1) $\int_{-\infty}^{\infty} \left(x e^{-bx^2}\right)^2 \left[-\delta(x-a) - \delta(x+a)\right] dx = -2\left(a e^{-ba^2}\right)^2$

$\int_{-\infty}^{\infty} x^2 e^{-2bx^2} \longrightarrow \frac{-d}{d\alpha}\int_{-\infty}^{\infty} e^{-\alpha x^2} = \frac{-d}{d\alpha}\sqrt{\frac{\pi}{\alpha}} = \frac{1}{2}\frac{\sqrt{\pi}}{\alpha^{3/2}} = \frac{1}{2}\frac{\sqrt{\pi}}{(2b)^{3/2}}$

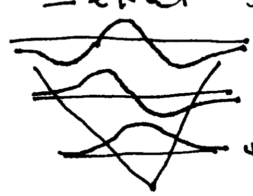
for $\alpha = 2b$

First excited state energy $< -.065$

$f = e^{-bx^2}$ or $e^{-b(x-a)^2} + e^{-b(x+a)^2}$ symmetric

2) — 2^{nd} ord
— RR
— exact

RR must be above exact and hence closer to exact

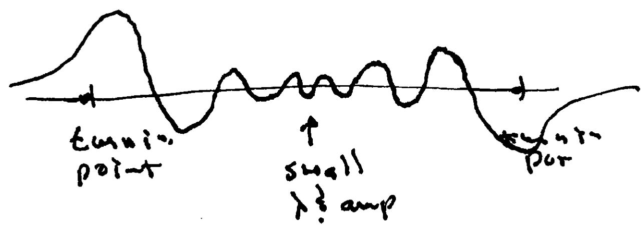
3) 

ψ_3 even

ψ_2 odd

ψ_1 even

zero: $\langle \psi_1 | \psi_2 \rangle$ $\langle \psi_3 | x | \psi_1 \rangle$



Key

4) A B
C D

	a	b	some
A	-2	2	0
B	-2	1	$-\frac{1}{4}$
C	-1.1	+1.1	$-\frac{1}{2}$ ← will tunnel out
D	-1	+1	$-1/2$

$n = 6 \Rightarrow 5$ zeros

5) starting in state ψ_a @ $t=0$, $|c_b|^2$ = prob in state b

energy E_a, energy E_b

$H'_{ba} = \langle \psi_b | H' | \psi_a \rangle$ time dep

$H' = V\cos(\omega t)$ freq of perturbation $\qquad \omega_0 = \frac{E_b - E_a}{\hbar}$

$V_{ab} = \langle \psi_a | V | \psi_b \rangle$

$P_{a\to b} = |c_b|^2$

6) $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$ $\qquad H' = \lambda \langle n | x^2 | n \rangle = \lambda \frac{\hbar}{2m\omega}(2n+1) \xrightarrow[n=0]{} \lambda \frac{\hbar}{2m\omega}$

$\frac{\hbar}{2m\omega}(a_+^2 + a_-^2 + \overbrace{a_+a_-}^{n} + \overbrace{a_-a_+}^{n+1})$

in 2^{nd} order gs only connects to $n=2$: $H' = \lambda \frac{\hbar}{2m\omega}\langle 2 | a_+^2 | 0 \rangle = \frac{\lambda \hbar}{\sqrt{2} m\omega}$ ($\to \sqrt{2}$)

$E^2 = -\frac{\left(\frac{\lambda \hbar}{\sqrt{2} m\omega}\right)^2}{2\hbar\omega}$

7) Let $\psi_{12} = A \qquad \psi_{21} = B$

$$V_{AA} = \int \psi_{12}\left[\lambda\,\delta(x - L/3)\,\delta(y - L/4)\right]\psi_{12}\,dx\,dy = \lambda\left(\frac{2}{L}\right)^2 \underbrace{\sin^2\frac{\pi}{3}}_{\frac{3}{4}}\ \underbrace{\sin^2\frac{2\pi}{4}}_{1}$$

$$= \lambda\left(\frac{2}{L}\right)^2 \frac{3}{4}$$

$$V_{BB} = \int \psi_{21}\left[\lambda\,\delta(x - \tfrac{L}{3})\,\delta(y - \tfrac{L}{4})\right]\psi_{21}\,dx\,dy = \lambda\left(\frac{2}{L}\right)^2 \underbrace{\sin^2\frac{2\pi}{3}}_{3/4}\ \underbrace{\sin^2\frac{\pi}{4}}_{\frac{1}{2}}$$

$$= \lambda\left(\frac{2}{L}\right)^2 \frac{3}{8}$$

$$V_{AB} = \int \psi_{12}\left[\lambda\,\delta(x - \tfrac{L}{3})\,\delta(y - \tfrac{L}{4})\right]\psi_{21}\,dx\,dy = \lambda\left(\frac{2}{L}\right)^2 \sin\frac{\pi}{3}\sin\frac{2\pi}{3}\,\sin\frac{2\pi}{4}\sin\frac{\pi}{4}$$

$$= \lambda\left(\frac{2}{L}\right)^2 \frac{3}{4}\frac{1}{\sqrt{2}}$$

$$V = \lambda\left(\frac{2}{L}\right)^2 \frac{3}{4}\begin{bmatrix} 1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2}\end{bmatrix} \rightarrow \det\begin{bmatrix} 1-x & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2}-x\end{bmatrix} = (1-x)\left(\tfrac{1}{2}-x\right) - \tfrac{1}{2}$$

$$= x^2 - \tfrac{3}{2}x = x\left(x - \tfrac{3}{2}\right)$$

eigenvalues : 0 & $\frac{3}{2}$

$\downarrow$

$\begin{pmatrix} 1 \\ -\sqrt{2}\end{pmatrix} \rightarrow$ pick $\perp$: $\begin{pmatrix} \sqrt{2} \\ 1\end{pmatrix}$ check: $\begin{pmatrix} 1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2}\end{pmatrix}\begin{pmatrix} \sqrt{2} \\ 1\end{pmatrix} = \frac{3}{2}\begin{pmatrix} \sqrt{2} \\ 1\end{pmatrix}$

good wavefunction $\quad \psi_a - \sqrt{2}\,\psi_b \qquad E' = 0$

$\sqrt{2}\,\psi_a + \psi_b \qquad E' = \lambda\left(\frac{2}{L}\right)\frac{3}{4}\,\frac{3}{2}$

5 zeros