Class 30

$u=33$, $K \to .34$ $\quad \delta_3$ has step $\Rightarrow P_3 \to 3$ zeros

$u=42$, $K \to 2.2$ $\quad \underbrace{\delta_4 \text{ has step}} \Rightarrow P_4 \to 4$ zeros

$u=39$, $K=.4$ $\quad \delta_1$ $\quad$ ↳ steps are important as then we know at some intermediate K $\delta = \pi/2$ (max effect) happened

resonances correspond to quasi bound states; in the WKB approx quasi bound states result from fitting the proper number of $\frac{\lambda}{2}$ between turning pts. The wavelength depends on total KE inside the well $\quad KE = E - V = E + U_0$

KE relates to λ inside well; E: $\frac{\hbar^2 k^2}{2m}$ $\qquad$ Note: $V = -U_0$ ← PE

KE to get proper # of $\frac{\lambda}{2}$ is more-or-less fixed so $U_0 \uparrow$ (greater depth of potential well) requires $E\downarrow$ (hence smaller K)

		$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	
	$u=50$ Bound states:	2	2	1	1	1	} same!
	$u=51$ Pi malt:	2	2	1	1	1	
guess	$u=42$ Bound states	2	2	1	1	0	} same!
	$u=42$ Pi malt	2	2 (just occurred)	1	1	0	

class 31 fit — the contribution to σ from a partial wave is $\underset{B}{\frac{4\pi}{K^2}}(2\ell+1)\underbrace{\sin^2\delta_\ell}_{1}$ $\quad$ so expect $B = .2 \to .1.8$, $A = 4\pi \to 4.6\pi$ (fix $B=.2$)

partial waves

$5\text{ MeV} = \frac{\hbar^2 k^2}{2m} \to K = \sqrt{\frac{2(mc^2)\,5\text{MeV}}{(\hbar c)^2}} = .491\ 1/\text{fm}$ $\quad$ ($mc^2 = 939.5$ MeV; $\hbar c = 197$ MeV·fm; fm $= 10^{-15}$ m)

note $1\text{ barn} = 100\text{ fm}^2$

math: $\sigma = 18.56\ (\text{fm})^2$

$\frac{d\sigma}{d\Omega} = |f|^2 = 1.133\ \frac{(\text{fm})^2}{\text{sr}}$ @ 90°

fraction covered $= 18.56\ (\text{fm})^2 \cdot 10^{21}/(\text{cm})^2 = 18.56\ \frac{10^{-30}\,10^{21}}{10^{-4}} = .1856 \times 10^{-3}$

scatter rate $= (.1856 \times 10^{-3})(2\times10^{10}/\text{sec}) = 3.71 \times 10^6/\text{sec}$

$d\sigma = 1.133\ \frac{(\text{fm})^2}{\text{sr}} \cdot \frac{\pi(.01)^2}{1^2} = 3.56 \times 10^{-4}\ (\text{fm})^2$

fraction covered $= 3.56\times10^{-4}(\text{fm})^2 \cdot \frac{10^{21}}{\text{cm}^2} = 3.56\times10^{-9}$

↳ $\times(2\times10^{10}) = 71/\text{sec}$

11.4 → $V(r) = \alpha\,\delta(r-a)$

1born

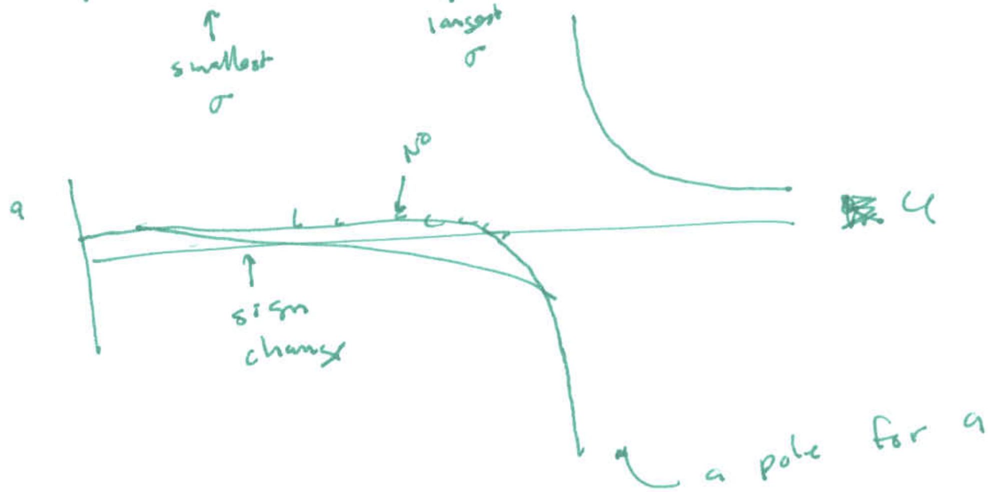
$$f(\theta) = \frac{-2m}{\hbar^2 q}\,\alpha\, a \sin(qa) \longrightarrow \frac{-2m\alpha a^2}{\hbar^2} = -a\beta$$

agree for small β

$q = 2k\sin\frac{\theta}{2}$

In the low energy limit 11.4 → $f = \frac{-a\beta}{1+\beta}$

	59	60	61	61.5	62	63
a	small	small	grows	grows	big	
	+	−	−	−	+	+
		smallest σ		largest σ		

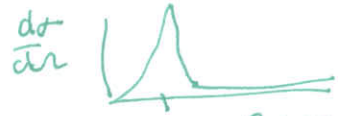


at low energy $\sigma \to 4\pi a^2$ ← |a| large ⇒ large σ

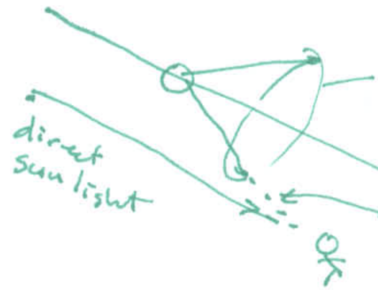
"scattering length" → 175,000

+Nobel → 2880

Class 32



somewhere along the ray that points from your eye towards the halo there must be material that has scattered light towards you. (Of course that material has also scattered light in directions that do not connect with your eye.



added states: added $\ell = 0$, $\ell = 2$

	4	30	33	36	39	42	45	48	51	54	57	60	63
scatter length sign		+	+	+	+	+	+	+	+	+	+	−	+

magnitude mostly constant, getting smaller — clearly smaller — large

Class 33

11.10 – Note "soft sphere" is just a repulsive square well. We did attractive square well in class, so all that is really required is $U_0 \to -V_0$

$c = \cos\theta$, $dc = -\sin\theta\, d\theta$

$$\text{FT: } V_0 \int e^{i\vec{q}\cdot\vec{r}_0}\, d^3r_0 = 2\pi V_0 \int_0^a \underbrace{\int_{-1}^{1} e^{iqr_0c}\, dc}_{\frac{1}{iqr_0}\left(e^{iqr_0} - e^{-iqr_0}\right) = \frac{2}{qr_0}\sin(qr_0)}\; r_0^2\, dr_0$$

$$= \frac{2\pi V_0}{q}\, 2 \int_0^a \sin(qr_0)\, r_0\, dr_0 = \frac{4\pi V_0}{q^3} \int_0^{qa} \underbrace{\theta \sin\theta}_{\frac{d}{d\theta}(-\theta\cos\theta) + \cos\theta}\, d\theta$$

$$= \frac{4\pi V_0}{q^3}\Big[\sin(qa) - qa\cos(qa)\Big]$$

$$f = \frac{-m}{2\pi\hbar^2}\,\text{FT} = -\frac{2m\, V_0 a^3}{q^2\hbar^2 a^2}\left[\frac{\sin(qa)}{qa} - \cos(qa)\right]$$

$$\frac{\sin\theta}{\theta} - \cos\theta \simeq 1 - \frac{1}{3!}\theta^2 - \left(1 - \frac{1}{2!}\theta^2\right) = \frac{1}{3}\theta^2$$

$$\xrightarrow[\theta \simeq 0]{} \frac{-2m V_0 a^3}{\hbar^2 q^2 a^2}\,\frac{1}{3} q^2 a^2 = \frac{-m}{2\pi\hbar^2} V_0 \frac{4}{3} a^3 \checkmark$$

$$\frac{1}{(1+(q/q_0)^2)^2} = F(q) = \int e^{i\vec{q}\cdot\vec{r}} \rho(r)\, d^3r$$

$$\Rightarrow \quad \frac{1}{(2\pi)^3} \int e^{-iq\cdot r} F(q)\, d^3q = \rho(r)$$

$$\parallel$$

$$\frac{1}{(2\pi)^2} \int_0^\infty \int_{-1}^{1} e^{-iqrc} F(q)\, q^2\, dq\, dc$$

$$\frac{1}{-iqr}\left(e^{-iqr} - e^{+iqr}\right) = \frac{2}{qr}\sin(qr)$$

$$= \frac{2}{(2\pi)^2 r} \int_0^\infty \frac{\sin(qr)\, q\, dq}{(1+(q^2/q_0^2))^2} = \frac{2}{(2\pi)^2 r^3} \underbrace{\int_0^\infty \frac{\sin\theta\ \theta\, d\theta}{\left(1+\frac{\theta^2}{(q_0 r)^2}\right)^2}}_{\text{mathematica}}$$

$$\frac{(q_0 r)^3\, \pi}{4\, e^{q_0 r}}$$

$$= \frac{q_0^3\, e^{-q_0 r}}{8\pi\, \cancel{e^{-q}}}$$

check $\quad 1 = \int \rho(r)\, 4\pi r^2 dr = \frac{1}{2} q_0^3 \int e^{-q_0 r}\, r^2 dr = \frac{1}{2} \underbrace{\int e^{-x} x^2 dx}_{2!} = 1$ ✓

Find rms $\quad \langle r^2 \rangle = \int r^2 \rho(r)\, dV = -\nabla_q^2 F(q)\Big|_{q=0}$

$$= -\frac{1}{q^2}\partial_q\, q^2\, \underbrace{\partial_q \frac{1}{(1+\frac{q^2}{q_0^2})^2}}_{-2\left(1+\frac{q^2}{q_0^2}\right)^{-3}\frac{2q}{q_0^2}}\Bigg|_{q=0}$$

$$\underbrace{\qquad}_{\frac{-4}{q_0^2}\left(1+\frac{q^2}{q_0^2}\right)^{-3} q^3}$$

$$= +\frac{12}{q_0^2}$$

Alt: $\quad \langle r^2 \rangle = 4\pi \frac{q_0^3}{8\pi} \int_0^\infty r^4 e^{-q_0 r} dr = \frac{1}{2}\frac{1}{q_0^2} \overbrace{\int_0^\infty x^4 e^{-x} dx}^{4!} = \frac{12}{q_0^2}$

$$\text{rms} = \frac{\sqrt{12}}{q_0} = .8\ \text{Fm}$$

4-30a (f) — we expect the charge density inside a gold nucleus to be approx const. $F(q)$ is Fourier transform of ρ part b is Fourier transform of a square well potential — ie Fourier transform of a solid ball exactly like ρ [except for sign]

$$44^\circ = .768^r = \frac{4.5}{KR} \rightarrow R = \frac{4.5}{(.768)\, 2.1\ \text{fm}^{-1}} = 2.8\ \text{fm}$$

close

Compare approx result $1.2\, A^{1/3} = 3$ fm

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11-13