6.2 — sho — $\mathbb{H} = \underbrace{\frac{p^2}{2m} + \frac{1}{2}k}_{H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2}(1+\varepsilon)x^2 \longrightarrow V = \frac{1}{2}\underset{\uparrow\, m\omega^2}{k}\, x^2\, \varepsilon$

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$|n\rangle \longleftrightarrow \hbar\omega(n+1/2)$

$V = \varepsilon \frac{1}{2} m\omega^2 x^2 \longleftarrow x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$

$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

$x^2 = \frac{\hbar}{2m\omega}(a_+ + a_-)^2$

$= \frac{\hbar}{2m\omega}\left(a_+^2 + a_+a_- + a_-a_+ + a_-^2\right)$

$\mathbb{E}_1 = \langle n|V|n\rangle = \varepsilon \frac{1}{2} m\omega^2 \frac{\hbar}{2m\omega} \langle n|\underset{0}{a_+^2} + \underbrace{\underset{\sqrt{n}^2}{a_+a_-} + \underset{\sqrt{n+1}^2}{a_-a_+}}_{2n+1} + \underset{0}{a_-^2}|n\rangle$

$= \varepsilon \frac{\hbar\omega}{2}(n+1/2)$

$\mathbb{E}_2 = \sum_{m\neq n} \frac{\langle m|V|n\rangle^2}{E_n - E_m}$

$m = n+2$ connects via a_+^2
$m = n-2$ connects via a_-^2
all other $m \Rightarrow 0$

$= \frac{|\langle n+2|V|n\rangle|^2}{-2\hbar\omega} + \frac{|\langle n-2|V|n\rangle}{2\hbar\omega}$

$\langle n+2|V|n\rangle = \varepsilon \frac{1}{2} m\omega^2 \frac{\hbar}{2m\omega} \langle n+2|\underset{\sqrt{(n+2)(n+1)}}{a_+^2} + \cdots\ |n\rangle$

$\langle n-2|V|n\rangle = \underbrace{\varepsilon \frac{1}{2} m\omega^2 \frac{\hbar}{2m\omega}}_{\frac{\varepsilon\hbar\omega}{4}} \langle n-2| \cdots \underset{\sqrt{(n-1)n}}{a_-^2}|n\rangle$

$\mathbb{E}_2 = \frac{\left(\frac{\varepsilon\hbar\omega}{4}\right)^2}{2\hbar\omega}\left\{\frac{(n+2)(n+1)}{-1} + \frac{(n-1)(n)}{1}\right\}$

$= \frac{\varepsilon^2\hbar\omega}{32}\left\{-4n-2\right\} = -\frac{\varepsilon^2\hbar\omega}{8}(n+1/2)$

Compare $\underset{\substack{\uparrow \\ \text{exact}}}{\mathbb{E}} = \hbar\omega\sqrt{1+\varepsilon}\,(n+1/2) = \hbar\omega(n+1/2)\left[1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2\right]$ ✓

6.3

non interacting $\Rightarrow$ product wavefunctions & $E = E_1 + E_2$

$$u_n = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n}{L} x\right) \qquad E = \frac{\hbar^2}{2m}\left(\frac{\pi n}{L}\right)^2$$

gs: $\Psi(x_1, x_2) = u_1(x_1)\, u_1(x_2) = \frac{2}{L} \sin\left(\frac{\pi}{L} x_1\right) \sin\left(\frac{\pi}{L} x_2\right)$

excited $= \frac{1}{\sqrt{2}}\left(u_1(x_1)\, u_2(x_2) + u_2(x_1)\, u_1(x_2)\right)$

$$E_1 \text{ ground} = \langle gs | V | gs \rangle = \iint \left(\sqrt{\tfrac{2}{L}}^{\,2} \sin\left(\tfrac{\pi}{L} x_1\right) \sin\left(\tfrac{\pi}{L} x_2\right)\right)^2 (-aV_0 \delta)$$

$$= -aV_0 \int \left(\sqrt{\tfrac{2}{L}}^{\,2} \sin\left(\tfrac{\pi}{L} x_1\right) \sin\left(\tfrac{\pi}{L} x_1\right)\right)^2 dx_1$$

$$= -\frac{aV_0\, 2^2}{L^2} \underbrace{\int_0^L \sin^4\left(\tfrac{\pi}{L} x_1\right) dx_1}_{\text{math} \rightarrow \frac{3}{8} L} = -\frac{aV_0}{L}\,\frac{3}{2}$$

$$E_1 \text{ excited} = \langle \text{excited} | V | \text{excited} \rangle$$

$$= \frac{1}{2} \iint \left(u_1(x_1) u_2(x_2) + u_2(x_1) u_1(x_2)\right)^2 (-aV_0 \delta)$$

$$= \frac{1}{2} \int_0^L \left(u_1(x_1) u_2(x_1) + u_2(x_1) u_1(x_1)\right)^2 (-aV_0)$$

$$= -2aV_0 \int_0^L u_1^2(x_1)\, u_2^2(x_1)\, dx_1$$

$$= -2aV_0 \int_0^L \frac{2}{L} \sin\left(\tfrac{\pi}{L} x_1\right) \frac{2}{L} \sin\left(\tfrac{2\pi}{L} x_1\right) dx_1$$

$$= -\frac{8aV_0}{L^2} \underbrace{\int_0^L \sin\left(\tfrac{\pi}{L} x\right) \sin\left(\tfrac{2\pi}{L} x\right) dx}_{\text{math} \rightarrow \frac{L}{4}}$$

$$= -\frac{2aV_0}{L}$$

Old exam 346t313 #6 #31

degenerate wf: $\psi_{12} = \frac{2}{L}\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)$ $\psi_{21} = \frac{2}{L}\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{\pi y}{L}\right)$

seek: $\langle\psi_{12}|V|\psi_{12}\rangle$ $\langle\psi_{12}|V|\psi_{21}\rangle$ $V = \lambda\,\delta(x-\frac{L}{3})\,\delta(y-\frac{L}{4})$

$\langle\psi_{21}|V|\psi_{21}\rangle$

$|\psi_{12}|^2\Big|_{\substack{x=L/3\\ y=L/4}}$ $\psi_{12}\psi_{21}\Big|_{\substack{x=L/3\\ y=L/4}}$ $|\psi_{21}|^2\Big|_{\substack{x=L/3\\ y=L/4}}$

$$\psi_{12}\Big|_{\substack{x=L/3\\ y=L/4}} = \frac{2}{L}\sin\left(\frac{\pi}{L}\frac{L}{3}\right)\sin\left(\frac{2\pi}{L}\frac{L}{4}\right) = \frac{2}{L}\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{2}\right) = \frac{2}{L}\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{L}$$

$$\psi_{21}\Big|_{\substack{x=L/3\\ y=L/4}} = \frac{2}{L}\sin\left(\frac{2\pi}{L}\frac{L}{3}\right)\sin\left(\frac{\pi}{L}\frac{L}{4}\right) = \frac{2}{L}\sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = \frac{2}{L}\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}$$
$$= \frac{1}{L}\sqrt{\frac{3}{2}}$$

so $\langle\psi_{12}|V|\psi_{12}\rangle = \lambda\left(\frac{\sqrt{3}}{L}\right)^2 = \frac{3\lambda}{L^2}$ $\langle\psi_{12}|V|\psi_{21}\rangle = \lambda\frac{\sqrt{3}}{L}\frac{1}{L}\sqrt{\frac{3}{2}} = \lambda\frac{3}{\sqrt{2}}\frac{1}{L^2}$

$\langle\psi_{21}|V|\psi_{21}\rangle = \lambda\left(\frac{1}{L}\sqrt{\frac{3}{2}}\right)^2 = \frac{\lambda}{L^2}\frac{3}{2}$

$$\text{matrix} = \frac{\lambda}{L^2}\begin{pmatrix} 3 & 3/\sqrt{2} \\ 3/\sqrt{2} & 3/2 \end{pmatrix}$$

$$= \frac{\lambda}{L^2}\frac{3}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{2}-x & 1 \\ 1 & \frac{1}{\sqrt{2}}-x \end{pmatrix} = (\sqrt{2}-x)\left(\frac{1}{\sqrt{2}}-x\right) - 1$$
$$= x^2 - \left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)x$$
$$= x\left(x - \left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)\right)$$

$x = 0$ eigenvector $= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}}\psi_{12} - \psi_{21}$

$E = E_0 + 0$

$x = \sqrt{2}+\frac{1}{\sqrt{2}}$ eigenvector $= \begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix} \longrightarrow \sqrt{2}\,\psi_{12} + \psi_{21}$

$E = E_0 + \left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)\frac{\lambda}{L^2}\frac{3}{\sqrt{2}}$

Remark: If you plot this ψ
you find its put a zero
on top of the δ function