normalized solutions to infinite square well: $\psi_n = \sqrt{\frac{2}{L}} \sin(k_n x)$

$k_n = \frac{n\pi}{L}$

$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2}{2mL^2} n^2$

$\omega_n = E_n/\hbar$

$H\psi_n = E_n \psi_n$ ← TISE

$\Psi_n = \psi_n e^{-i\omega_n t}$ ← use superposition for general solutions

$i\hbar \partial_t \Psi_n = H \Psi_n$ ← TDSE

Characteristics of these solutions (will generalize!)

→ alternate even/odd (about well midpoint) with ground state even

→ up a step in energy; add a node

→ orthogonal & normalized: $\langle \psi_n | \psi_m \rangle = \delta_{nm}$

→ Complete - any function can be expressed as a linear combination of $\{\psi_n\}$ - Fourier

Fouriers Trick: $f(x) = \sum c_n \psi_n \Rightarrow \langle \psi_m | f \rangle = c_m$

→ $|c_n|^2$ can be thought of as the probability particle will be measured to have energy E_n ("ie in state ψ_n")

$\sum |c_n|^2 = 1$; $\langle H \rangle = \langle E \rangle = \sum |c_n|^2 E_n$

→ initial value problem: given $\psi(x)$ starts ($t=0$) as $f(x)$ what is its future ie $\Psi(x,t)$

$\Psi(x,t) = \sum c_n \psi_n e^{-i\omega_n t}$

← constants

Eg: $\Psi = \frac{1}{\sqrt{2}} (\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t})$

← normalization determined from $\langle \Psi | \Psi \rangle = 1$

$|A|^2 = \frac{1}{2}$ ⇒ choice ⇒ $A = \frac{1}{\sqrt{2}}$

Lemma: $\langle \psi_1 | \psi_2 \rangle^* = \langle \psi_2 | \psi_1 \rangle$

result: probability sloshes back & forth at angular frequency $= (\omega_2 - \omega_1)$

$\langle x \rangle = \frac{L}{2} - \frac{32}{9\pi^2} L \cos[(\omega_2 - \omega_1)t]$

$\langle \psi_1 | x | \psi_2 \rangle = \frac{-16}{9\pi^2} L$