1) (a) $L_x = \frac{1}{2}(L_+ + L_-)$ $\qquad L_\pm Y_{\ell m} = \hbar\sqrt{\ell(\ell+1) - m(m\pm1)}\, Y_{\ell\, m\pm1}$

$L_x N(-Y_{11} + Y_{1-1})$

$= \frac{N}{2}(-L_- Y_{11} + L_+ Y_{1-1})$ $\qquad L_+ Y_{1-1} = \hbar\sqrt{2}\, Y_{10} \qquad L_- Y_{11} = \hbar\sqrt{2}\, Y_{10}$

$= \frac{N}{2}\hbar\sqrt{2}(-Y_{10} + Y_{10}) = 0$

(b) $L_z\, iN(-Y_{22} + Y_{2-2}) = iN\hbar(-2Y_{22} - 2Y_{2-2}) = -2iN\hbar(Y_{22} + Y_{2-2})$

$\lambda = \frac{-2iN_{xy}\hbar}{N_{x^2-y^2}}$

(c) orthonormal $\Rightarrow$ $1 = N^2(2 + \underset{|1+i|^2}{2} + 2 + \underset{|1-i|^2}{2}) \Rightarrow N = \frac{1}{\sqrt{8}}$

Prob $L_z = 0 \rightarrow \frac{2}{8} + \frac{2}{8}$ $\qquad$ Prob $L_z = +\hbar \rightarrow \frac{2}{8}$ $\qquad$ Prob $L_z = -\hbar \rightarrow \frac{2}{8}$

Prob $L^2 = 0 = \frac{2}{8}$ $\qquad$ Prob $L^2 = 2\hbar^2 \rightarrow \frac{6}{8}$

(d) L_z eigen: just P_z

Not L^2 eigen: just hybrid $sp3$ (has $\ell=0$ & $\ell=1$)

(2) $J^2 = L^2 + S^2 + 2L\cdot S = L^2 + S^2 + L_+S_- + L_-S_+ + 2L_zS_z$

$L^2 \rightarrow 6\hbar^2$, $S^2 \rightarrow \frac{3}{4}\hbar^2$, $L_+S_- \rightarrow$ only $Y_0\chi^+$, $L_-S_+ \rightarrow$ only $Y_1\chi_-$, $2L_zS_z \rightarrow$ only $Y_{21}\chi_-$ $\quad 2\times1(-\frac{1}{2}) = -\hbar^2$

$L_+Y_{20} = \hbar\sqrt{6}\, Y_{21} \qquad L_-Y_{21} = \hbar\sqrt{6}\, Y_{20} \qquad S_+\chi_- = \hbar\chi_+ \qquad S_-\chi_+ = \hbar\chi$

overall factor of $\hbar^2$: $L^2+S^2 \Rightarrow (6+\frac{3}{4})\left[\sqrt{\frac{3}{5}}\, Y_{21}\chi_- - \sqrt{\frac{2}{5}}\, Y_{20}\chi_+\right]$

L_+S_- $\qquad - \sqrt{6}\sqrt{\frac{2}{5}}\, Y_{21}\chi_-$

L_-S_+ $\qquad + \sqrt{6}\sqrt{\frac{3}{5}}\, Y_{20}\chi_+$

$2L_zS_z$ $\qquad - Y_{21}\chi_-$

$\sqrt{\frac{3}{5}}\, Y_{21}\chi_-\left(\frac{27}{4} - 2 - 1\right) - \sqrt{\frac{2}{5}}\, Y_{20}\chi^+\left(\frac{27}{4} - 3\right)$

$\rightarrow \frac{15}{4} = \frac{3}{2}\cdot\frac{5}{2} \qquad j = 3/2$

$L^2 \rightarrow \hbar^2\ell(\ell+1)$ with $\ell = 1$

$S^2 \rightarrow \hbar^2 s(s+1)$ with $s = 1/2$

$J_z = \hbar\frac{1}{2}$

(3) $n \to$ orbit semi major axis

$\ell \to$ angular momentum

$m \to$ z component of angular momentum

100 → 1s

200 → 2s

211, 210, 21−1 } → 2p

300 → 3s

311, 310, 31−1 } 3p

32x 3d

(4) $L_x = y P_z - z P_y$

$$[L_x, P_y] = [y P_z, P_y] - \overbrace{[z P_y, P_y]}^{0}$$

$$= [y, P_y] P_z = -\frac{\hbar}{i} P_z = i\hbar P_z$$

(5) spacial state is antisym under particle exchange, spin state is symmetric → overall antisym which is correct for fermions

$$-\frac{\hbar^2}{2m}\left[\partial_{x_1}^2 e^{i(5(x_1+x_2))}\sin(3(x_1-x_2)) + \partial_{x_2}^2 e^{i(5(x_1+x_2))}\sin(3(x_1-x_2))\right]$$

$\downarrow \partial x$

$i5\, e^{i()}\sin + 3 e^{i()}\cos$

$\downarrow \partial x$

$-5^2 e^{i()}\sin + i5\,3 e^{i()}\cos + i\,5\cdot 3\, e^{i()}\cos - 3^2 e^{i()}\sin$

$-5^2 e^{i()}\sin - 5\cdot 3\, e^{i()}\cos - i5\cdot 3\, e^{i()}\cos - 3^2 e^{i()}\sin$

$-2\times(5^2+3^2)$

$$\Rightarrow E = \frac{\hbar^2}{m}(5^2+3^2)$$

note $P = 10$, $p = 3$

$$\frac{\hbar^2}{2m}\left[\frac{10^2}{2} + \frac{3^2}{1/2}\right]$$

(6) $$\int_0^\infty \int_0^\pi \int_0^{2\pi} |\psi|^2 \overbrace{r^2\cos^2\theta}^{z^2}\, \underbrace{r^2 \underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega}\, dr}_{dV}$$

$r \quad \theta \quad \phi$

7) $\begin{pmatrix}1\\1\end{pmatrix} \leftrightarrow \frac{\hbar}{2}$ $\begin{pmatrix}1\\-1\end{pmatrix} \leftrightarrow -\frac{\hbar}{2}$

$\begin{pmatrix}1\\i\end{pmatrix} \leftrightarrow \frac{\hbar}{2}$ $\begin{pmatrix}0\\1\end{pmatrix} \leftrightarrow -\frac{\hbar}{2} \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$

$N^2(5^2+1^2+3^2+1^2) = N^2 36 = 1 \longrightarrow N = \frac{1}{6}$

$S_z : \begin{cases} +\frac{\hbar}{2} & \text{Prob} = \left|\frac{5i+1}{6}\right|^2 = \frac{26}{36} \\ -\frac{\hbar}{2} & \text{Prob} = \left(\frac{3i-1}{6}\right)^2 = \frac{10}{36} \end{cases}$

$\frac{36+16}{72}$

$S_y : \begin{cases} +\hbar/2 & \sqrt{\text{Prob}} = \frac{1}{\sqrt{2}}\frac{1}{6}(1 \; -i)\begin{pmatrix}5i+1\\3i-1\end{pmatrix} = \frac{1}{\sqrt{2}\,6}\begin{pmatrix}5i+1+3+i\\6i+4\end{pmatrix} \\ -\hbar/2 & \sqrt{\text{Prob}} = \frac{1}{\sqrt{2}}\frac{1}{6}(-i, \; 1)\begin{pmatrix}5i+1\\3i-1\end{pmatrix} = \frac{1}{\sqrt{2}\,6}\begin{pmatrix}5-i+3i-1\\4+2i\end{pmatrix} \end{cases}$

$\downarrow$

$\frac{1}{72}(16+4)$

$\begin{pmatrix}0\\1\end{pmatrix} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle = \downarrow = \chi_-$