$$\int_{-\infty}^{+\infty} e^{-\alpha x^2-\beta x}\, dx = \sqrt{\frac{\pi}{\alpha}}\, e^{\beta^2/4\alpha} \qquad \int_0^{+\infty} x\, e^{-\alpha x^2}\, dx = \frac{1}{2\alpha} \qquad \int_0^{\infty} x^n\, e^{-\alpha x}\, dx = n!/\alpha^{n+1}$$

$$H\psi = i\hbar\partial_t\psi \qquad H\psi = E\psi \qquad H = \frac{p^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\,\partial_x^2 + V(x) \qquad p = -i\hbar\partial_x \qquad [p,x] = -i\hbar$$

$$\partial_t\, \psi^*(x,t)\psi(x,t) = -\partial_x J \qquad \text{where current } J = \frac{\hbar}{2im}\,(\psi^*\,\partial_x\psi - \psi\,\partial_x\psi^*) = \frac{\hbar}{2im}\,\psi^*\,\overleftrightarrow{\partial_x}\,\psi$$

$$\frac{d}{dt}\langle\psi|A|\psi\rangle = \langle\psi|\partial_t A|\psi\rangle + \langle\psi|i[H,A]/\hbar|\psi\rangle \qquad \sigma_A\sigma_B = \Delta A\,\Delta B \geq \frac{1}{2}\,|\langle\psi|i[A,B]|\psi\rangle|$$

Free Particle: $u_p(x) = \frac{1}{\sqrt{2\pi\hbar}}\, e^{ipx/\hbar}$ or $u_k(x) = \frac{1}{\sqrt{2\pi}}\, e^{ikx}$ where $p = k\hbar$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{ikx}\, g(k)\, dk \qquad g(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\, f(x)\, dx$$

Particle-in-a-box with $V(x) = 0$ for $0 < x < L,$ but $V(x) = \infty$ elsewhere

$$E_n = \frac{(\hbar k)^2}{2m} \qquad u_n(x) = \sqrt{\frac{2}{L}}\,\sin(kx) \qquad \text{where} \qquad k = \frac{n\pi}{L} \qquad n = 1,2,3\ldots$$

3-d: $|n_x n_y n_z\rangle = u_{n_x}(x)u_{n_y}(y)u_{n_z}(z)$ $E = \frac{(\hbar k)^2}{2m}$ where $\vec{k} = \langle n_x\pi/L_x, n_y\pi/L_y, n_z\pi/L_z\rangle$

fermi ($s{=}\frac{1}{2}$) gas: $\mathcal{N} = 2\times\frac{(2mE)^{3/2}}{6\pi^2\hbar^3}\, V$ $\quad E_F = \frac{\hbar^2}{2m}\,\left(3\pi^2 n\right)^{2/3}$ $\quad P = \frac{2}{3}\,\langle E\rangle\, n = \frac{2}{5}\, E_F\, n$

Delta Function: $V(x) = -\alpha\delta(x) \Longrightarrow \Delta\psi' = -\frac{2m\alpha}{\hbar^2}\,\psi(0)$ $\quad \psi(x) = \sqrt{\kappa}\, e^{-\kappa|x|}$ where: $\kappa = m\alpha/\hbar^2$ $\quad E = -\frac{\kappa^2\hbar^2}{2m}$

Harmonic Oscillator with $V(x) = \frac{1}{2}\, m\omega^2 x^2$ $\quad E_n = \hbar\omega\,(n + \frac{1}{2})$ $\quad n = 0,1,2\ldots$

$|n\rangle = u_n(x) = N_n H_n(\xi)\, e^{-\frac{1}{2}\xi^2}$ where $\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$ and H_n is an n^{th} degree polynomial

$$a_- = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{p}{m\omega}\right) = \frac{1}{\sqrt{2}}\,(\xi + \partial_\xi) \qquad a_+ = a_-^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - i\frac{p}{m\omega}\right) = \frac{1}{\sqrt{2}}\,(\xi - \partial_\xi)$$

$$[a_-, a_+] = 1 \qquad [H, a_\pm] = \pm\hbar\omega a_\pm \qquad H = \hbar\omega\left(\tfrac{1}{2} + a_+a_-\right) \qquad a_-\,|n\rangle = \sqrt{n}\,|n-1\rangle \qquad a_+\,|n\rangle = \sqrt{(n+1)}\,|n+1\rangle$$

3-d: $\left(n_x + n_y + n_z + \frac{3}{2}\right) = \left(2n_r + \ell + \frac{3}{2}\right)$ $\quad \psi = \sqrt{\frac{2\cdot n_r!}{(\ell + \frac{1}{2} + n_r)!}}\; r'^{\ell}\, L_{n_r}^{\ell+\frac{1}{2}}(r'^2)\, e^{-r'^2/2}\, Y_{\ell m}(\theta,\phi)$

Angular Momentum: $\vec{L} = \vec{r}\times\vec{p}$ $\quad [L_i, V_j] = i\hbar\epsilon_{ijk}V_k$ where vector $\vec{V} = \vec{r}, \vec{p}, \vec{L}$ $\quad |\ell m\rangle = Y_{\ell m}(\theta,\phi)$

$$\vec{L}^2\,|\ell m\rangle = \ell(\ell+1)\,\hbar^2\,|\ell m\rangle \qquad L_z\,|\ell m\rangle = m\hbar\,|\ell m\rangle \qquad L_\pm\,|\ell m\rangle = \sqrt{\ell(\ell+1) - m(m\pm1)}\;\hbar\;|\ell\, m\pm1\rangle$$

$$L_\pm = L_x \pm iL_y \qquad [L_+, L_-] = 2\hbar L_z \qquad [L_z, L_\pm] = \pm\hbar L_\pm \qquad [\vec{L}^2, L_\pm] = 0$$

Spin $\frac{1}{2}$: $\vec{S} = \frac{\hbar}{2}\,\vec{\sigma}$ $\quad |\frac{1}{2}\,\frac{1}{2}\rangle = \chi_+ = \uparrow = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad |\frac{1}{2}\, -\frac{1}{2}\rangle = \chi_- = \downarrow = \begin{pmatrix}0\\1\end{pmatrix}$

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad \sigma_i^2 = 1 \qquad \sigma_x\sigma_y = i\sigma_z = -\sigma_y\sigma_x$$

Clebsch-Gordan: $|jm\rangle = \sum C(jm;\ell m_\ell, sm_s)\,|\ell m_\ell\rangle\,|sm_s\rangle$ know how to use table!

Radial Equation: $\psi(r,\theta,\phi) = Y_{\ell m}(\theta,\phi)R(r)$ $\quad R(r) = \frac{u(r)}{r}$

$$\left[\frac{-\hbar^2}{2m}\left(\partial_r^2 + \frac{2}{r}\partial_r\right) + \frac{\hbar^2\ell(\ell+1)}{2mr^2} + V(r)\right] R = E\, R \qquad \left[\frac{-\hbar^2}{2m}\partial_r^2 + \frac{\hbar^2\ell(\ell+1)}{2mr^2} + V(r)\right] u = E\, u$$

Free: $R(r) = j_\ell(kr)$ $\quad E = \frac{(\hbar k)^2}{2m}$ $\quad$ in box: $kR =$ zero of j_ℓ

H atom: $H = \frac{p^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 r}$ $\quad E_n = -\frac{1}{2}\, mc^2\,\frac{(Z\alpha)^2}{n^2} = -\frac{1}{2}\,\frac{Z^2e^2}{4\pi\epsilon_0 a_0 n^2} \approx -13.6\text{ eV}\;\frac{Z^2}{n^2}$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} \approx .53\text{ Å} \qquad \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137} \qquad n = n_r + \ell + 1 \qquad \rho = \sqrt{\frac{8m|E|}{\hbar^2}}\; r = \frac{2Zr}{na_0}$$

$|n\ell m\rangle = R_{n\ell}(\rho)\, Y_{\ell m}(\theta,\phi)$ where $R_{n\ell} = N_{n\ell}\,\rho^\ell\, L_{n_r}^{2\ell+1}(\rho)\, e^{-\frac{1}{2}\rho}$ $\quad N_{n\ell} = \frac{2}{n^2}\sqrt{\frac{(n-\ell-1)!}{(n+\ell)!}}$

Spectroscopic Notation: orbital: s,p,d,f,g $\quad$ term: ${}^{2S+1}L_J$ $\quad$ atomic: 1s,2s,2p,3s,3p,4s,3d $\quad$ nuclear: 1s,1p,1d,2s,1f,2p,1g

Spin-statistics: fermion: $s = \frac{1}{2}, \frac{3}{2}, \ldots$ $\quad$ boson: $s = 0,1,2,\ldots$

$$u(x_i) = (N!)^{-1/2}\begin{vmatrix} f(x_1) & f(x_2) & f(x_3) & \cdots \\ g(x_1) & g(x_2) & g(x_3) & \cdots \\ h(x_1) & h(x_2) & h(x_3) & \cdots \\ \vdots & \vdots & \vdots & \end{vmatrix}$$

2-particle CM Coordinates:

$$\vec{\mathbf{R}} = \frac{m_1}{M}\,\vec{\mathbf{r}}_1 + \frac{m_2}{M}\,\vec{\mathbf{r}}_2 \qquad \vec{\mathbf{r}}_1 = \vec{\mathbf{R}} + \frac{m_2}{M}\,\vec{\mathbf{r}} \qquad M = m_1 + m_2$$

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2 \qquad \vec{\mathbf{r}}_2 = \vec{\mathbf{R}} - \frac{m_1}{M}\,\vec{\mathbf{r}} \qquad \mu = \frac{m_1m_2}{M}$$

$$\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{P^2}{2M} + \frac{p^2}{2\mu}$$