1. A system with unperturbed eigenstates and energies ϕ_n and and E_n, respectively, is subjected to a time dependent perturbation:

$$H'(t) = \frac{e^{-t^2/\tau^2}}{\sqrt{\pi}\tau} \, A$$

where A is a *time independent operator* (not a constant) and τ is a constant.

   (a) If initially ($t = -\infty$) the system is in its ground state ϕ_0, show that, to first order, the probability amplitude that at $t = +\infty$ the system will be in its m^{th} state ($m \neq 0$) is:

$$c_m = -\frac{iA_{m0}}{\hbar} \, e^{-(E_0 - E_m)^2 \tau^2 / 4\hbar^2}$$

   where

$$A_{m0} = \langle \phi_m | A | \phi_0 \rangle$$

   (b) The limit $\tau^2 \Delta E^2 / 4\hbar^2 \gg 1$ is called the *adiabatic limit*. Why do all transition probabilities tend to zero in this limit?

   (c) Consider the limit of an impulsive perturbation: $\tau \to 0$. Show that the probability P that the system makes <u>any</u> transition whatsoever out of the ground state is:

$$P = \frac{1}{\hbar^2} \, \left[\langle \phi_0 | A^2 | \phi_0 \rangle - \langle \phi_0 | A | \phi_0 \rangle^2 \right]$$

   Hint: there is a trick to summing over almost all states!

   (d) Show the impulsive approximation is equivalent to:

$$H'(t) = A\delta(t)$$

   (e) Integrate the time dependent Schrödinger's equation over a small time interval including $t = 0$ to show:

$$\psi(t = 0^+) = \left(1 + \frac{iA}{2\hbar}\right)^{-1} \left(1 - \frac{iA}{2\hbar}\right) \psi(t = 0^-)$$

   Thus if $\psi(x,t) = \phi_0 e^{-iE_0 t/\hbar}$ initially, then for $t > 0$:

$$\psi(x,t) = \sum c_m \phi_m e^{-iE_m t/\hbar}$$

   where:

$$c_m = \langle \phi_m | \left(1 + \frac{iA}{2\hbar}\right)^{-1} \left(1 - \frac{iA}{2\hbar}\right) | \phi_0 \rangle$$

   Show that this reduces to first-order perturbation if the perturbation is weak.