$$|c_b|^2 = \frac{|\tilde V_{ba}|^2}{4\hbar^2}\frac{\sin^2(\Xi t)}{\Xi^2} \qquad \Xi = \frac{\omega_0-\omega}{2} \qquad V_{ab} = \tilde V_{ab}\cos(\omega t)$$

$$\omega_0 = \omega_b - \omega_a$$

$p \leftarrow$ dipole

Electric dipole: $V = -E_0\cos(\omega t)\,(\overbrace{qz}^{p})$

Seek: absorption / stimulated emission from incoherent (random) light of a thermal (Planck) source.

energy density $= u = \frac{\varepsilon_0}{2}E_0^2$

energy at a range of freq: $\rho\, d\omega$

$$P_{\substack{a\to b\\ b\to a}} = \frac{E_0^2|\vec p|^2}{4\hbar^2}\frac{\sin^2\Xi t}{\Xi^2} \to \frac{|\vec p|^2}{\hbar^2\varepsilon_0}\int\rho\,\underbrace{\frac{\sin^2\Xi t}{\Xi^2}\,d\Xi}_{\pi t}$$

$\rho\, d\omega\,\frac{2}{\varepsilon_0}$

$2\,d\Xi$

$\rho(\omega_0)$

seek avg over all possible polarizations

put $\vec p$ on z axis $\quad \vec p\cdot\hat n = |p|\cos\theta$

$$\frac{\iint\cos^2\theta\sin\theta\,d\theta\,d\phi}{\int\sin\theta\,d\theta\,d\phi} = \frac{1}{3}$$

$$P = \frac{|p|^2}{3\hbar^2\varepsilon_0}\rho(\omega)\,\pi t$$

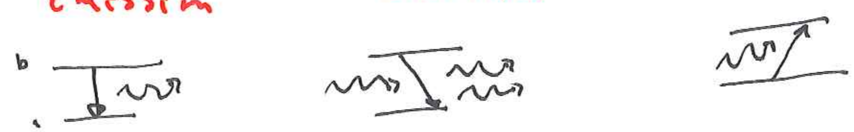
Einsteins Argument

$$\frac{dN_b}{dt} = -N_bA - N_bB_{ba}\rho(\omega) + N_aB_{ab}\rho(\omega) = 0$$

spontaneous emission

stimulated emission

absorption



atoms in thermal equilibrium with light: $\frac{N_b}{N_a} = e^{-\frac{\Delta E}{kT}}$ $\quad \hbar\omega_0$

$$\text{Planck: } \rho(\omega) = \frac{\hbar}{\pi^2c^3}\frac{\omega^3}{e^{\hbar\omega/kT}-1}$$

$$\rho(\omega_0) = \frac{N_bA}{N_aB_{ab}-N_bB_{ba}} = \frac{A}{\frac{N_a}{N_b}B_{ab}-B_{ba}}$$

$e^{+\hbar\omega/kT}$

so $B_{ba}=B_{ab}$ & $\frac{A}{B} = \frac{\hbar}{\pi^2c^3}\omega^3$

$$\Rightarrow \quad A = \frac{\hbar}{\pi^2 c^3}\omega^3 \left[B = \frac{\pi |\bar{p}|^2}{3\hbar^2 \varepsilon_0} \right]$$

$$= \frac{\omega_0^3 |p|^2}{3\pi\varepsilon_0 \hbar c^3}$$

Unit check / estimate : $\hbar\omega_0 = \frac{1}{2}\alpha^2 mc^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$p \sim e\,a = e\,\frac{r_c}{\alpha}$$

$$\frac{(\alpha^2 mc^2)^3 \, e^2 \left(\frac{r_c}{\alpha}\right)^2}{3\pi\varepsilon_0 \hbar^4 c^3} \sim \alpha^{6+1-2} \left(\frac{mc}{\hbar}\right)^3 \frac{c^3}{c^2} r_c^2$$

$$= \alpha^5 \frac{c}{r_c}$$

$$\tau = \frac{1}{A} = \frac{r_c}{\alpha^5 c} \sim 10^{-10}\,\mathrm{s}$$

Classical: $\frac{e^2 a^2}{6\pi\varepsilon_0 c^3}\tau = \alpha^2 mc^2$

$$a = \frac{e^2}{4\pi\varepsilon_0 a^2 m} = \alpha \frac{\hbar c}{m} \frac{1}{(r_c/\alpha)^2}$$

$$= \alpha^3 \frac{c^2}{r_c}$$

$$\alpha \frac{\hbar}{c^2}\left(\alpha^3 \frac{c^2}{r_c}\right)^2 \tau = \alpha^2 mc^2$$

$$\tau = \frac{1}{\alpha^5} \frac{m}{\hbar} r_c^2$$

$$= \frac{r_c}{\alpha^5 c}$$