

Fourier Trans

① unit sphere $\rho = \frac{3}{4\pi R^3}$ $\leftarrow q = 2k \sin \frac{\theta}{2}$ eventually

$$\begin{aligned}
 FT &= \frac{3}{4\pi R^3} \int_0^R \int_0^\pi \int_0^{2\pi} 1 e^{i\vec{k}\cdot\vec{r}} r^2 dr d\theta d\phi \\
 &= \frac{3}{4\pi R^3} \int_0^R \sin(kr) r dr = \frac{3}{k^3 R^3} \int_0^{kR} \sin(u) u du \\
 &= \frac{3}{k^3 R^3} \left[\sin u - u \cos u \right]_0^{kR} = 3 \left[\frac{\sin(kR)}{k^3 R^3} - \frac{\cos(kR)}{k^2 R^2} \right]
 \end{aligned}$$

$\xrightarrow{2\pi}$
 $\xrightarrow{\frac{e^{ikr} - e^{-ikr}}{ikr}}$

Dörst
 430.11

Note: $\frac{\sin x}{x} - \cos x = \left(1 - \frac{x^2}{3!}\right) - \left(1 - \frac{x^2}{2!}\right) = \frac{x^2}{3} \rightarrow F(0) = 1$

next order: $\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right) = -\frac{x^4}{30}$

$\rightarrow F = 1 - \frac{1}{10} (kR)^2$

Note $\langle r^2 \rangle = \frac{\int r^4 dr 4\pi}{\int r^2 dr 4\pi} = \frac{\frac{1}{5} R^5}{\frac{1}{3} R^3} = \frac{3}{5} R^2$

$\frac{1}{6} \langle r^2 \rangle = \frac{1}{10} R^2$

② Yukawa / Rutherford $V = \frac{ZeZ'e^2}{4\pi\epsilon_0} \frac{e^{-\mu r}}{r}$

$$\begin{aligned}
 FT &= \frac{ZeZ'e^2}{4\pi\epsilon_0} \int_0^\infty \int_0^\pi \int_0^{2\pi} e^{-\mu r} r e^{i\vec{k}\cdot\vec{r}} dr d\theta d\phi \\
 &= \frac{ZeZ'e^2}{\epsilon_0 k} \int_0^\infty e^{-\mu r} \sin(kr) dr \\
 &= \frac{ZeZ'e^2}{\epsilon_0 k^2} \text{Im} \left(e^{-(\mu - ik)r} \right) \rightarrow \text{Im} \left[\frac{1}{\mu - ik} \right]
 \end{aligned}$$

$\xrightarrow{2\pi}$
 $\xrightarrow{\frac{e^{ikr} - e^{-ikr}}{ikr}}$

$\approx \frac{k}{\mu^2 + k^2}$

$x = \frac{\mu}{2\pi k^2} \rightarrow -\frac{ZeZ'e^2}{4\pi\epsilon_0} \frac{k^2 h^2}{2\mu} \rightarrow -\frac{ZeZ'e^2}{4\pi\epsilon_0 E} (2 \sin^2 \frac{\theta}{2})^2$

Fig. 4-5 The best-fitting Fermi density for gold gives this excellent agreement with experiment. (After Hofstadter [5].)

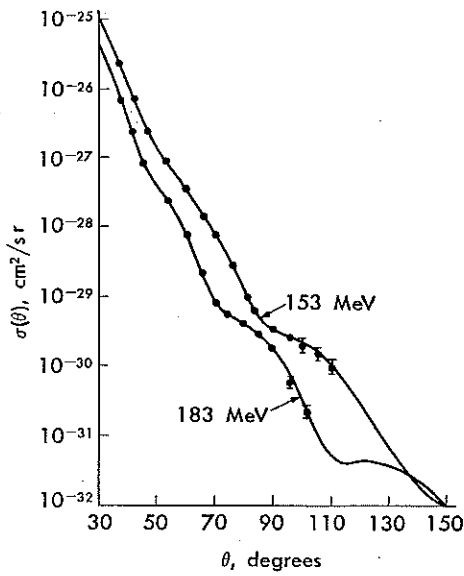


Fig. 4-8 Electron elastic-scattering form factor for ^{16}O . The experimental data of McCarthy and Sick [24] are shown by dots. The full line is the theoretical fit obtained by assuming a closed-shell configuration and using harmonic-oscillator wave functions; the dashed curve is obtained from Woods-Saxon wavefunctions. (After Donnelly and Walker [25].)

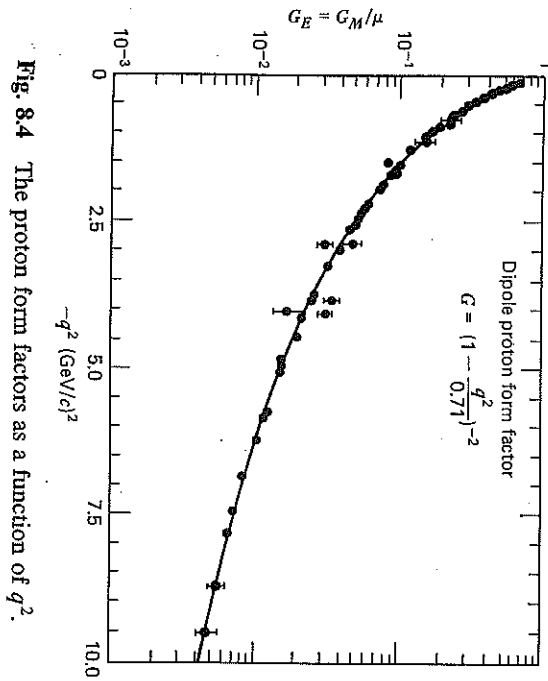
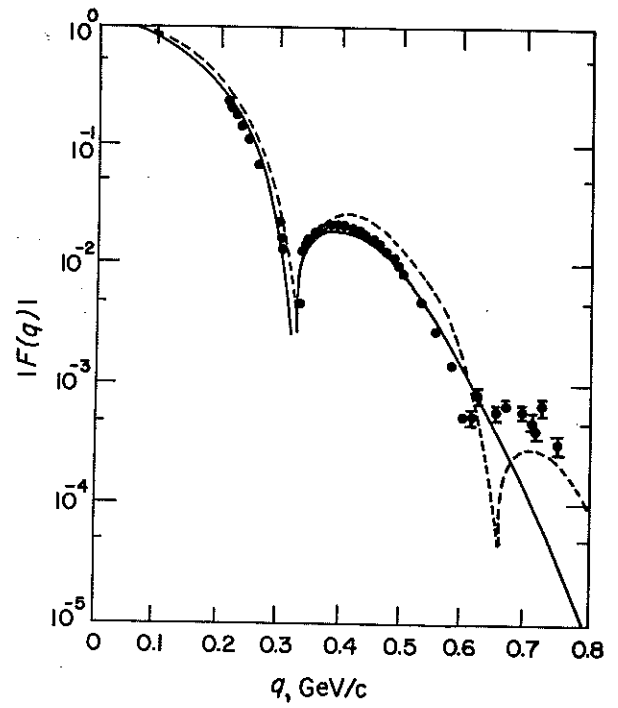


Fig. 8-4 The proton form factors as a function of q^2 .

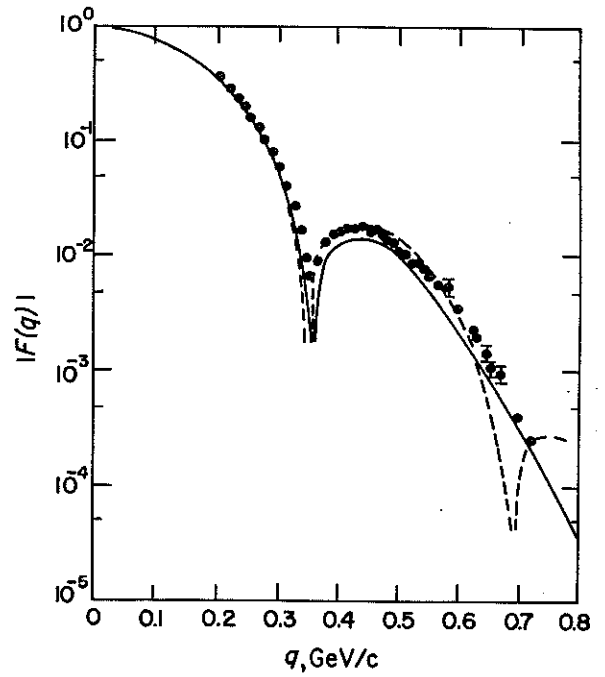


Fig. 4-9 Same as in Fig. 4-8, but for ^{12}C . (After Donnelly and Walker [25].)

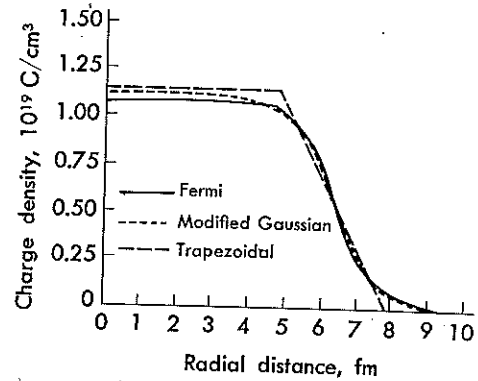
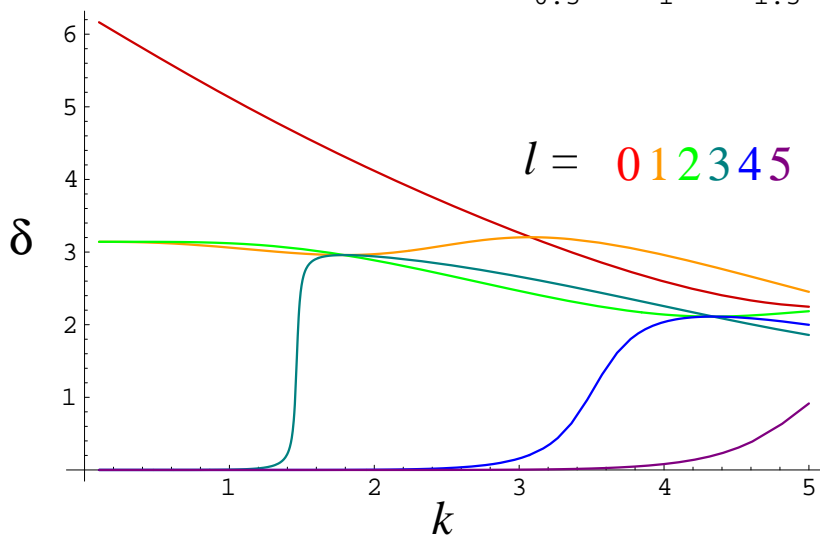
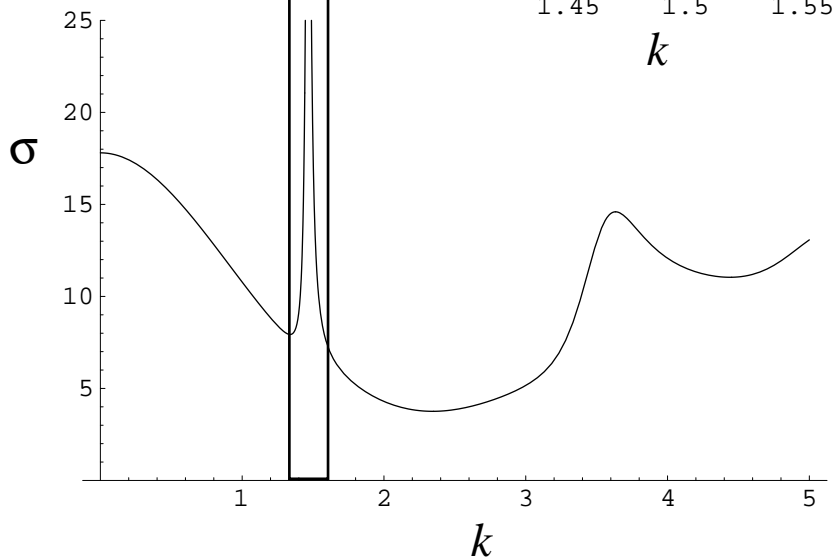
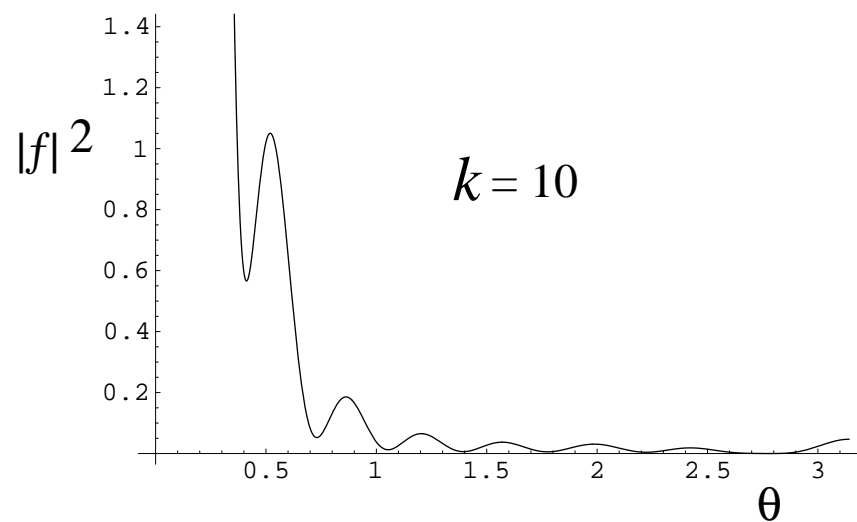
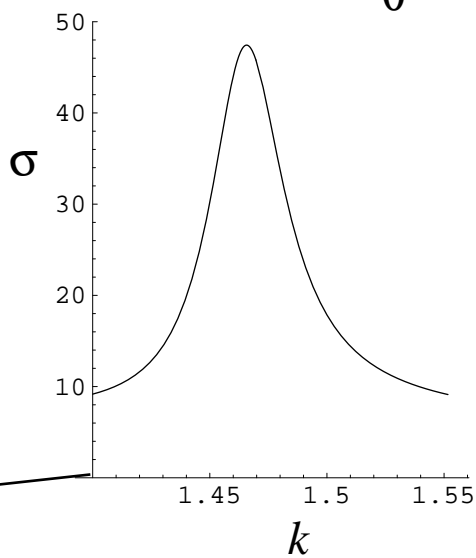
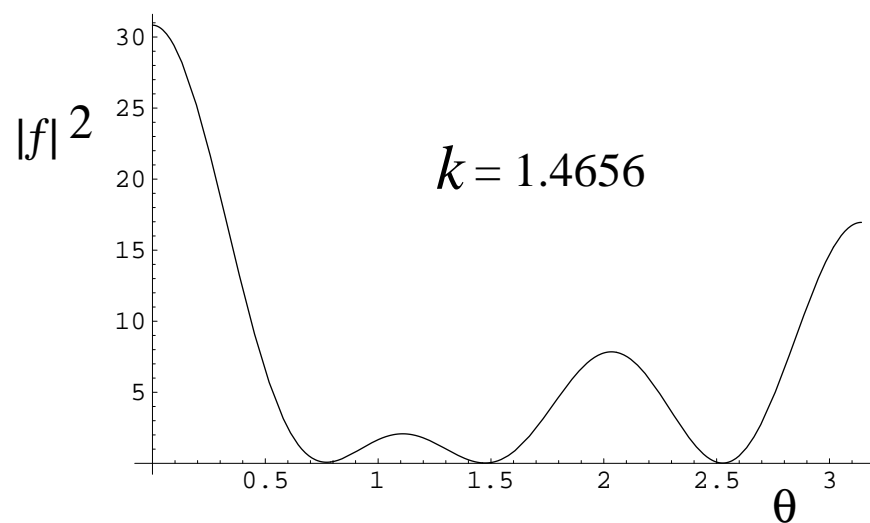
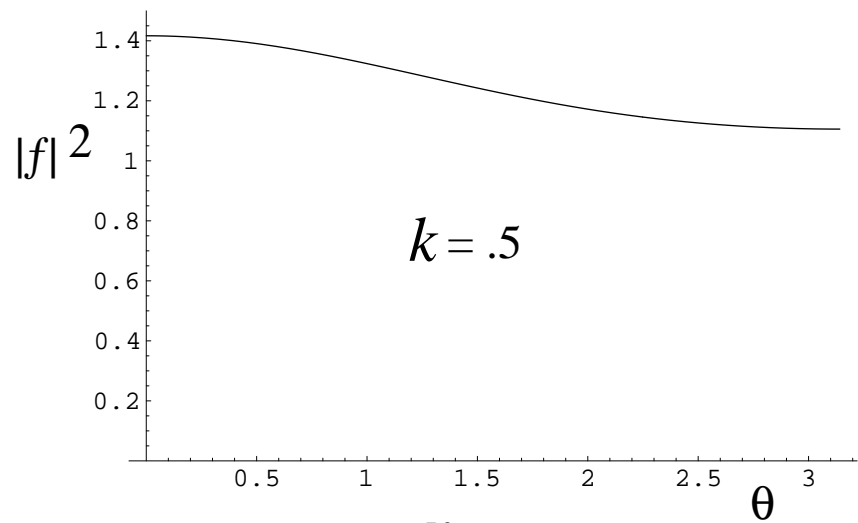


Fig. 4-6 Densities for gold, all of which fit the electron-scattering data. (After Hofstad [5].)



Partial Wave vs Born Approximation

