1 a) Only eigenvalues of $Q$ will be observed.
if $q$ is an eigenvalue with eigenfunction $\psi_q$, the prob of observing $q$ is $|\langle \psi_q | \psi \rangle|^2$

b) $\int_a^b |\psi|^2 dx$

2) $1 = N^2 \int \psi^* \psi \, dx = N^2 (25 + 2 + 9) = N^2 \cdot 36 \Rightarrow N = \frac{1}{6}$

$\text{Prob} = \left| \frac{1+i}{6} \right|^2 = \frac{2}{36} = \frac{1}{18}$

$\langle \psi H \psi \rangle = \frac{1}{36} \left( 5^2 E_1 + 2 u_2 + 9 E_3 \right)$

$= \hbar\omega \left[ \underbrace{\left( \frac{25}{36} \cdot 1 + \frac{2}{36} \cdot 2 + \frac{9}{36} \cdot 3 \right)}_{\to 2.06} + 1/2 \right]$

$\psi = 5 u_1 e^{-i\frac{3}{2}\omega t} + (1+i) e^{-i\frac{5}{2}\omega t} + 3 e^{-i\frac{7}{2}\omega t}$

3) $x = \sqrt{\frac{\hbar}{2m\omega}} (a_+ + a_-)$ $\quad p = \sqrt{\frac{\hbar m\omega}{2}} i (a_+ - a_-)$

$\langle x \rangle = \frac{1}{2} \langle u_0 e^{-i\omega t/2} + i u_1 e^{-\frac{3}{2}i\omega t} | \sqrt{\frac{\hbar}{2m\omega}} (a_+ + a_-) | u_0 e^{-i\omega t/2} + i u_1 e^{-i\frac{3}{2}\omega t} \rangle$

$\to u_1 e^{-i\omega t/2} + i\sqrt{2} u_2 e^{-i\frac{3}{2}\omega t}$

$\to 0 + i u_0 e^{-i\frac{3}{2}\omega t}$

$= \frac{1}{2} \sqrt{\frac{\hbar}{2m\omega}} \left\{ i e^{-i\omega t} - i e^{+i\omega t} \right\} = \sqrt{\frac{\hbar}{2m\omega}} \left\{ \frac{e^{i\omega t} - e^{-i\omega t}}{2i} \right\}$

$= \sqrt{\frac{\hbar}{2m\omega}} \sin(\omega t)$

$\langle p \rangle = \frac{1}{2} \langle u_0 e^{-i\omega t/2} + i u_1 e^{-i\frac{3}{2}\omega t} | \sqrt{\frac{\hbar m\omega}{2}} i (a_+ - a_-) | u_0 e^{-i\omega t/2} + i u_1 e^{-i\frac{3}{2}\omega t} \rangle$

$\to u_1 e^{-i\omega t/2} + i\sqrt{2} u_2 e^{-i\frac{3}{2}\omega t}$

$\to 0 + i u_0 e^{-i\frac{3}{2}\omega t}$

$= \frac{1}{2} \sqrt{\frac{\hbar m\omega}{2}} i \left\{ -i e^{-i\omega t} - i e^{+i\omega t} \right\} = \sqrt{\frac{\hbar m\omega}{2}} \cos(\omega t)$

both $x$ & $p \sim a_+$ & $a_- \Rightarrow u_{n+1}$ & $u_{n-1}$ but not $u_n$

$\langle x \rangle = 0$ since $x > 0$ as much as $x < 0$

$\langle p \rangle = 0$ since osc back & forth

4) $1 = N^2 \int_0^{L/2} 1 \, dx = N^2 \frac{L}{2} \Rightarrow N = \sqrt{\frac{2}{L}}$

$$\text{prob} = |\langle u_1|\psi\rangle|^2 = \left| \sqrt{\tfrac{2}{L}}^{\,2} \int_0^{L/2} \underbrace{\sin\left(\tfrac{\pi}{L}x\right) \cdot 1}_{-\frac{L}{\pi}\cos\left(\frac{\pi}{L}x\right)\Big|_0^{L/2} \, = \, \frac{L}{\pi}} \, dx \right|^2$$

$$= \left| \frac{2}{\pi} \right|^2 \sim .4$$

$$\psi = \sum \underbrace{\alpha_i}_{\langle u_i|\psi\rangle} u_i \, e^{-iE_i t/\hbar}$$

5) $(px - xp)f = \frac{\hbar}{i}\{\partial_x(xf) - x\partial_x f\} = \frac{\hbar}{i} f$

$(Vp - pV)f = \frac{\hbar}{i}\{V\partial_x f - \partial_x(Vf)\} = \left(-\frac{\hbar}{i}\partial_x V\right) f$

$[p^2, x] = p[p,x] + [p,x]p = 2\frac{\hbar}{i} p$ ✓

$\left[\frac{p^2}{2m} + V, x\right] = \left[\frac{p^2}{2m}, x\right] = \frac{1}{2m} 2 \frac{\hbar}{i} p = \frac{\hbar}{i}\frac{p}{m}$

$[A, BC] = ABC - BCA = ABC - BAC + BAC - BCA$

$= [A,B]C + B[A,C]$

6)
$$\begin{pmatrix} a & 2b & & \\ 2b & a & 2b & \\ & 2b & a & 2b \\ & & 2b & a \end{pmatrix} \begin{pmatrix} 2 \\ 1+\sqrt5 \\ 1+\sqrt5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2a + 2b(1+\sqrt5) \\ 4b + a(1+\sqrt5) + 2b(1+\sqrt5) \\ \text{same \#2} \\ \text{same \#1} \end{pmatrix}$$

Now: $2a + 2b(1+\sqrt5) = 2\cdot(a + b(1+\sqrt5))$ ✓

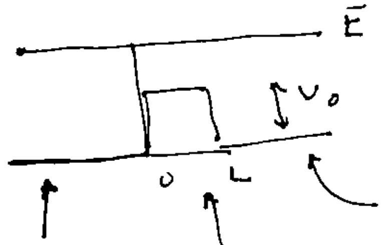
$(a + b(1+\sqrt5))(1+\sqrt5) = a(1+\sqrt5) + b\underbrace{(1+\sqrt5)^2}_{6+2\sqrt5} = a(1+\sqrt5) + b(4+2+2\sqrt5)$ ✓

$1 = N^2|v_1|^2 = N^2\underbrace{(2^2 + (1+\sqrt5)^2 + (1+\sqrt5)^2 + 2^2)}_{28.94} \Rightarrow N = \frac{1}{\sqrt{28.94}} = .186$

$v_1 \cdot v_2 = 4 + (1+\sqrt5)(1-\sqrt5) - (1+\sqrt5)(1-\sqrt5) - 4 = 0$

$$\alpha_1 = \frac{\langle v_1 \cdot \psi\rangle}{\langle v_1 \cdot v_1\rangle} = \frac{2 + 2(1+\sqrt5) + 3(1+\sqrt5) + 4\cdot 2}{\underset{4 + (6+2\sqrt5)2 + 4}{2^2 + (1+\sqrt5)^2 + (1+\sqrt5)^2 + 2^2}} = \frac{5(3+\sqrt5)}{4(5+\sqrt5)}$$

7)




$k^2 = \frac{2mE}{\hbar^2}$ $\qquad$ $q^2 = \frac{2m(E-V_0)}{\hbar^2}$

$-\psi'' = k^2\psi \longrightarrow e^{\pm ikx}$

$-\psi'' = k^2\psi$ $\qquad$ $-\psi'' = q^2\psi \longrightarrow e^{\pm iqx}$

unknown: R, A, B, T

$\psi$ & $\psi'$ continuous @ $x=0$ & $x=L$

$x=0$: $\quad 1+R = B$

$\qquad ik(1-R) = Aq$

$x=L$ $\quad A\sin(qL) + B\cos(qL) = Te^{ikL}$

$\qquad q\left(A\cos(qL) - B\sin(qL)\right) = ikTe^{ikL}$

Extra Credit: motion at constant speed from left to right

except: slows when crosses $x=0$

speeds when crosses $x=L$