Using your own head, notes, book, `www.physics.csbsju.edu`, *Mathematica* and questions to Kirkman, complete 4 of the following problems, one of each type (WKB, RR, TIPT, dTIPT). Your exam must be turned in by Wednesday, November 25, 5:30 P.M. unless you have made prior (before 23-Nov-2015) arrangements.

1. (dTIPT) Consider a particle-in-a-2d-box with $V(x,y) = 0$ for $0 < x < L$ and $0 < y < L$ and $V(x,y) = \infty$ elsewhere. The eigenenergies depend on two whole-number quantum numbers: n_x and n_y:

$$E = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2\right)$$

with corresponding orthonormal eigenfunction:

$$\psi_{n_x n_y}(x,y) = \frac{2}{L}\,\sin\left(n_x\pi x/L\right)\sin\left(n_y\pi y/L\right)$$

A perturbing 2d delta function potential is placed at the location $(x,y) = (L/4, 2L/3)$:

$$V' = \lambda\,\delta(x - L/4)\,\delta(y - 2L/3)$$

Consider the first-order energy shift of the degenerate pair: $\psi_{12}(x,y)$ & $\psi_{21}(x,y)$. Find the matrix representing V' in this degenerate sub-space. Find the matrix's eigenvalues and vectors. Report the "good" wavefunctions and the corresponding approximate eigenenergy (accurate to first-order) for those "good" wavefunctions. FYI: $\sin(\pi/3) = \sin(2\pi/3) = -\sin(4\pi/3) = \sqrt{3}/2$, $\sin(\pi/4) = 1/\sqrt{2}$, $\sin(\pi/2) = 1$

2. (dTIPT) For homework you considered the Stark Effect for H, i.e., how the energy levels of the H-atom are affected by an electric field. We chose an electric field in the $+z$ direction, which required a voltage: $-zE$. With a negative electron charge $-e$, the resulting perturbing potential energy was $+eEz$. For $n = 3$ we needed the nine degenerate states: $|300\rangle$, $|31\text{ -}1\rangle$, $|310\rangle$, $|311\rangle$, $|32\text{ -}2\rangle$, $|32\text{ -}1\rangle$, $|320\rangle$, $|321\rangle$, $|322\rangle$ where we are using the notation $|n\ell m\rangle$ and the 9×9 matrix $\langle 3\ell' m'|z|3\ell m\rangle$. The energy levels should not be any different if the electric field is in the $+x$ direction. (Note: see `assignments.txt: Class 29` (2013) or handout for further details/wavefunctions.)

   (a) Find the 9×9 matrix $\langle 3\ell' m'|x|3\ell m\rangle$.

   (b) Find its eigenvalues and eigenvectors. The eigenvalues should be exactly the same as for $\langle 3\ell' m'|z|3\ell m\rangle$. Note, however, that in both cases the eigenvalues are not distinct (e.g., there are three 0 eigenvalues).

   (c) What critical problem is not automatically solved if these eigenvalues are degenerate (i.e., not distinct)?

   (d) For $\langle 3\ell' m'|z|3\ell m\rangle$, because of the degeneracy, the critical problem was not automatically solved, nevertheless it was solved without any additional work because of a special factor. What was that factor? Why is that factor related to the taking the electric field in the z direction?

   (e) Provide evidence that the critical problem is still present in the $\langle 3\ell' m'|x|3\ell m\rangle$ eigenvectors provided by *Mathematica*.

3. (TIPT) Consider a particle-in-a-box with $V(x) = 0$ for $0 < x < L$ and $V(x) = \infty$ elsewhere with orthonormal energy eigenfunctions

$$E_n = \frac{(q\hbar)^2}{2m} \qquad u_n(x) = \sqrt{\frac{2}{L}}\sin(qx) \quad \text{where} \quad q = \frac{n\pi}{L}$$

A perturbing potential is placed at the center of the box:

$$V'(x) = \begin{cases} \lambda & |x - L/2| < b < L/2 \\ 0 & \text{elsewhere} \end{cases}$$

(where λ is a constant), is applied. Write down an expression for the first-order energy shifts of an arbitrary state. Claim: $\mathbb{E}_1$ (first-order correction for the energy) for the ground state much larger than $\mathbb{E}_1$ for the first excited state... Why? Generally speaking which states will have large $\mathbb{E}_1$ and which small $\mathbb{E}_1$? What states *connect* to the ground state in the second-order energy shift calculation? The second-order energy shift involves an infinite sum. Calculate one (non-zero) term in the sum. (Show your steps or print out enough *Mathematica* code so I can determine exactly what you did.)

4. (TIPT) Consider a particle-in-a-2d-box with $V(x, y) = 0$ for $0 < x < L$ and $0 < y < L$ and $V(x, y) = \infty$ elsewhere. The eigenenergies depend on two whole-number quantum numbers: n_x and n_y:

$$E = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2\right)$$

with corresponding orthonormal eigenfunction:

$$\psi_{n_x n_y}(x, y) = \frac{2}{L}\,\sin\left(n_x\pi x/L\right)\sin\left(n_y\pi y/L\right)$$

Consider the effect of a constant (uniform) perturbing potential:

$$V'(x, y) = \lambda$$

on an arbitrary eigen state of the unperturbed system: $\psi_{n_x n_y}(x, y)$. Exactly calculate the first-order, second-order, and third-order energy shifts. (The equation for third-order energy shift is a footnote on page 256.) This problem is not as hard as it sounds.

5. (RR) The file `morse.pdf` #1 & #2

6. (WKB) The file `morse.pdf` #5

7. (RR) The file `x4_rr+wkd.pdf` #1

8. (WKB) The file `x4_rr+wkd.pdf` #2