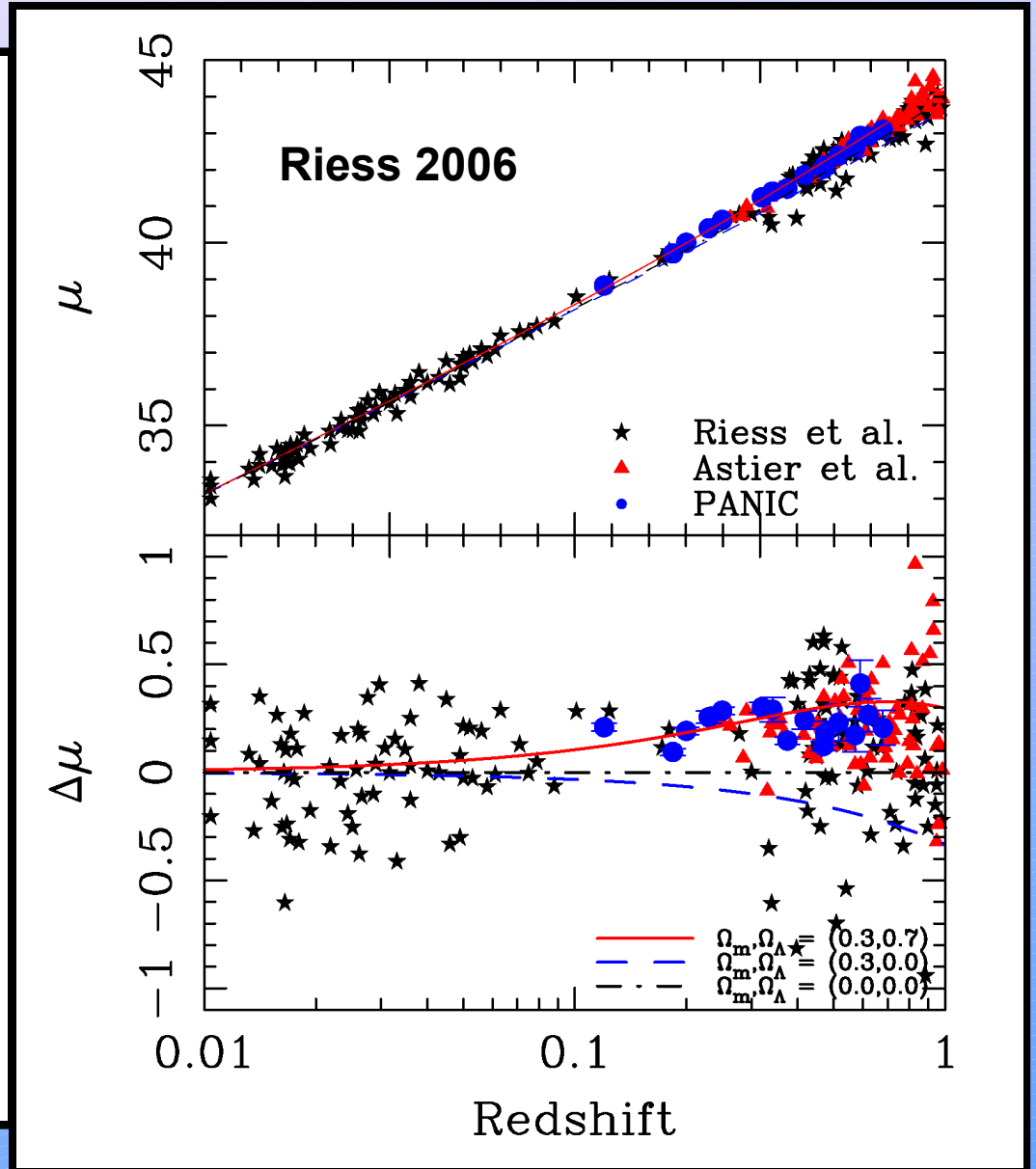
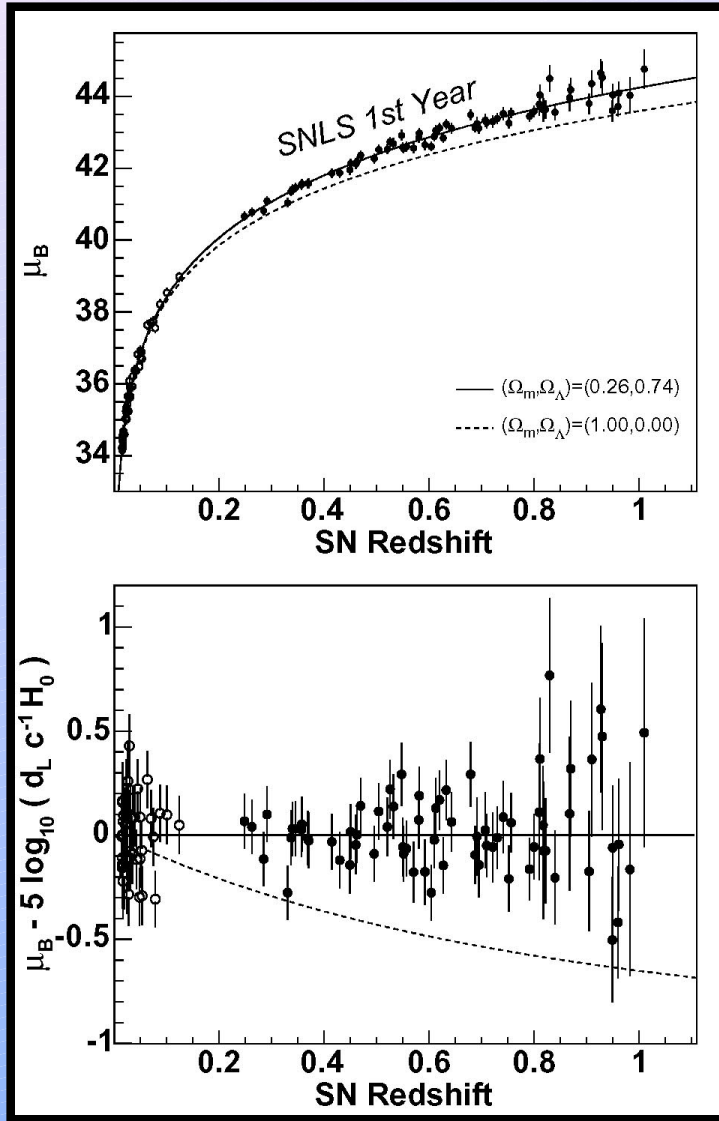
# Fundamental Plane of Cosmology







# Hubble Diagram SNe Ia - 2006

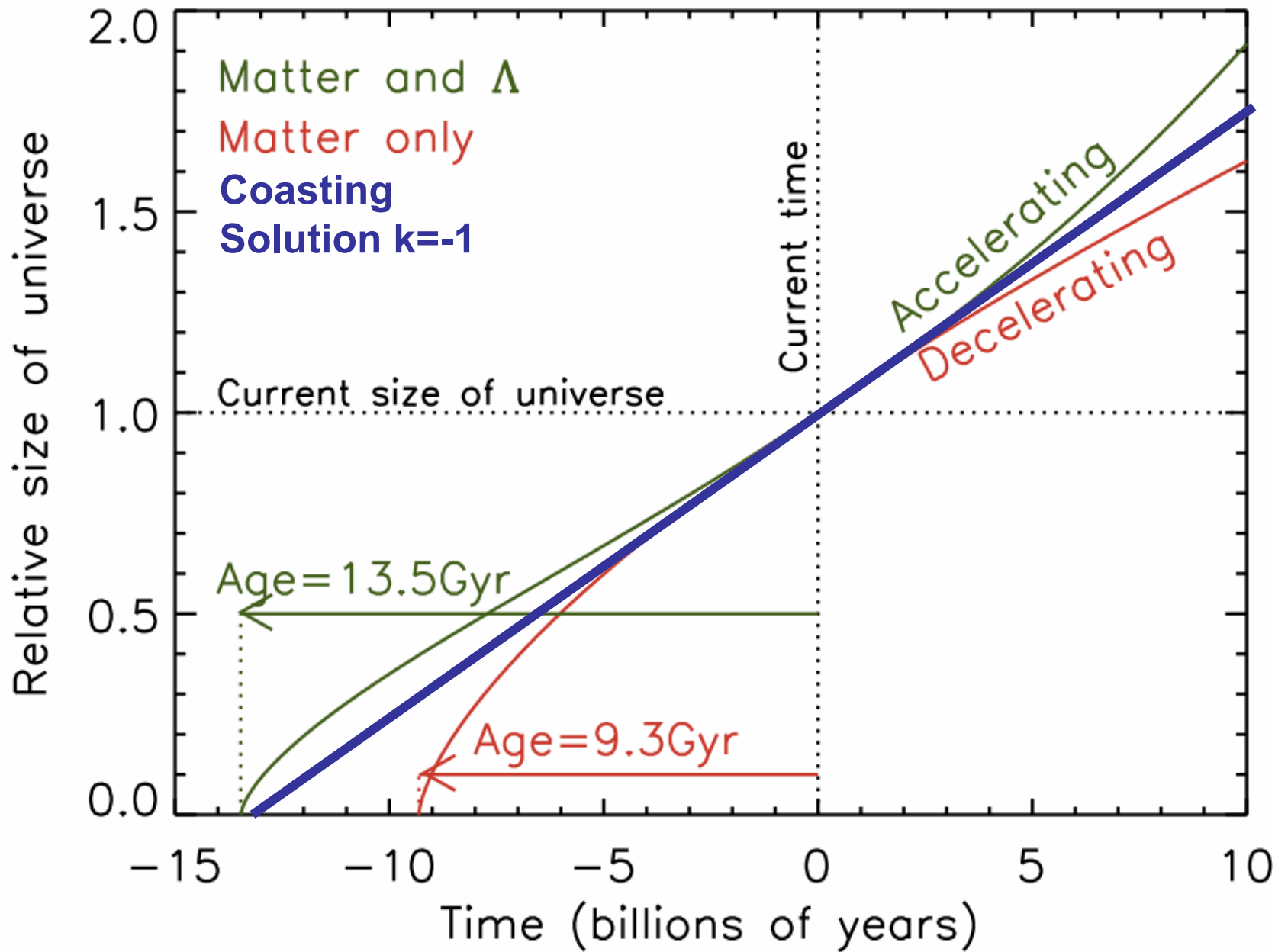


# Coasting Universe – a Reference Model $k = -1$

$$(\dot{a}/a)^2 = H_0^2 ( \Omega_k / a^2 ) ; \Omega_k = - kR_H^2/R_0^2$$

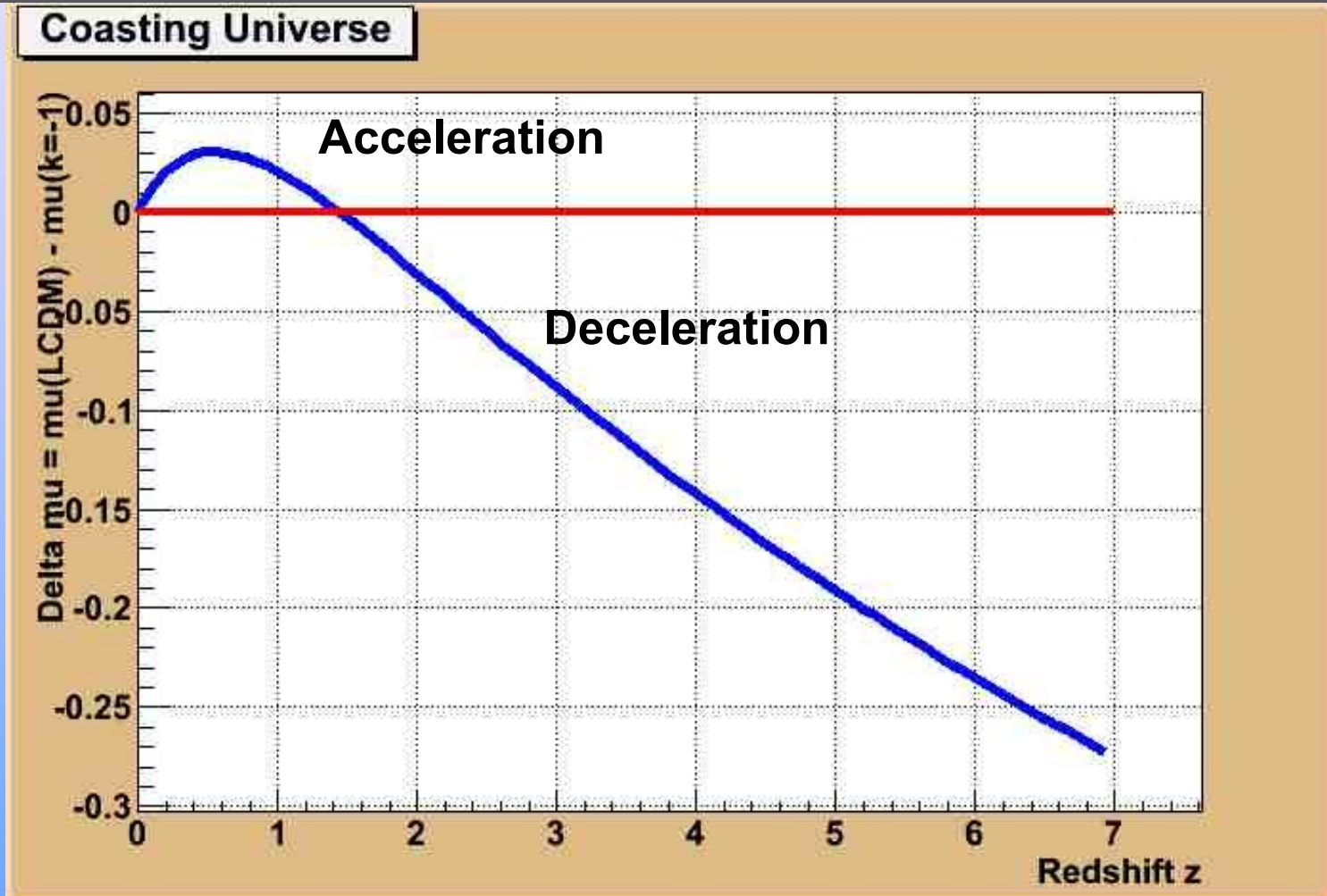
$$\rightarrow a(t) = R_0 (t/t_0) : \text{linear}$$

$$\rightarrow d_L(z) = (c/H_0) (z + z^2/2)$$



# Coasting Universe – Ref Model

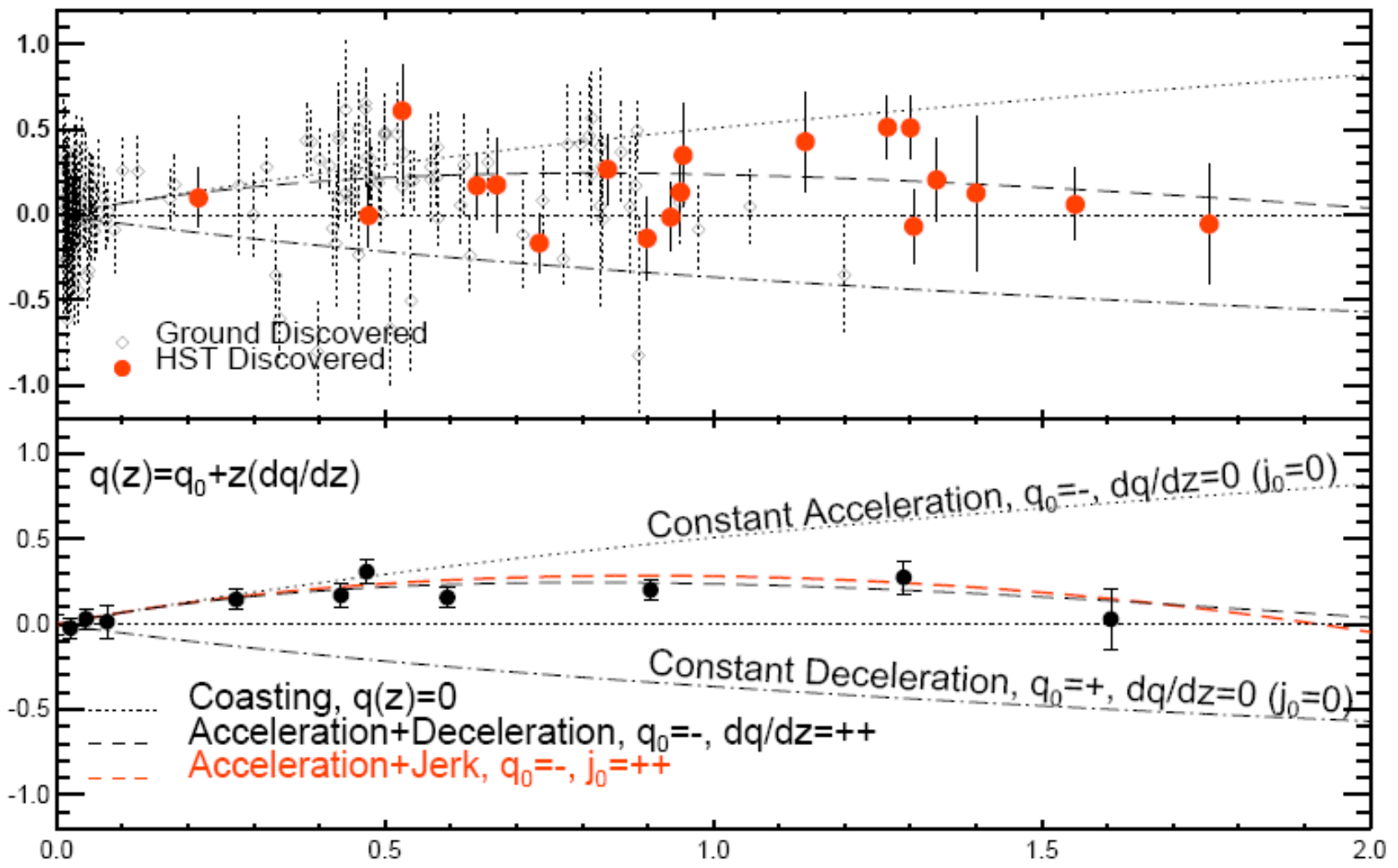
Neither Acceleration nor Deceleration



# Type Ia Supernovae

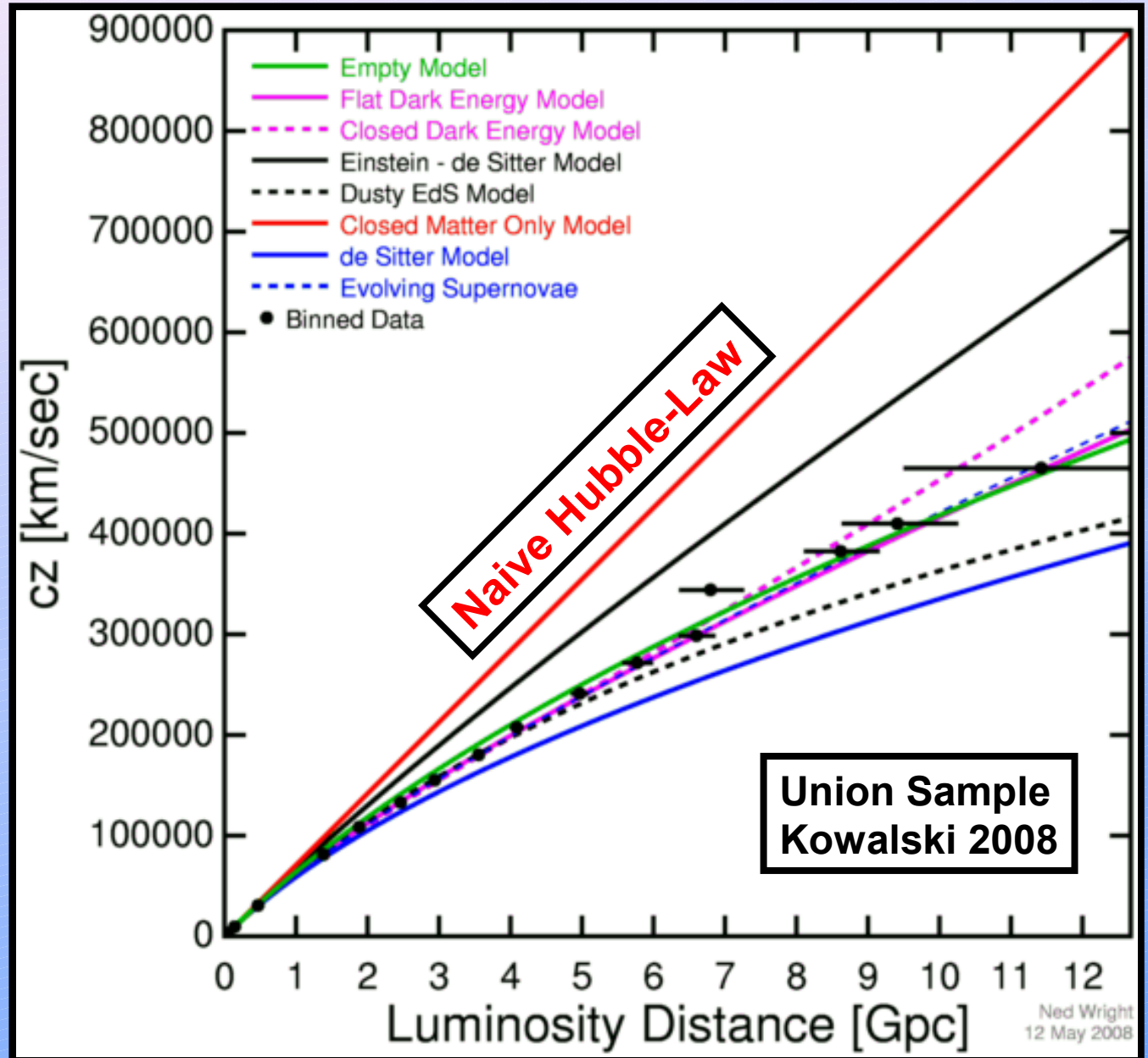
(Riess et al. 2006)

How faint each object appears (when viewed from Earth)

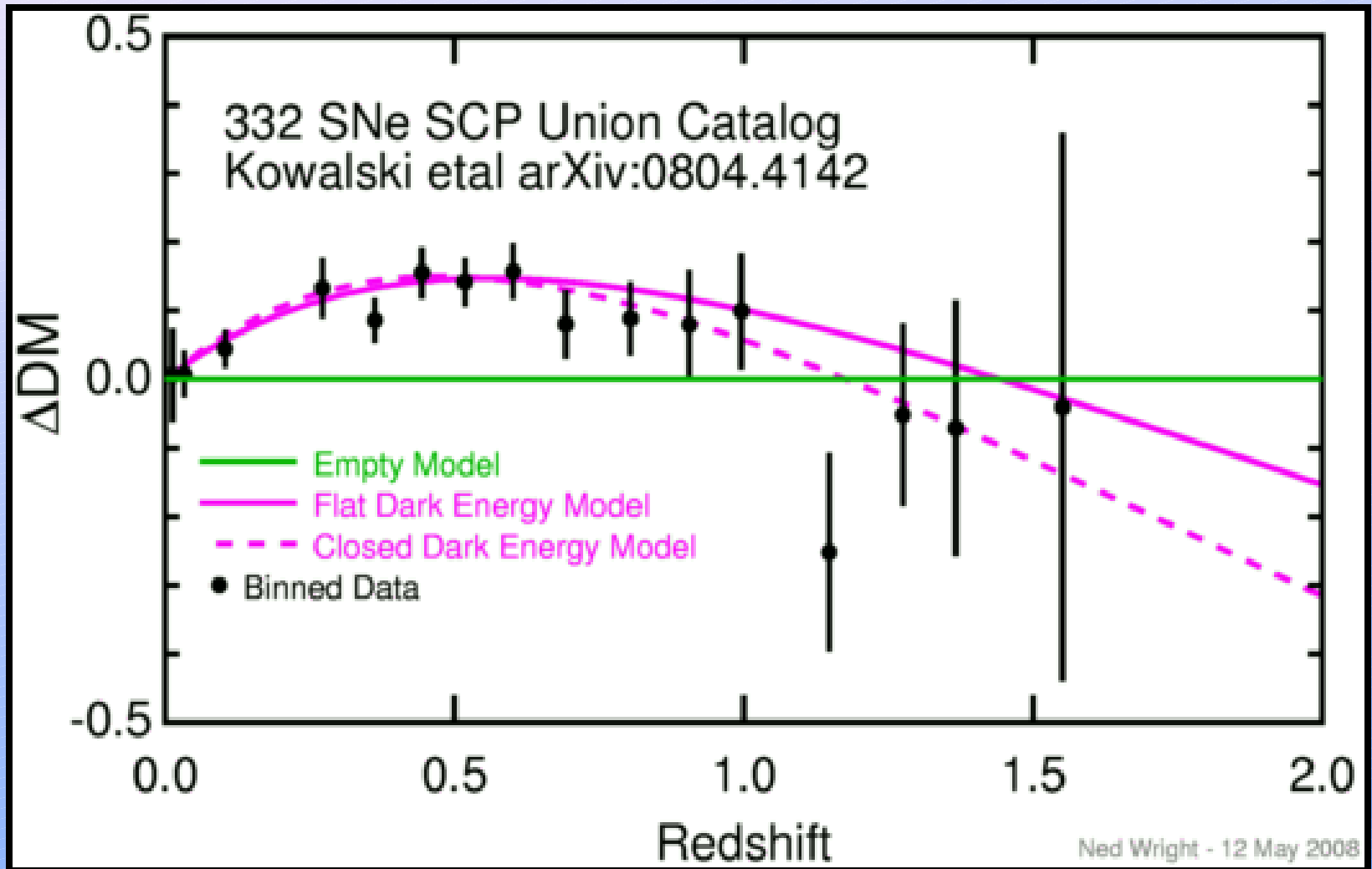


Redshift (amount universe "stretched" since the object's light was emitted)

# Observed Supernovae Determine World Models



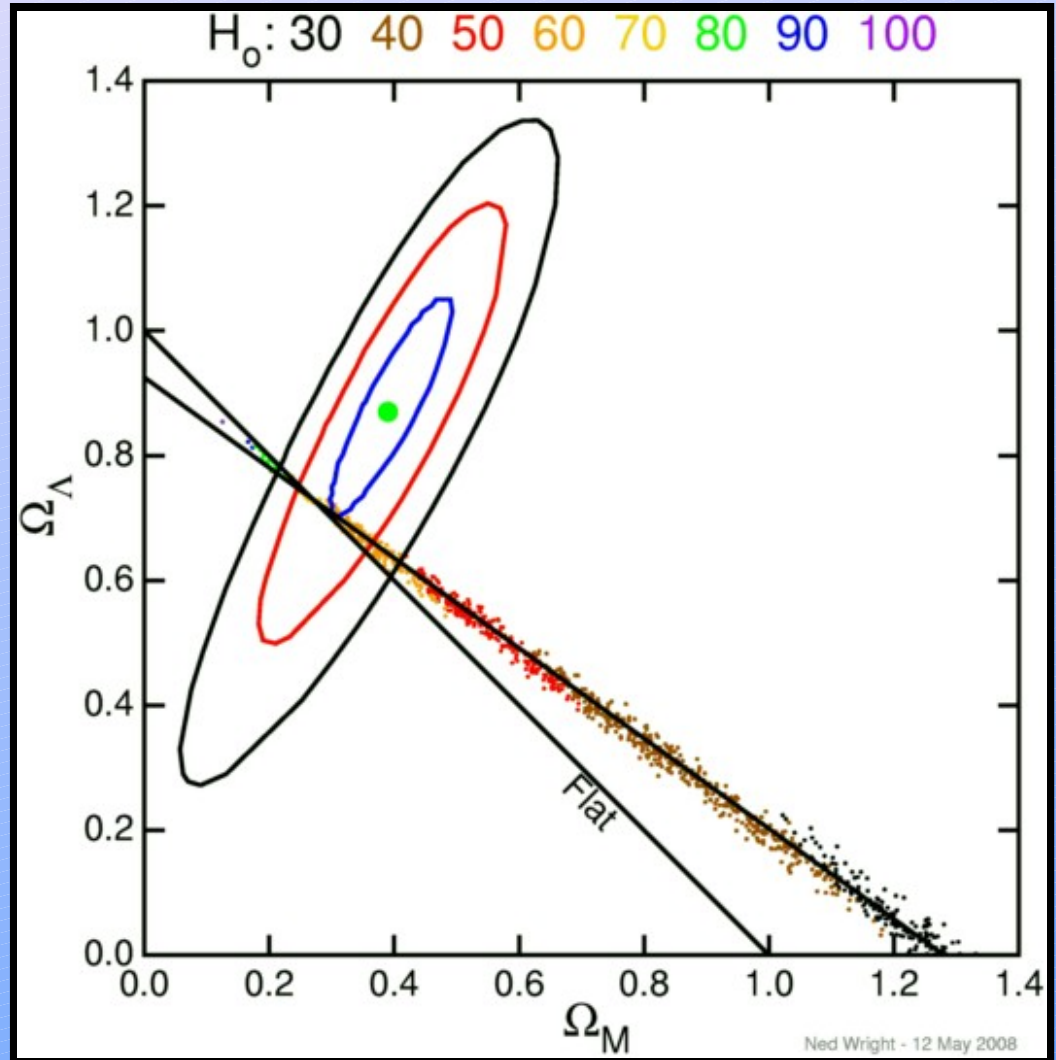
# Hubble Diagram SNe Ia 2008





# Union Sample 2008

The addition of high redshift supernovae has had two effects on the supernova error ellipse. The long axis of the ellipse has gotten shorter, and the slope of the ellipse has gotten higher. The best fit model has gotten closer to the CMB degeneracy track in absolute terms, and it has also gotten closer in terms of standard deviations in the [Kowalski \*et al.\* \(2008\)](#) dataset.

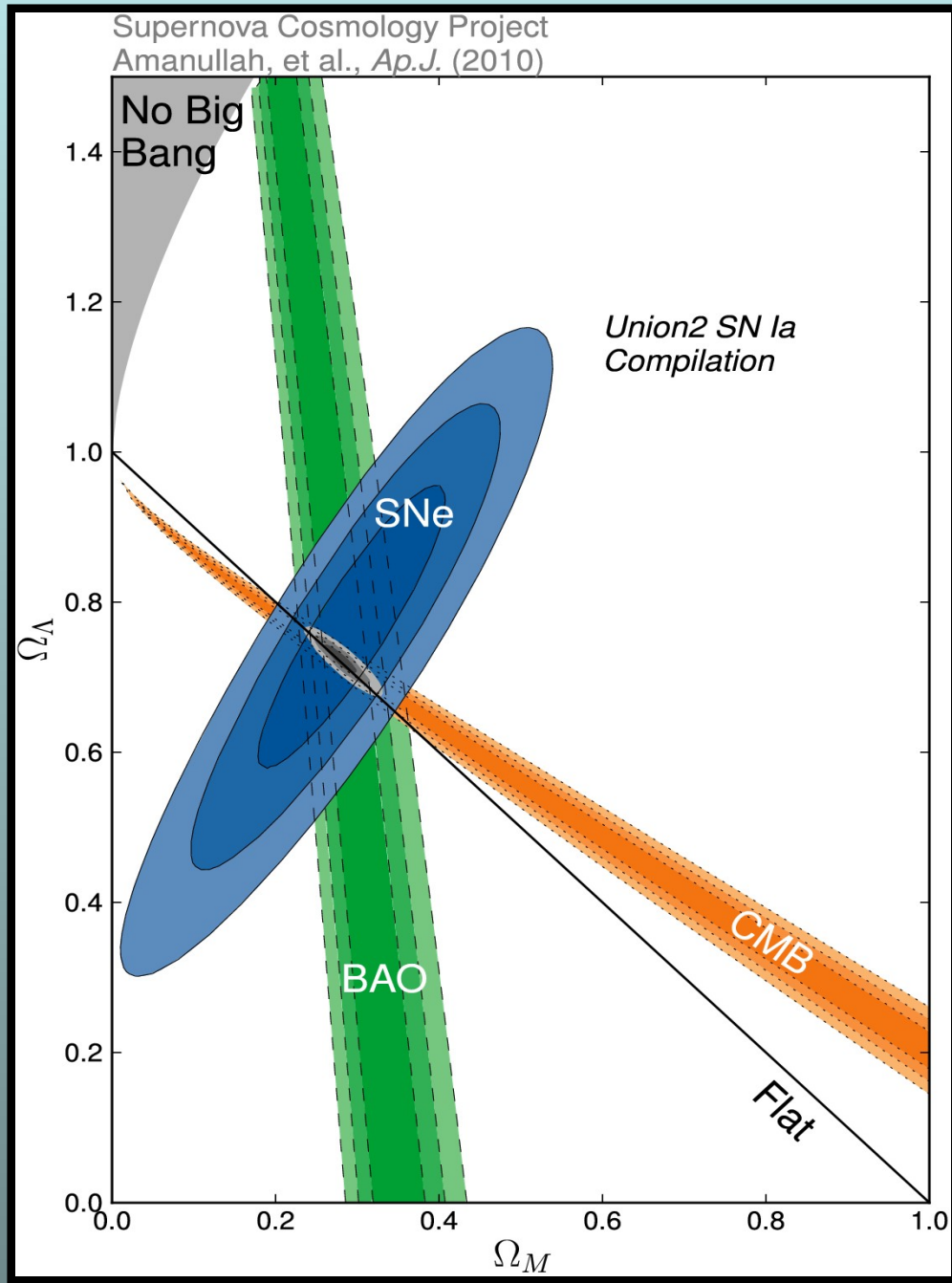


# Fundamental Plane

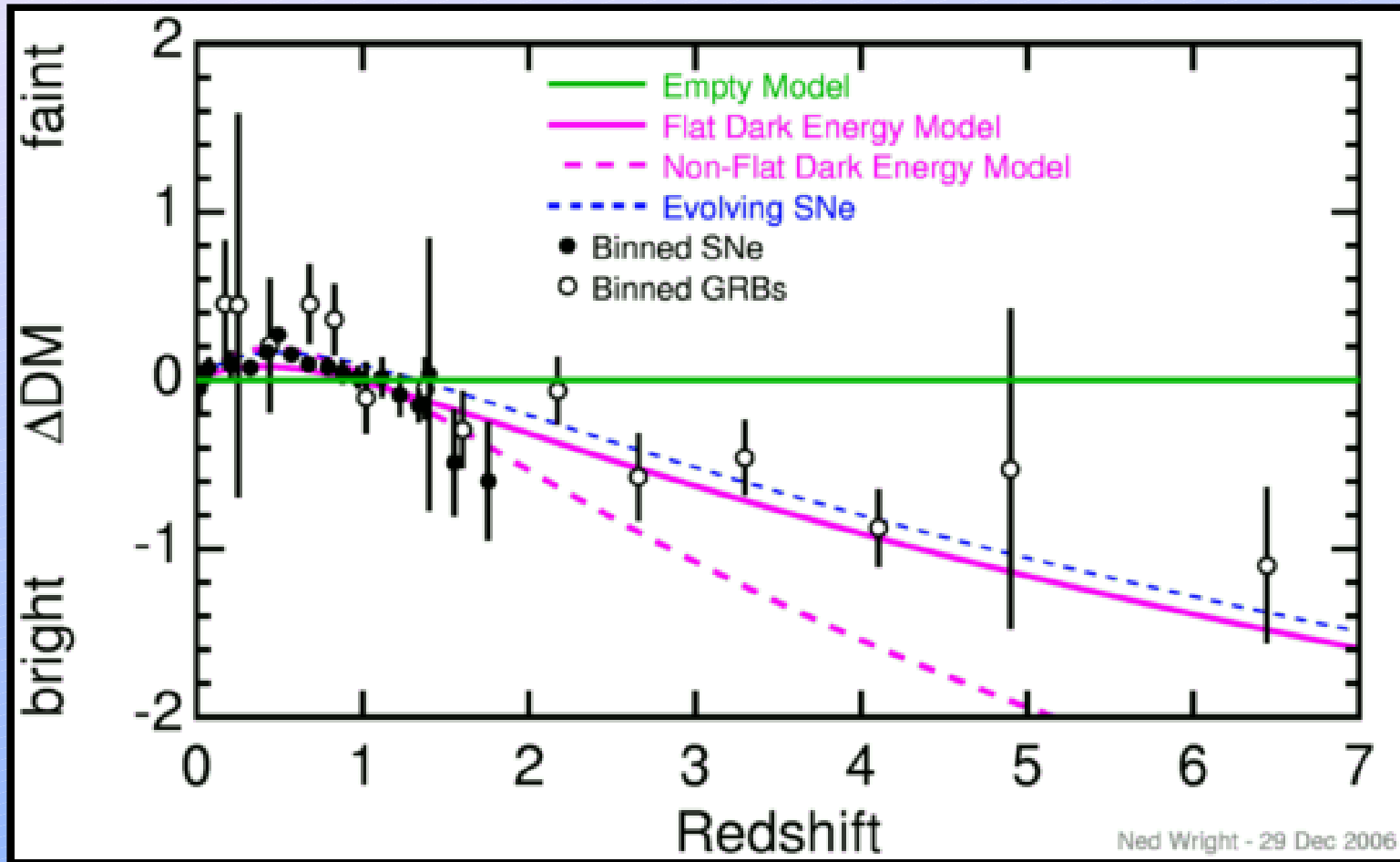
## SNe Ia – 2010

SCP Union2 Sample – 557 SNe

Amanullah et al. 2010



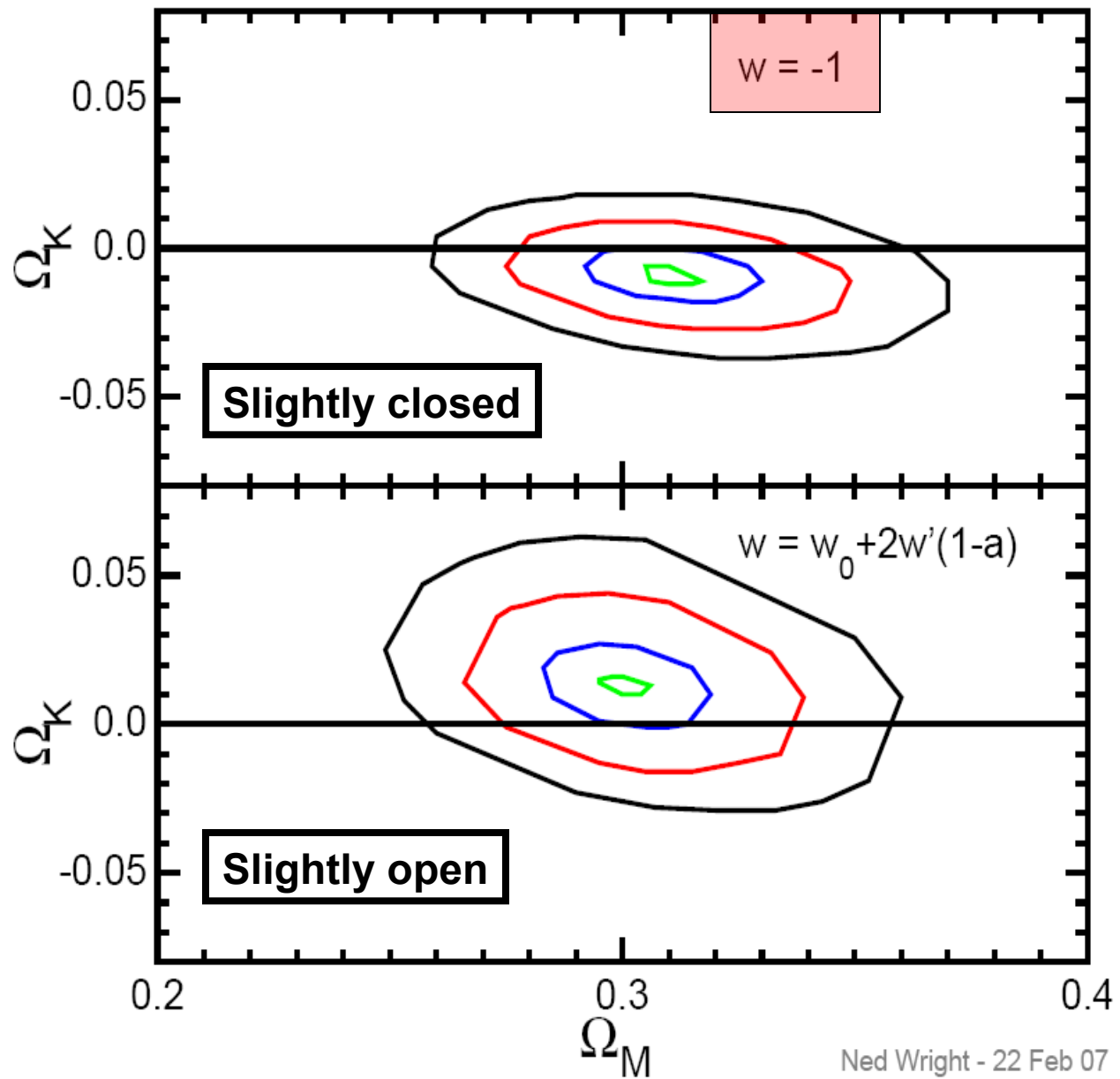
# GRBs in Expanding Universe



Ned Wright - 29 Dec 2006

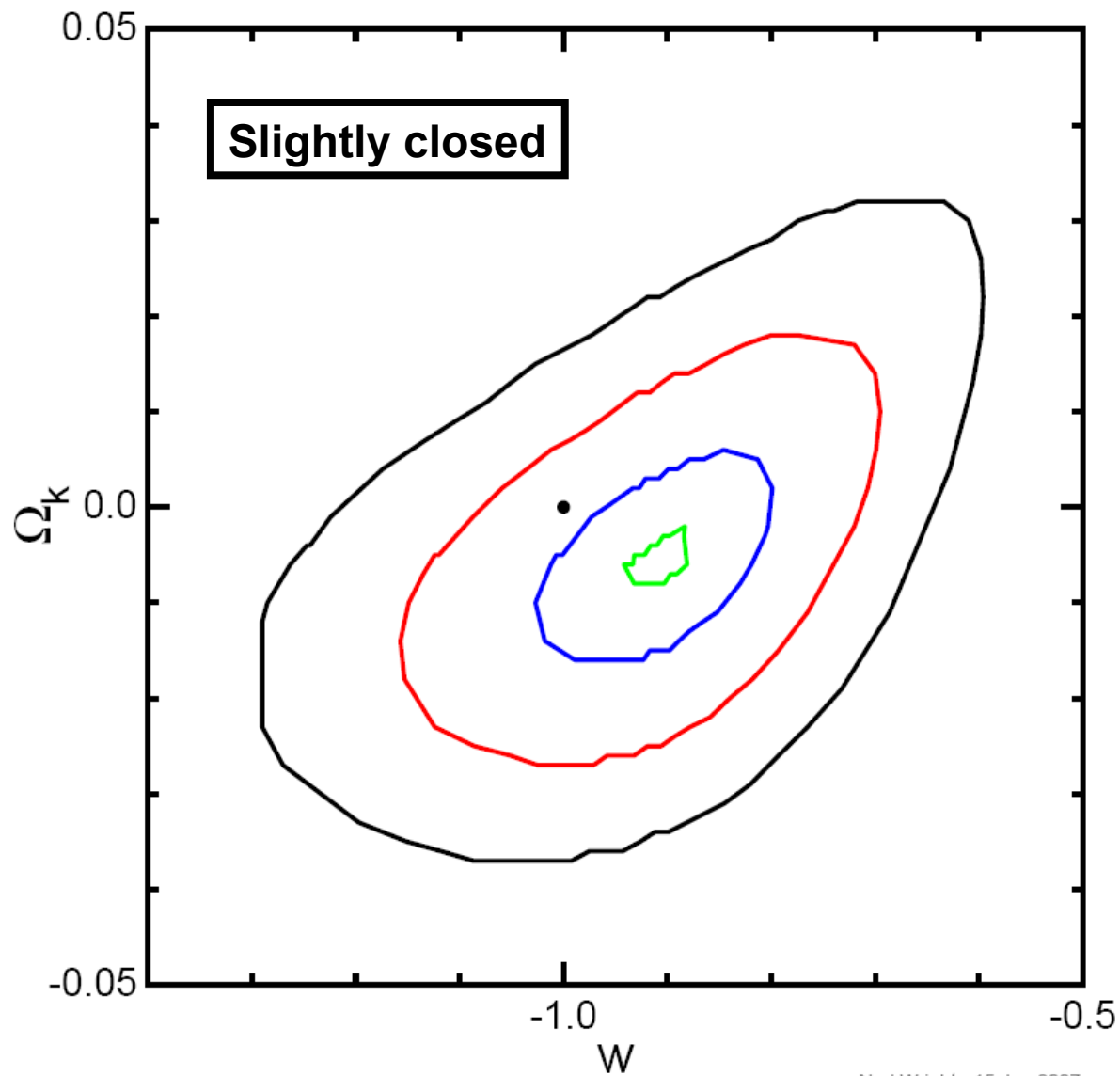
Data: Schaefer 2006

# Curvature Constraints



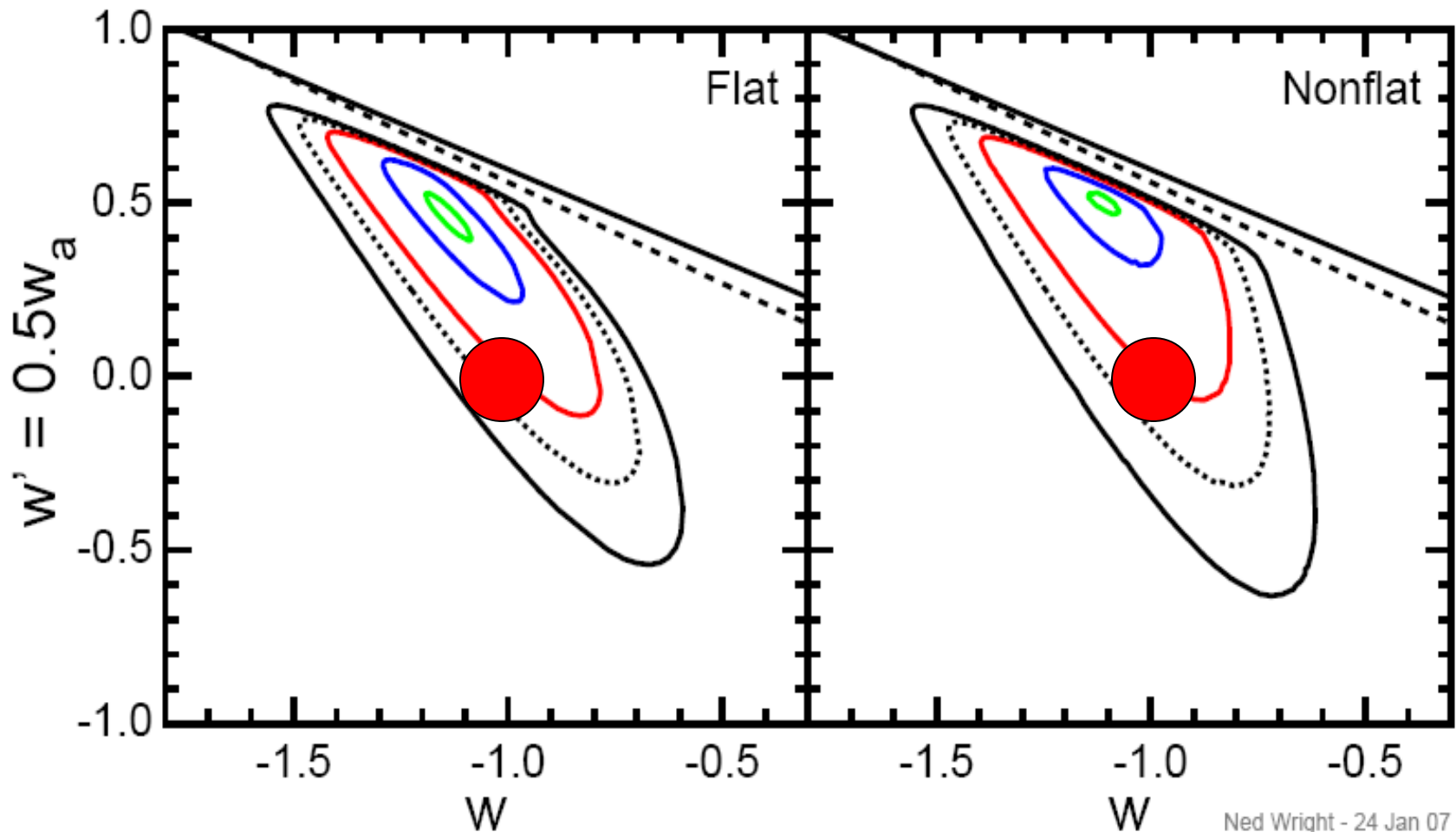
# Curvature vs DE Constraints

Wright 2007



Ned Wright - 15 Jan 2007

# Constraints on Vacuum EoS



# Supernova Projects

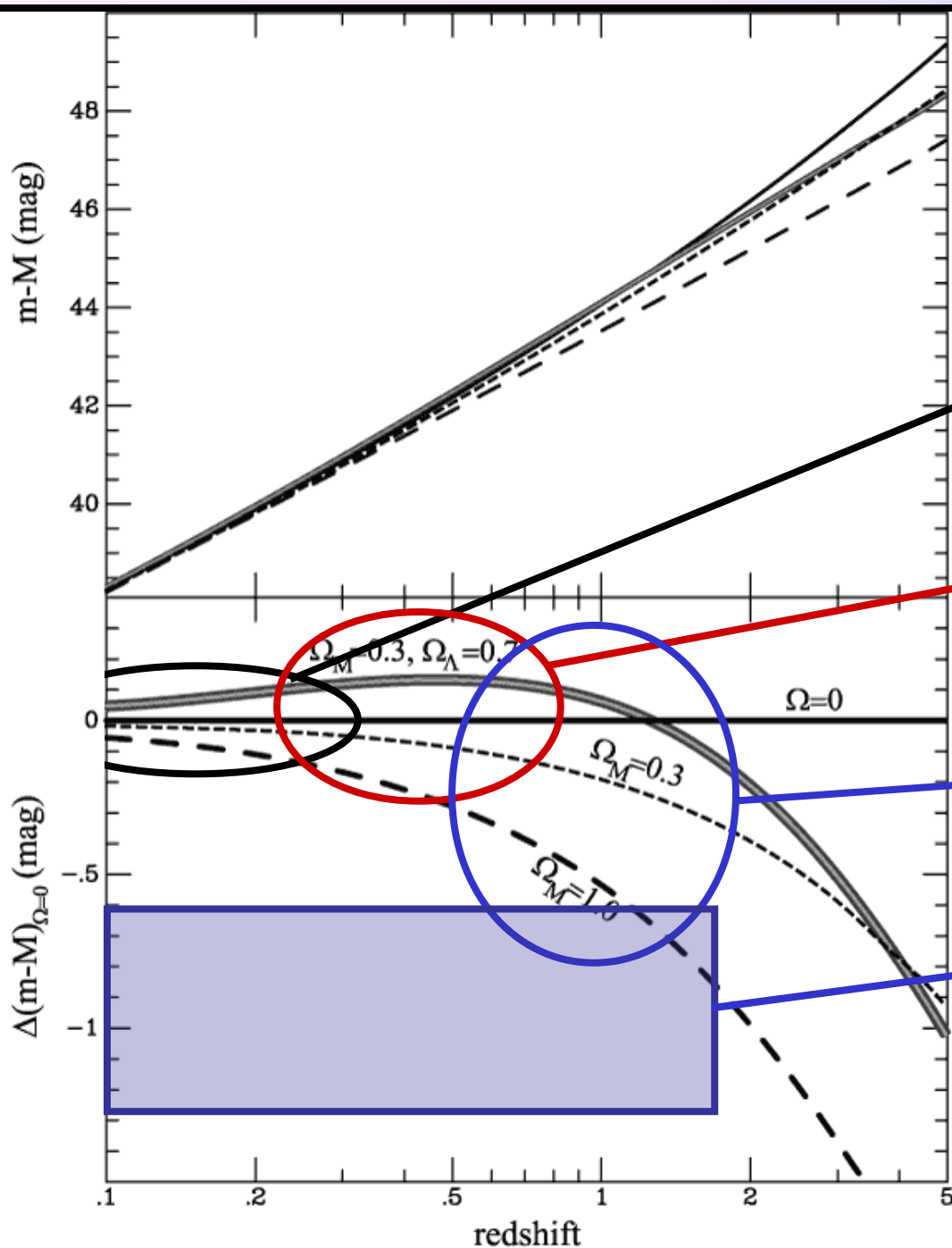
SN Factory  
Carnegie SN Project  
SDSSII

ESSENCE  
CFHT Legacy Survey

Higher-z SN Search  
(GOODS)

JDEM/LSST

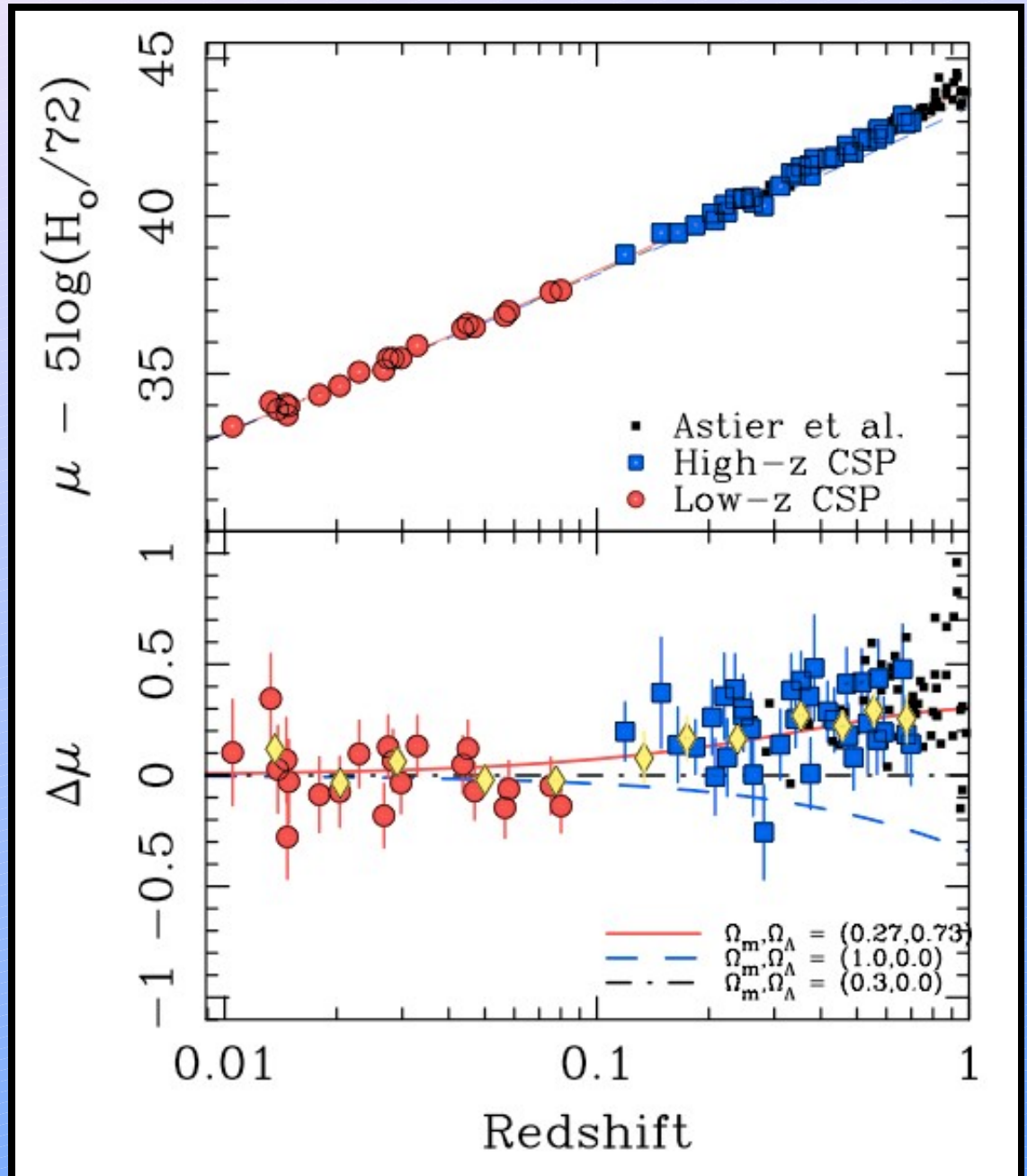
Plus the local searches:  
LOTOSS, CfA, ESC



# Carnegie SN Project

-

# Hubble Diagram to $z \sim 0.7$





# Essence

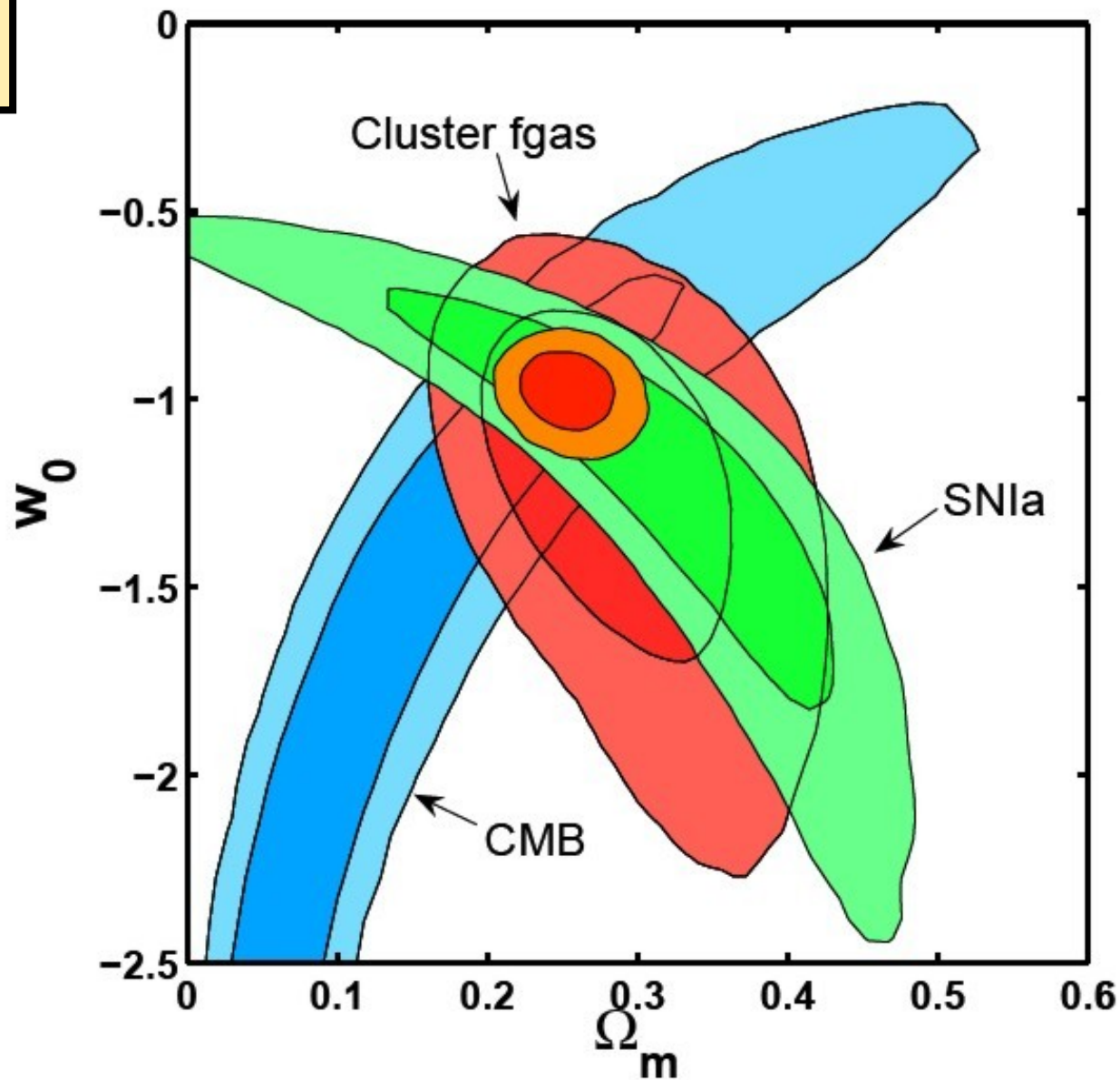
- World-wide collaboration to find and characterise SNe Ia with  $0.2 < z < 0.8$
- Search with CTIO 4m Blanco telescope.
- Spectroscopy with VLT, Gemini, Keck, Magellan
- Goal: Measure distances to 200 SNe Ia with an overall accuracy of 5%  
→ determine  $w$  to 10% overall.



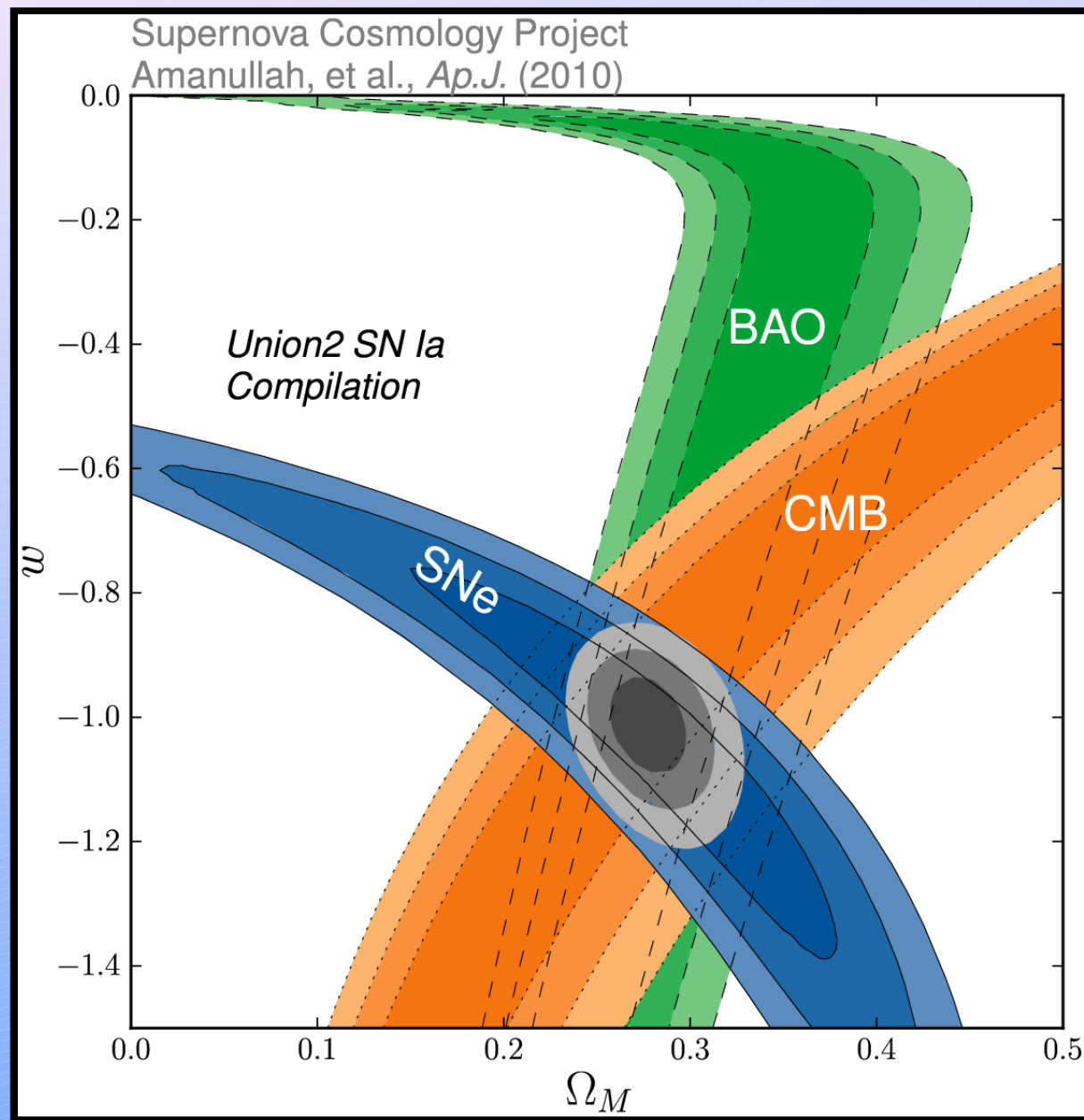
$$P_v = w \rho_v$$

$$w \sim w_0 + w_1 z$$

Is Dark Energy  
Vacuum Energy ?



# Is Dark Energy Vacuum Energy ?



# WMAP – 7 Years Results

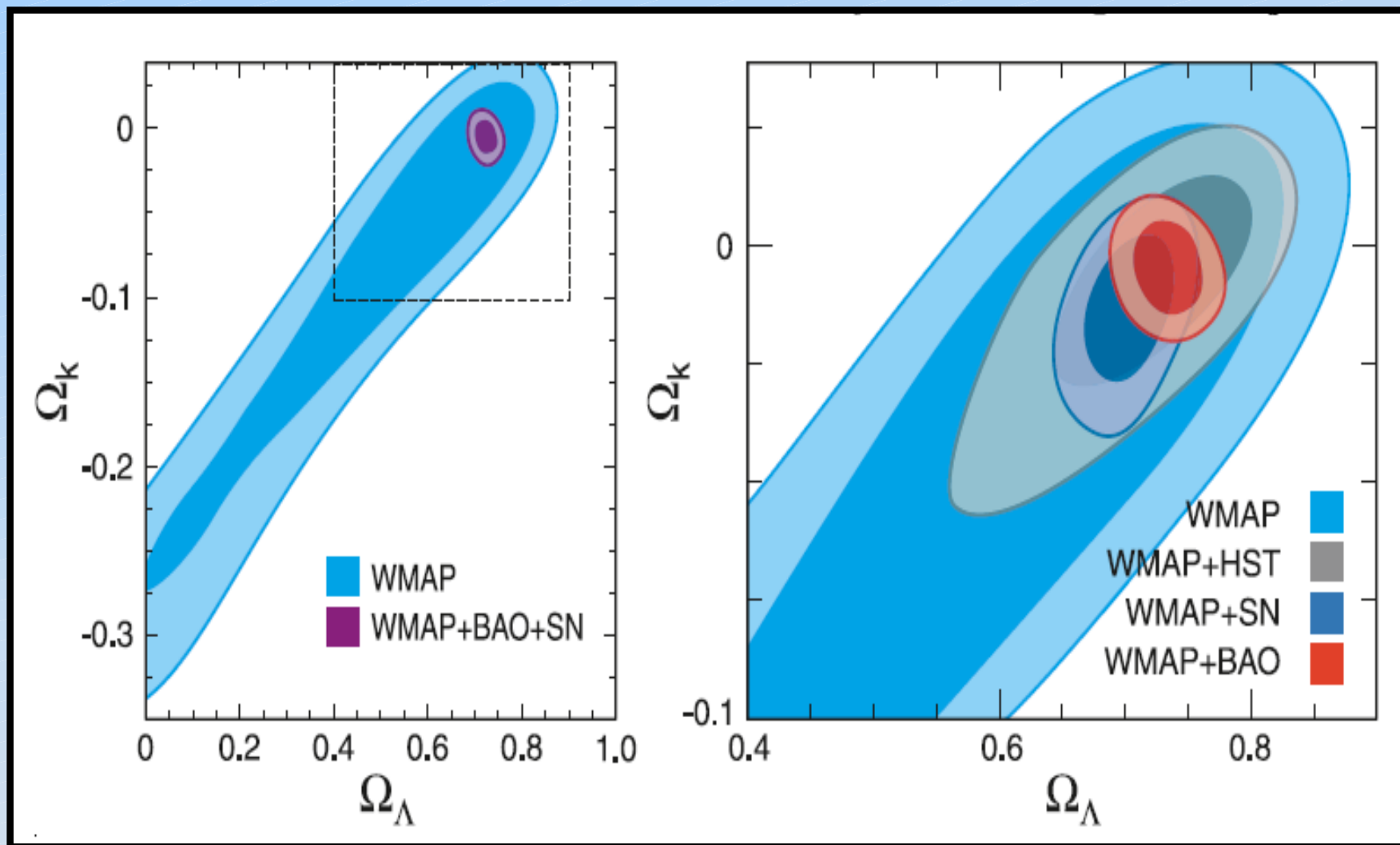
- **Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Sky Maps, Systematic Errors, and Basic Results**  
Jarosik, N., et.al., 2010, ApJSup, astro-ph
- **Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Galactic Foreground Emission**  
Gold, B., et.al., 2010, ApJSup, astro-ph
- **Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters**  
Larson, D., et.al., 2010, ApJSup, astro-ph
- **Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Are There Cosmic Microwave Background Anomalies?**  
Bennett, C., et.al., 2010, ApJSup, astro-ph
- **Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation**  
Komatsu, E., et.al., 2010, ApJSup, astro-ph

# WMAP – 7 Years Results

## 9 global FLRW Parameters:

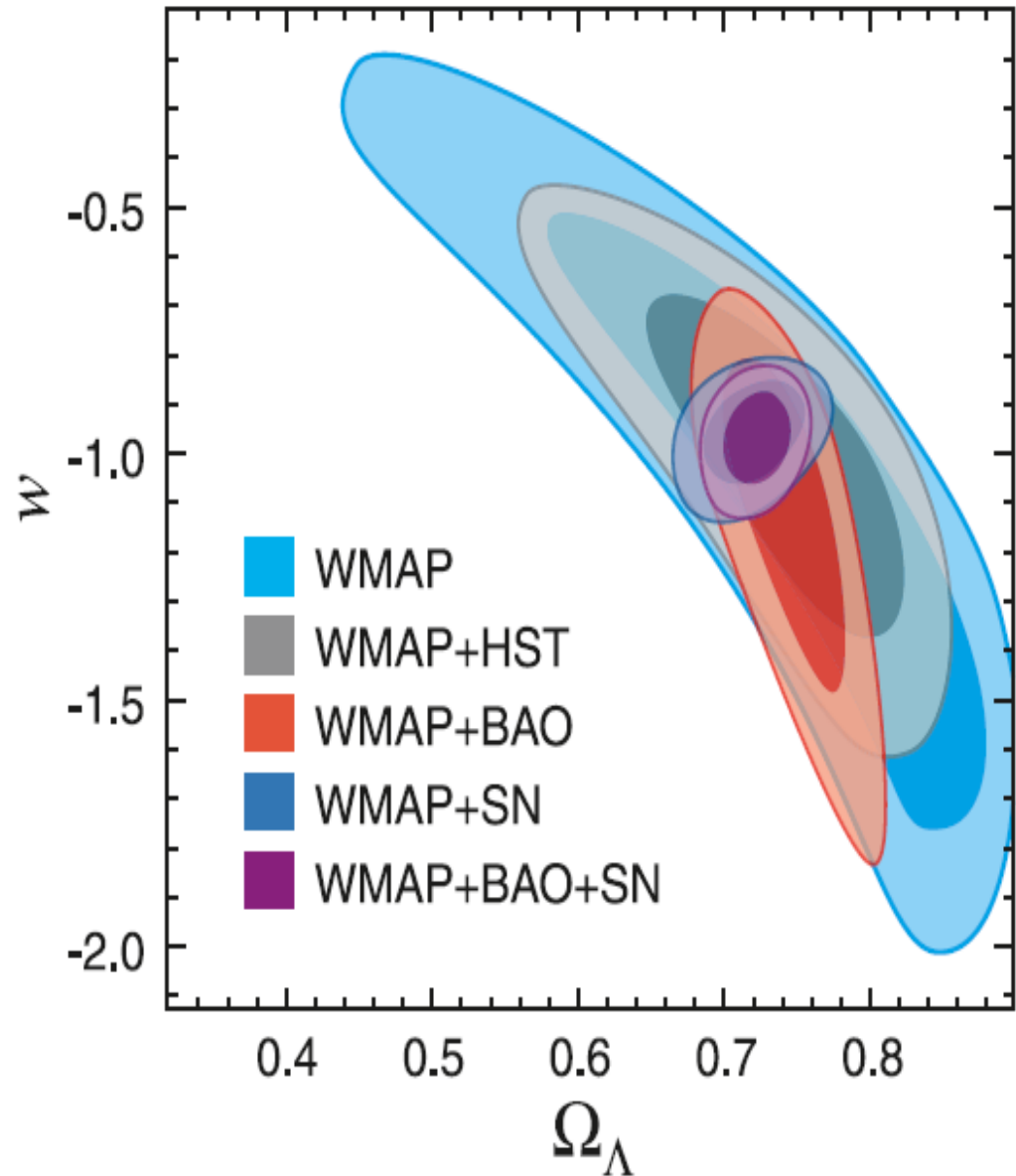
$H_0$	71+/-2 km/sec/Mpc	Expansion rate
$q_0$	-0.67 +/- 0.15	Deceleration parameter
$t_0$	13.75 +/- 0.17	Age of the Universe
$T_0$	2.725 +/- 0.001K	CMB Temperature
$\Omega_k$	- 0.01 +/- 0.005	Curvature $\rightarrow k = +1 ?$
$\Omega_B$	0.0449 /- 0.0028	Baryons
$\Omega_M$	0.266 +/- 0.029	DM+B
$\Omega_\nu$	< 0.002	Massive Neutrinos
$\Omega_{DE}$	0.734 +/- 0.029	Dark Energy DE

# WMAP7 Data – Curvature

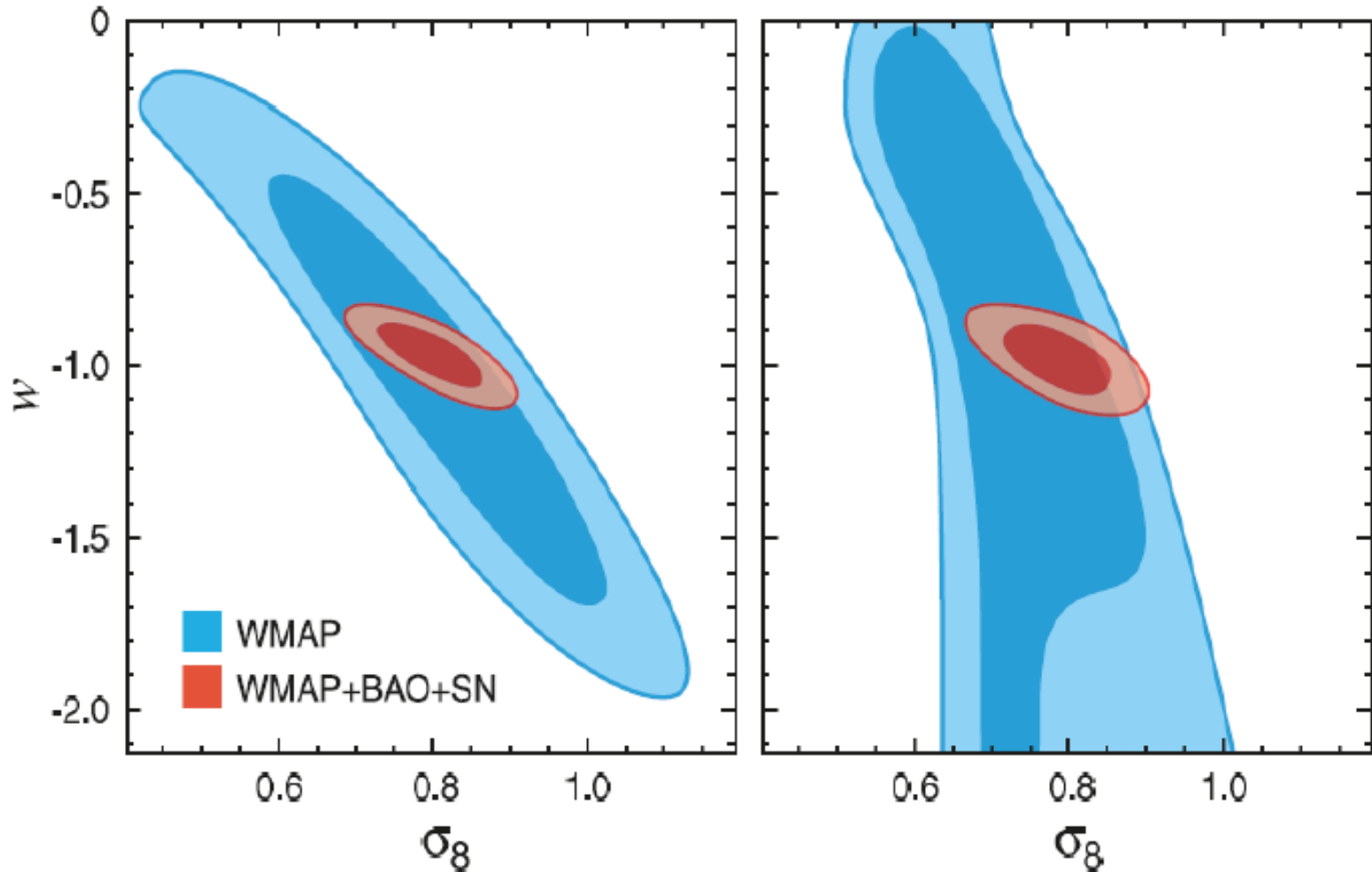


# WMAP7 Data DE w EoS

$$P_v = w \rho_v$$
$$w \sim w_0 + w_1 z$$

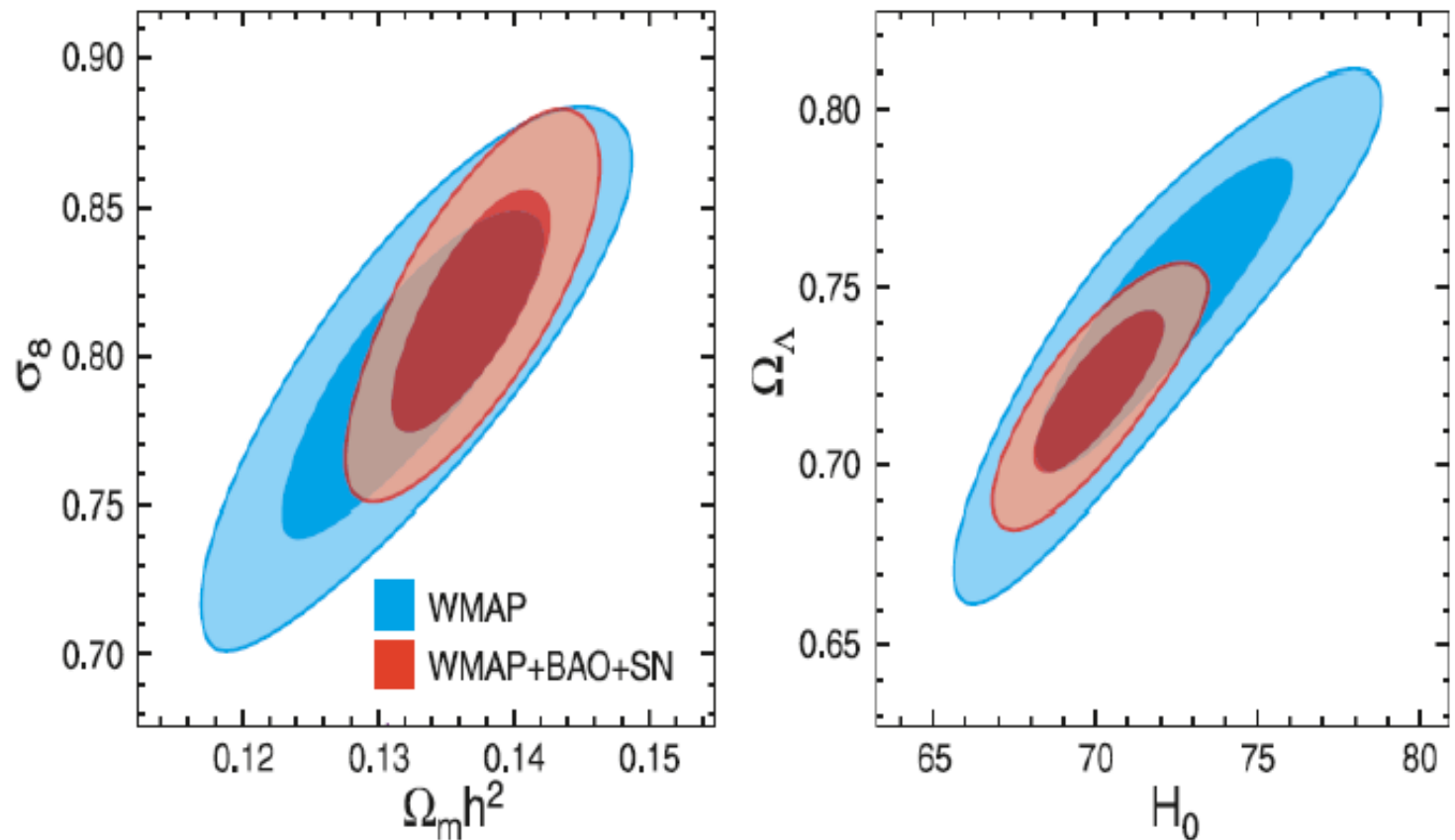


# WMAP7 Data – Fluctuations

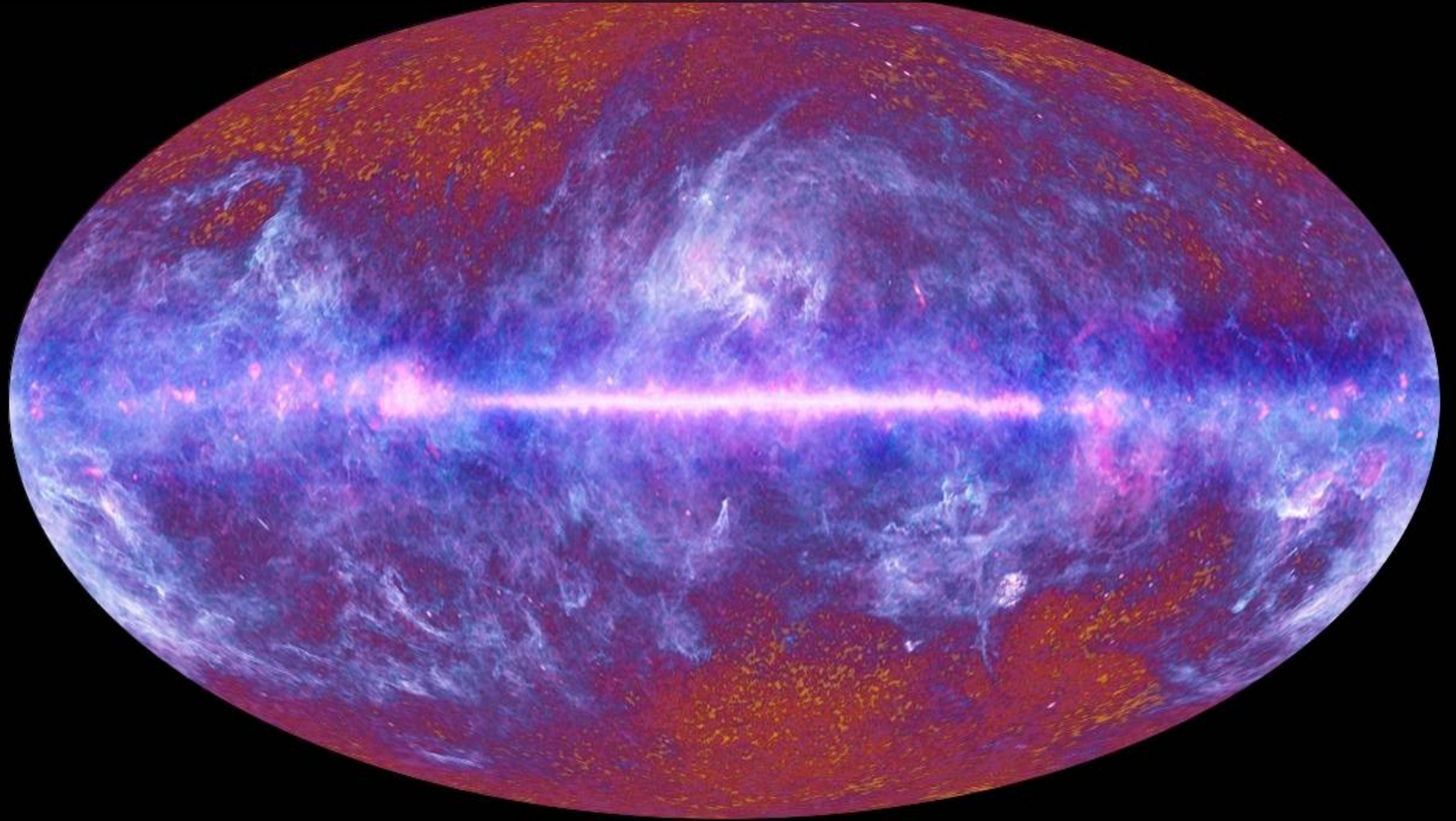




# WMAP7 Data – Hubble Const



# Future: Planck ~ 2012



The Planck one-year all-sky survey



(c) ESA, HFI and LFI consortia, July 2010

# The Observable Universe

© 2001 Brooks/Cole Publishing/ITP

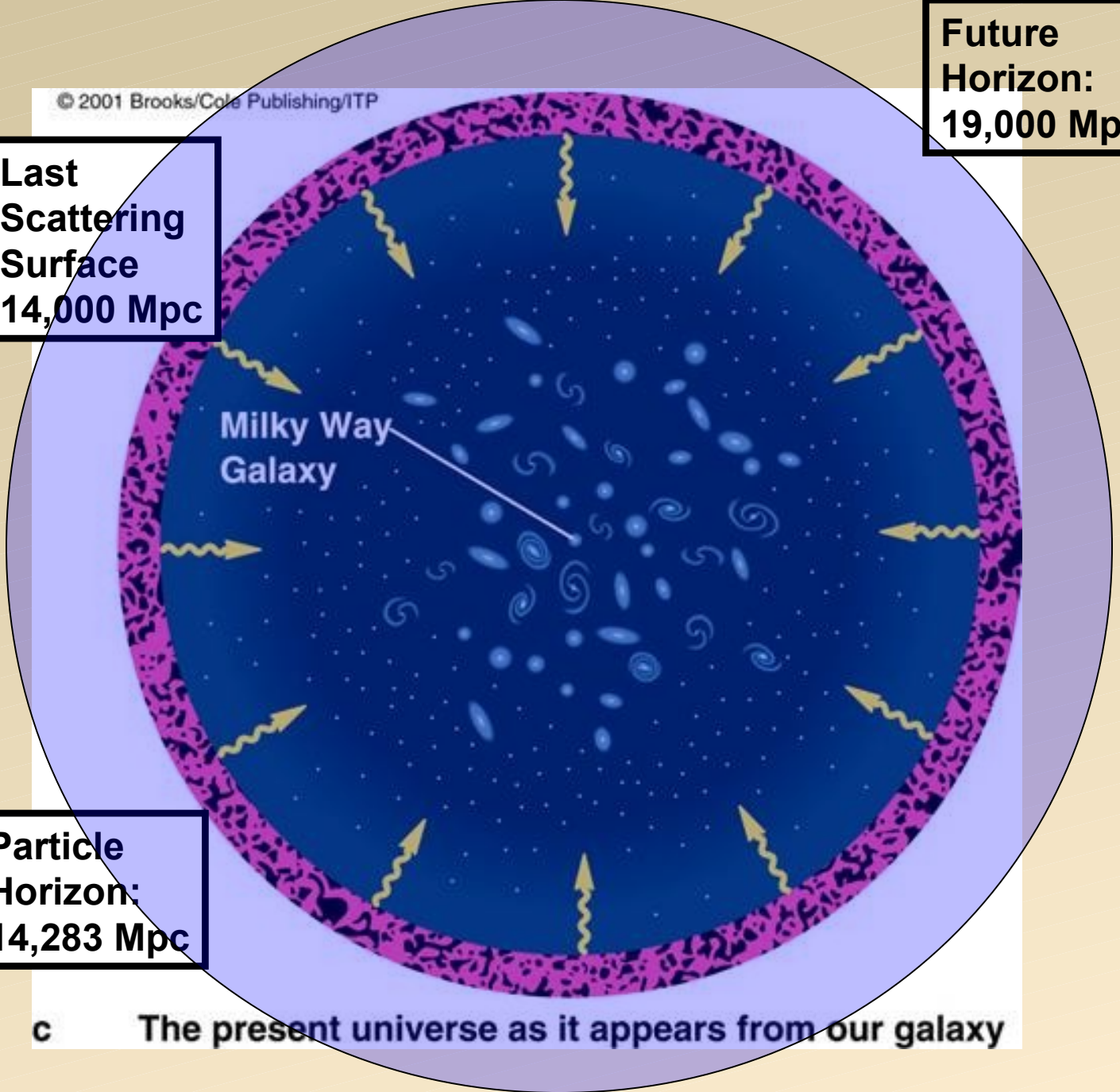
Last  
Scattering  
Surface  
14,000 Mpc

Future  
Horizon:  
19,000 Mpc

Particle  
Horizon:  
14,283 Mpc

Milky Way  
Galaxy

c The present universe as it appears from our galaxy



# Conformal Time

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)(d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2)), & k = +1 \\ ds^2 &= -dt^2 + a^2(t)(d\chi^2 + \chi^2(d\theta^2 + \sin^2 \theta d\phi^2)), & k = 0 \\ ds^2 &= -dt^2 + a^2(t)(d\chi^2 + \sinh^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2)), & k = -1 \end{aligned}$$

Use conformal time  $d\eta = dt/a$ :  $\eta(t_0) = 3.38$  present value

$$\eta(t) = \int_0^t \frac{dt}{a} = \int_0^{a(t)} \frac{da}{a^2 H(a)} = \frac{1}{H_0} \int_0^{a(t)} \left[ \Omega_r + \Omega_m a + \Omega_k a^2 + \Omega_\Lambda a^4 \right]^{-1/2} da \quad (4.178)$$

→ Particle horizon scale:

$$\chi R_{H_0} = 3.38 R_{H_0} = 14,300 \text{ Mpc}.$$

# Particle & Future Horizon

**Particle Horizon:** 
$$\tilde{r}_P = a(t) \int_0^t \frac{dt}{a(t)}$$

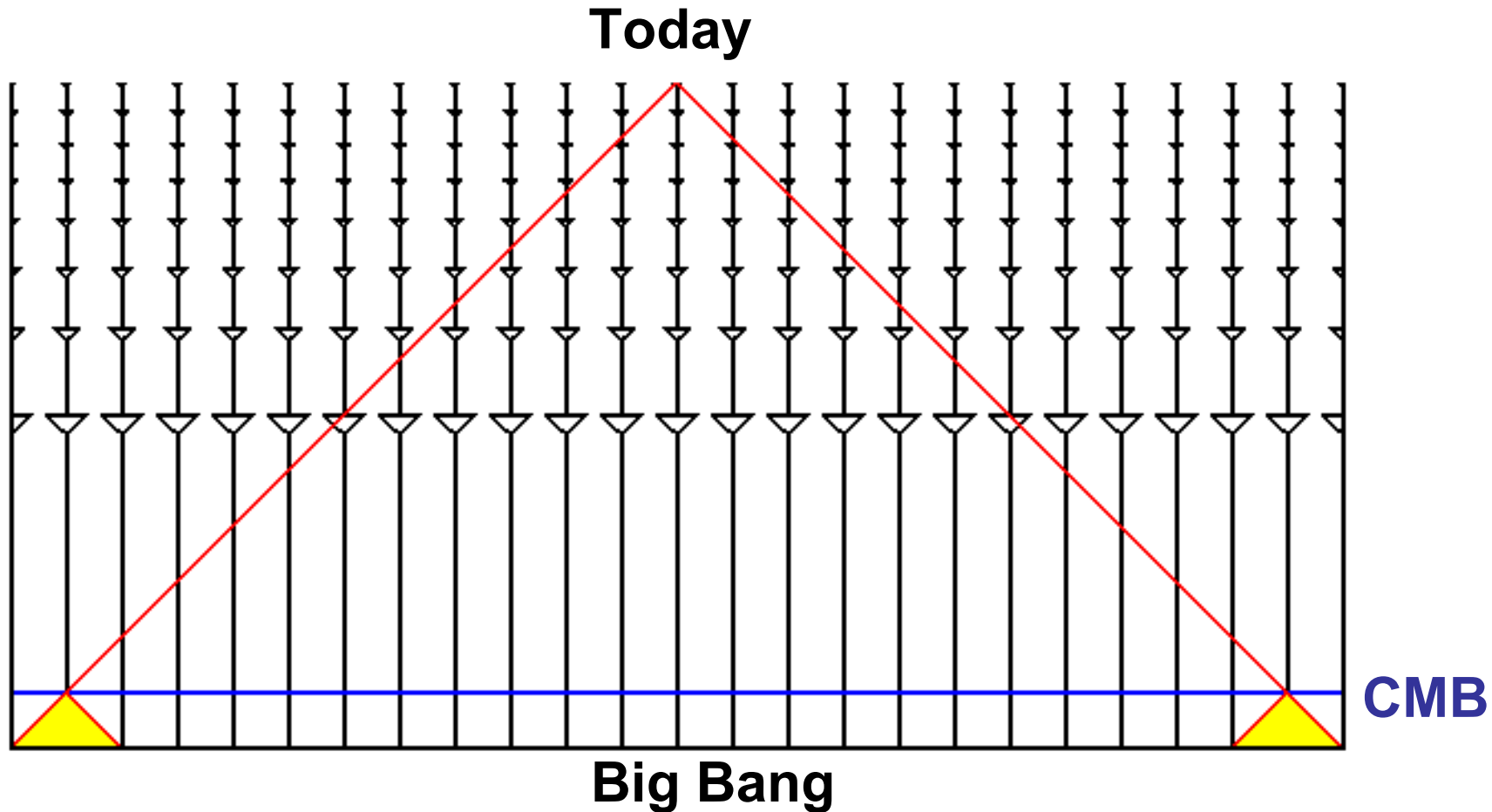
**Particle horizon** is the maximum distance from which particles could have traveled to the observer in the age of the universe. It represents the portion of the Universe, which we could have conceivably observed at the present day.

**Event Horizon:** 
$$\tilde{r}_E = a(t) \int_t^\infty \frac{dt}{a(t)},$$

A **cosmological horizon** marks a limit to the observability, and marks the **boundary** of a region that an observer cannot see into directly due to cosmological effects.

# Universe in Conformal Time

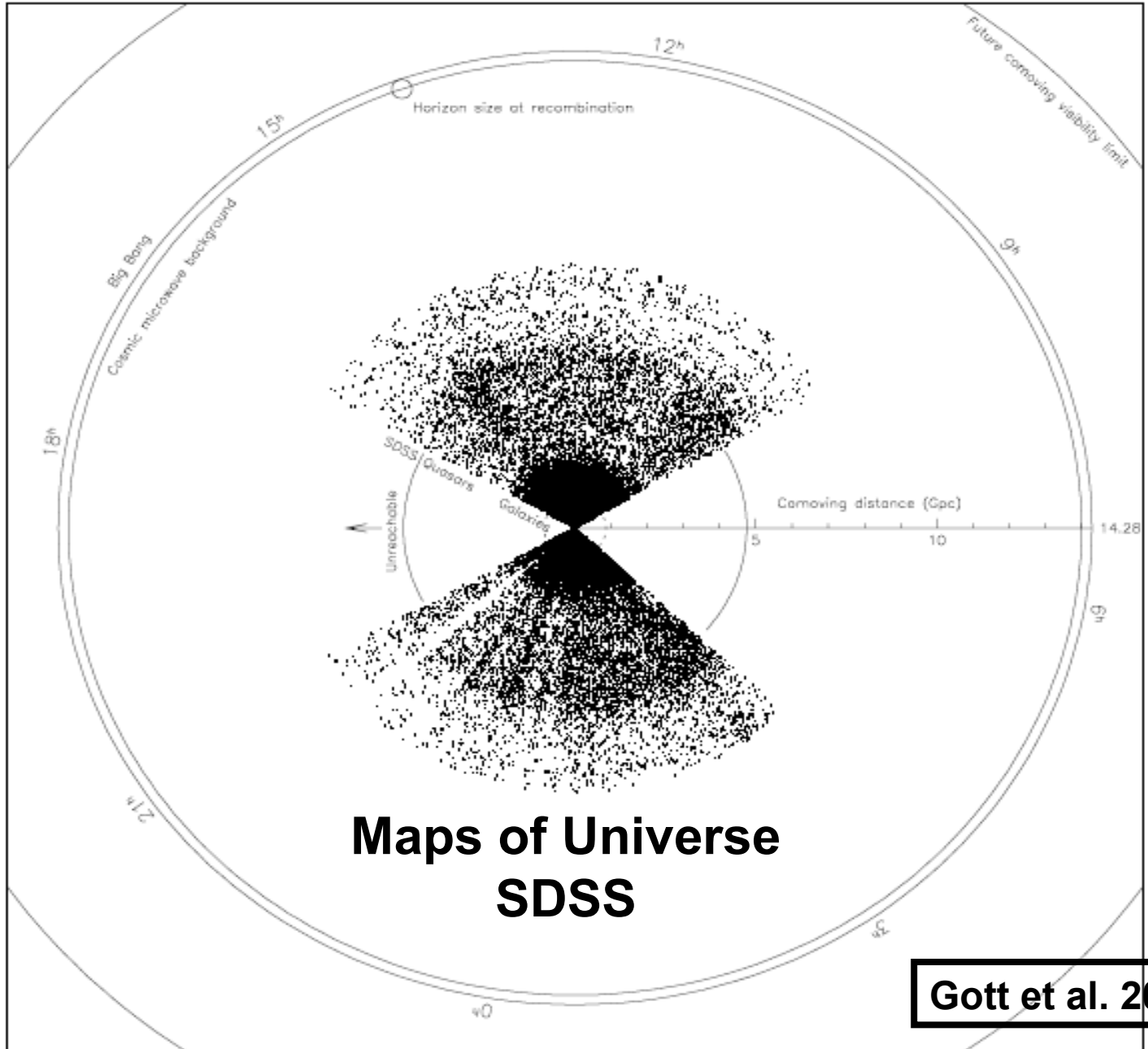
$$ds^2 = R^2(\eta) [d\eta^2 - d\sigma^2]$$



# Comoving Distance

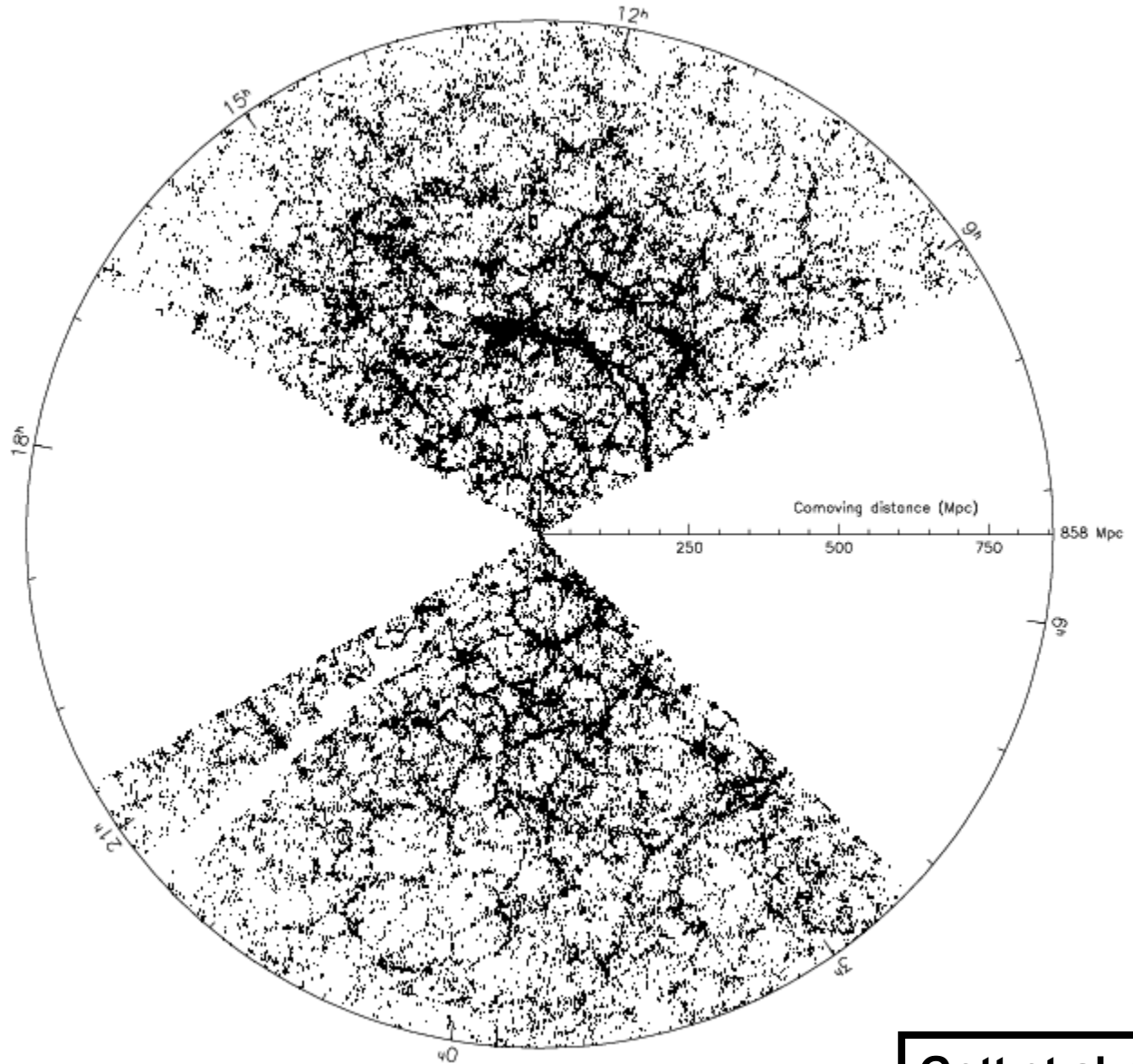
Redshift $z$	$r(z)$ (Mpc)	Remark
$\infty$	14,283	Big Bang (end of inflationary period)
3233	14,165	Equal matter and radiation density epoch
1089	14,000	Recombination
6	8,422	
5	7,933	
4	7,305	
3	6,461	
2	5,245	
1	3,317	
0.5	1,882	
0.2	809	
0.1	413	

# Universe in Comoving Distance





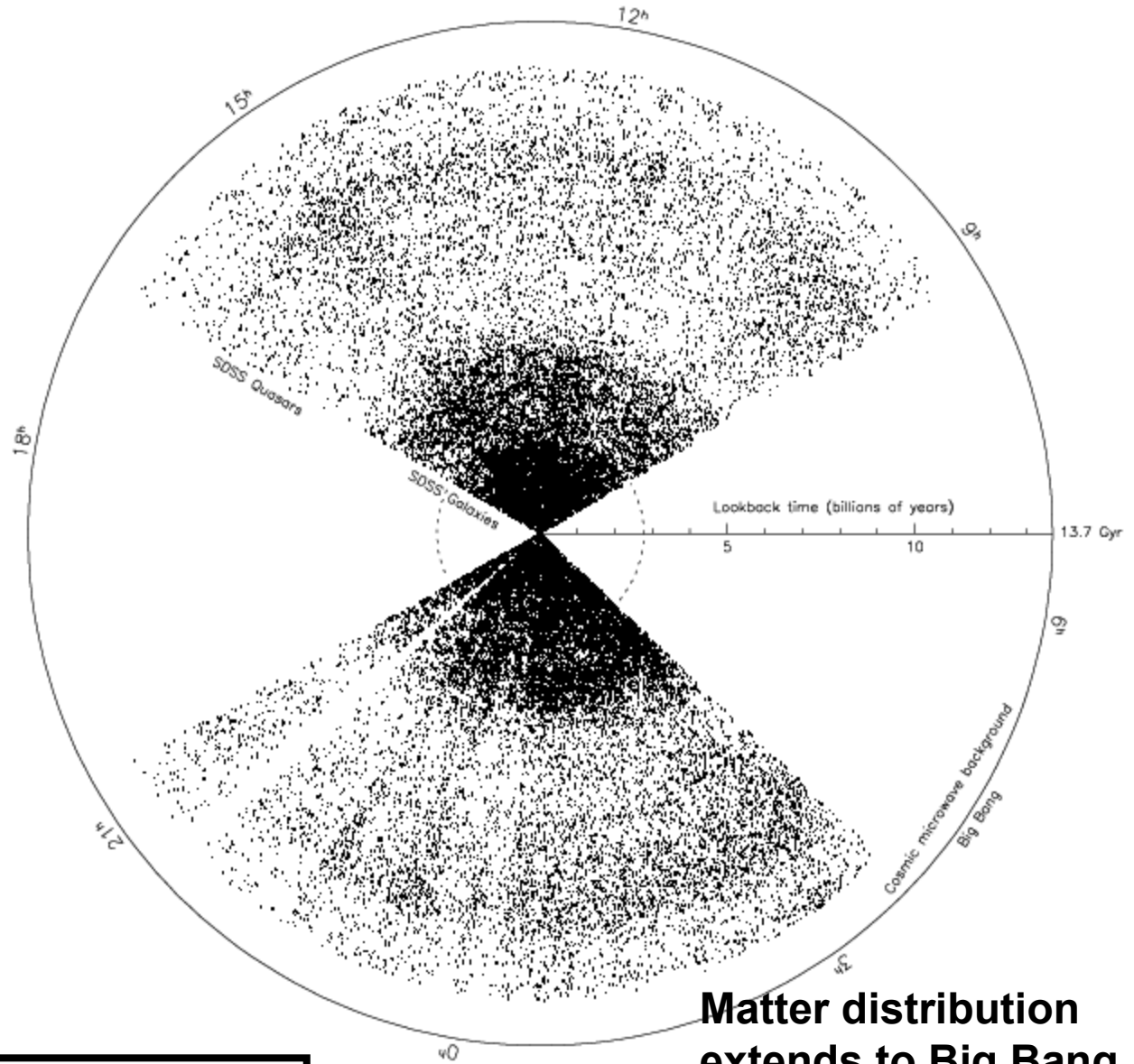
# Universe zoomed in Comoving Distance



Gott et al. 2003

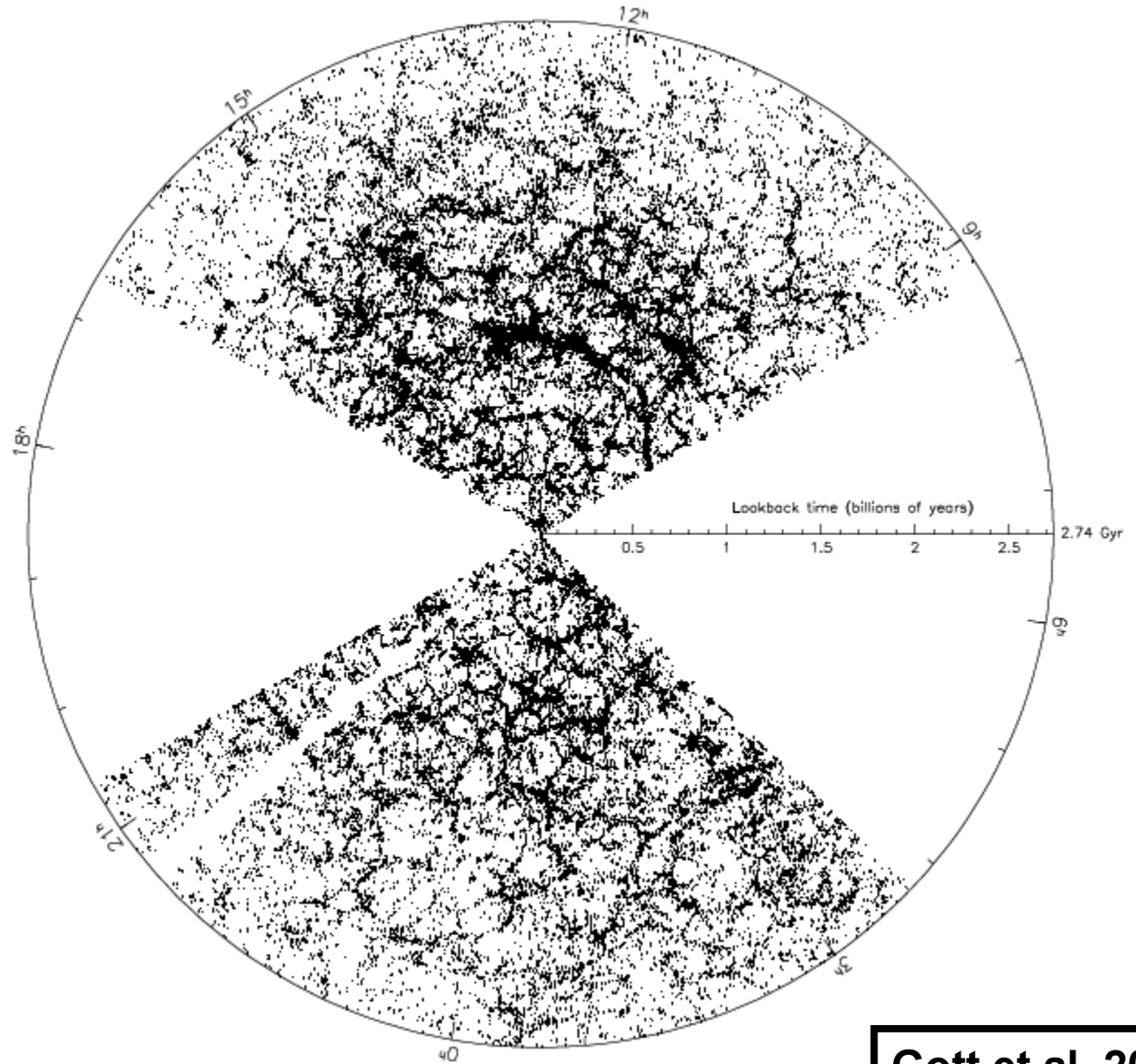
# Universe in in Look-Back Coords

Gott et al. 2003



Matter distribution  
extends to Big Bang  
→ Wrong impression

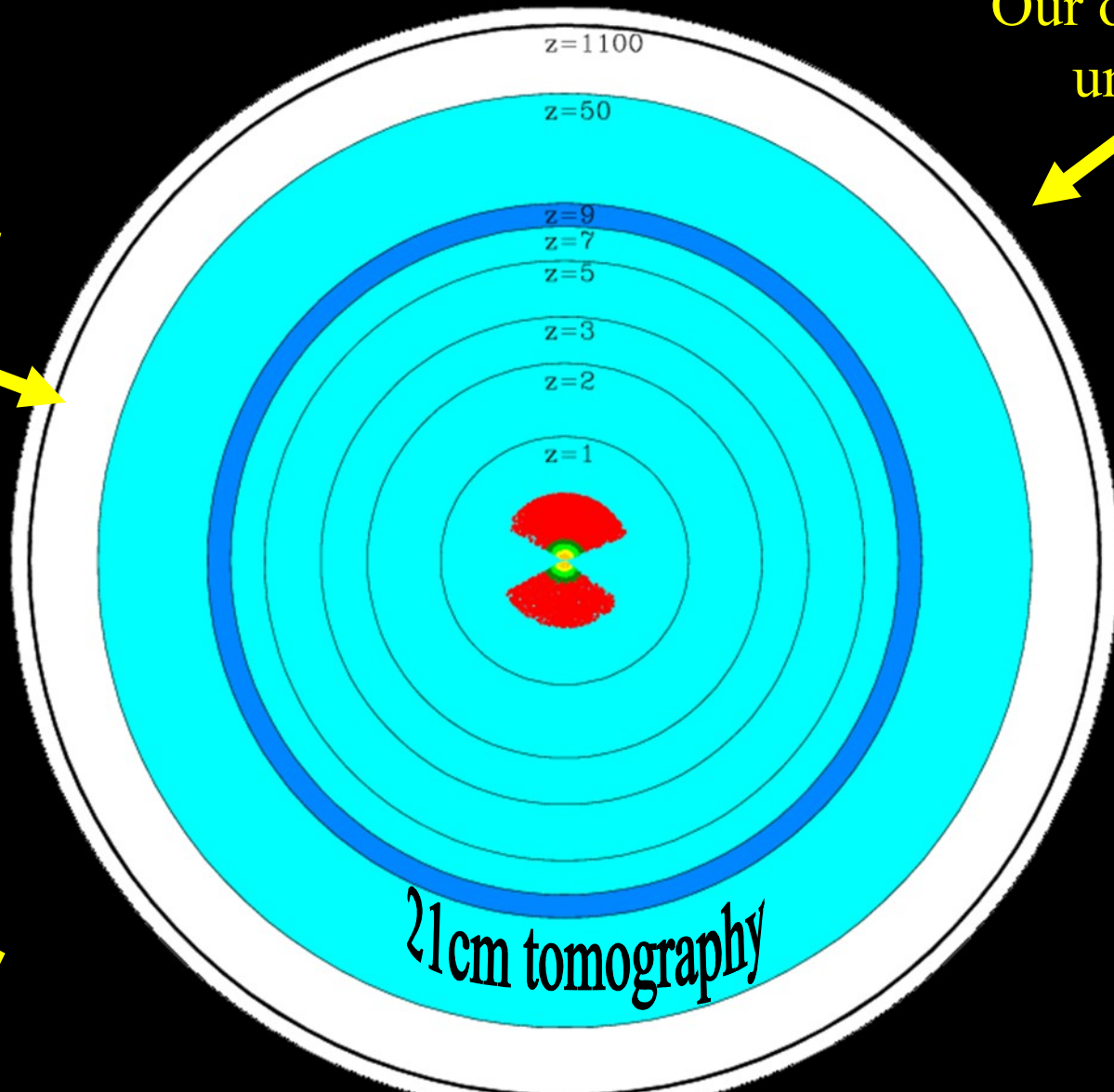
# Universe in In Look-Back Coords



Gott et al. 2003

Last scattering surface

Our observable universe



Future: 21 cm Tomography



# Unsolved Problem: What is Dark Energy ?

- **Constant of Nature**

= cosmological constant  $\Lambda$  à la Einstein? – No!

- **Vacuum Energy (Quantum effects)?**

to be expected, but at the moment not predictable

**Consequence of „quantized“ Space  
(Loop-Quantum Gravity LQ)? – yes ?**

- **Quintessence? New scalar Field  $\approx \Lambda(t)$  ?**

- **Some effect from existence of future Horizon?  
(~ Hawking radiation) - ???**

# Summary

- **ΛCDM Relativistic Cosmos** is a good approx.
- 2 Friedmann-Equations determine the **Expansion of the Universe for given EoS.**
- **Matter in the present Universe** consists of various components: **Baryons, Photons, Neutrinos, Dark Matter and Dark Energy (w).**
- Expressed in terms of **Omega-parameters.**
- Since 1997, Supernovae observations give evidence for an accelerated expansion → **Dark Energy is therefore required and  $\Omega_k \sim 0$ .**
- **EoS w** of Dark Energy is one of crucial problem.