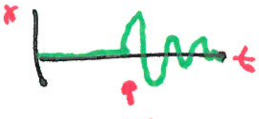
Recall Mechanics Greens Function ... solve SHO hit with hammer

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \delta(t-t') \leftarrow \text{hammer blow @ } t=t'$$

↖ Call solution $G(t,t') =$



then $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$ has solution:

$G = 0$ if $t' > t$

$$x(t) = \int_{-\infty}^{t} f(t')\,G(t,t')\,dt' = \int_{-\infty}^{\infty} f(t')\,G(t,t')\,dt'$$

as $$\left(\frac{d^2}{dt^2} + 2\beta\frac{d}{dt} + \omega_0^2\right)x = \int f(t')\underbrace{\left[\frac{d^2}{dt^2} + 2\beta\frac{d}{dt} + \omega_0^2\right]G}_{\delta(t-t')}\,dt'$$

$= f(t)$ ✓

Idea: write SE as

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \leftarrow \frac{\hbar^2 k^2}{2m}$$

$$\underbrace{\boxed{\frac{2m}{\hbar^2}V\psi}}_{\text{define as } Q} = (\nabla^2 + k^2)\psi$$

Solve:

$$(\nabla^2 + k^2)\psi = \delta^3(\vec{r} - \vec{r}_0)$$

↑ call solution $G(\vec{r},\vec{r}_0)$

Then $$\psi = \int_{\text{all space}} Q(\vec{r}_0)\,G(\vec{r},\vec{r}_0)\,d^3r_0$$ ↖ Free variable

$\therefore$ $$(\nabla^2 + k^2)\psi = \int Q\,(\nabla^2 + k^2)G\,d^3r_0$$

$$= \int Q\,\delta(\vec{r} - \vec{r}_0)\,d^3r_0 = Q(r)$$

Note: not quite this easy as Q involves ψ & we don't know ψ [if we did we would not be trying to solve for it]. Guess scattered wave is small fraction of beam $= e^{ikz} = \psi_0$

↖ This is solution to $V=0$ Schrodinger ... ie $(\nabla^2 + k^2)\psi_0 = 0$

ie homogeneous solution

iterate:

$$\psi_1 = \psi_0 + \int Q(r_0)\,G(\vec{r},\vec{r}_0)\,d^3r_0$$

↖ use ψ_0 here

This ψ will satisfy: $(\nabla^2+k^2)\psi_1 = \frac{2m}{\hbar^2} V \underbrace{\psi_0}_{\text{not } \psi}$

Perhaps improve by iteration:

$$\psi_2 = \psi_0 + \int \overset{\text{use } \psi_0}{Q(r_0)}\, G(r, r_0)\, d^3r_0$$

use $\psi_1 = \psi_0 + \int Q\, G\, d^3r_0$ here

We skip the beautiful complex variable / Cauchy Residue work that finds:

$$G(\vec{r}, \vec{r}_0) = -\frac{e^{i\boxed{|\vec{r}-\vec{r}_0|k}}}{4\pi \boxed{|\vec{r}-\vec{r}_0|}}$$

r_0 integrates over potential & $k = \frac{2\pi}{\lambda}$

if λ smaller than R this term will vary a lot

if r = macroscopic distance to detector $|\vec{r}-\vec{r}_0|$ is essentially the same

$$|\vec{r}-\vec{r}_0|^2 = (\vec{r}-\vec{r}_0)\cdot(\vec{r}-\vec{r}_0) = r^2 - 2r\cdot r_0 + r_0^2 \approx r^2\left(1 - 2\frac{\vec{r}\cdot\vec{r}_0}{r^2}\right)$$

$$|r-r_0| = r\sqrt{1 - 2\frac{\vec{r}\cdot\vec{r}_0}{r^2}} \approx r\left(1 - \frac{\vec{r}\cdot\vec{r}_0}{r^2}\right) = r - \hat{r}\cdot\vec{r}_0$$

direction to detector

nucleus scale

$$G = \frac{-e^{i|\vec{r}-\vec{r}_0|k}}{4\pi|r-r_0|} \approx -\frac{e^{ikr}}{4\pi r}\, e^{-i\underbrace{k\hat{r}}\cdot\vec{r}_0}$$

$k\hat{r} \equiv \vec{K}$ ... the way the scattered particle moving toward detector

Put together:

$$\psi_1 = e^{i\underbrace{kz}} - \frac{2m}{\hbar^2}\int V(r_0)\,\psi_0(r_0)\, e^{-i\vec{K}\cdot\vec{r}_0}\, \frac{e^{ikr}}{4\pi r}\, d^3r_0$$

more generally $\vec{k}'\cdot\vec{r}$ where $\vec{k}'$ is incoming wave

$$\underbrace{\frac{-m}{2\pi\hbar^2}\int V(r_0)\, e^{i(\vec{k}'-\vec{K})\cdot r_0}\, d^3r_0}_{f(\theta)}\ \frac{e^{ikr}}{r}$$

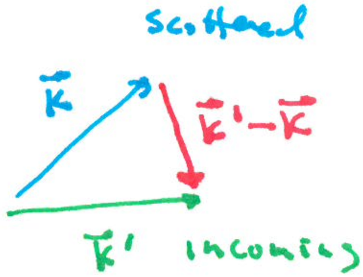
$e^{i\vec{k}'\cdot\vec{r}}$

$|\vec{k}'| = |\vec{K}|$ as energy conserved

Note: the Fourier Transform of V would be

$$\tilde{V}(q) = \int V(r_0)\, e^{i\vec{q}\cdot\vec{r}_0}\, d^3r_0$$

So $f(\theta) = \frac{-m}{2\pi\hbar^2}\tilde{V}(\underbrace{\overrightarrow{k'-k}}_{\text{called the "momentum transfer"}})$



Often Fourier Transform just depends on $|\vec{q}|$

$|\overrightarrow{k'-k}| = 2k\sin(\theta/2)$

Remark: if λ so large (i.e. q so small) that $\frac{r_0}{\lambda} \sim 0$

then $\tilde{V}(q) = \int V(r_0)\, e^0\, d^3r_0 = \int V(r_0)\, d^3r_0$

& $f(\theta) = \frac{-m}{2\pi\hbar^2}\int V(r_0)\, d^3r_0$ ← isotropic

If λ so small (i.e. q so large) that $\frac{r_0}{\lambda}$ oscillates rapidly over $V \rightarrow \tilde{V}$ small

For given k, largest q is $\theta = 180$ - backscatter - $\sin(\frac{\theta}{2}) = 1$

so generally speaking backscatter is small for large k.