$$V = x^5 - 25x^4 + 225x^3 - 900x^2 + 1575x - 945$$

$$v' = 5x^4 - 100x^3 + 675x^2 - 1800x + 1575$$

$$v'' = 20x^3 - 300x^2 + 1350x - 1800$$

$$xv'' + 2(\ell+1-x)v' + (\rho_0 - 2(\ell+1))V = 0$$

only these two terms will have x^5

only these two terms will have consta

Look at x^5 : $-2xv' \Rightarrow -10x^5$

$$(\rho_0 - 2(\ell+1))V \Rightarrow \underbrace{(\rho_0 - 2(\ell+1))}_{\text{must be } 10} x^5$$

Look at const: $2(\ell+1)v' \Rightarrow 2(\ell+1)1575$

$$\underbrace{(\rho_0 - 2(\ell+1))}_{10}(-945) = -9450$$

$$(\ell+1)3150 = 9450$$
$$\ell+1 = 3$$
$$\ell = 2$$

So: diff eq: $xv'' + 2(3-x)v' + 10v = 0$

$$\begin{array}{rcccccc}
xv'' = & & 20x^4 & -300x^3 & +1350x^2 & -1800x & \\
6v' = & & 30x^4 & -600x^3 & +4050x^2 & -10800x & +9450 \\
-2xv' = & -10x^5 & +200x^4 & -1350x^3 & +3600x^2 & -3150x & \\
10v = & 10x^5 & -250x^4 & +2250x^3 & -9000x^2 & +15750x & -9450 \\
\hline
 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

define $y = 2(\ell+1) \Rightarrow c_{j+1} = \left[\dfrac{2j + y - \rho_0}{(j+1)(j+y)}\right] c_j$

$j=0$ $\quad 1575 = \left[\dfrac{y-\rho_0}{y}\right](-945) \rightarrow \left[\dfrac{-10}{6}\right](-945)$

$j=1$ $\quad -900 = \left[\dfrac{2+y-\rho_0}{2(1+y)}\right](1575) \rightarrow \left[\dfrac{-8}{2\cdot 7}\right]1575$

$j=2$ $\quad 225 = \left[\dfrac{4+y-\rho_0}{3(2+y)}\right](-900) \rightarrow \left[\dfrac{-6}{3\cdot 8}\right](-900)$

$j=3$ $\quad -25 = \left[\dfrac{6+y-\rho_0}{4(3+y)}\right](225) \rightarrow \dfrac{-4}{4\cdot 9}\,225$

$j=4$ $\quad 1 = \left[\dfrac{8+y-\rho_0}{5(4+y)}\right](-25) \rightarrow \dfrac{-2}{5(4+y)}(-25) \rightarrow y = 6$

$j=5$ $\quad 0 = [10+y-\rho_0]\cdot 1 \longrightarrow \rho_0 - y = 10$ $\qquad$ start here web #11 $\int \psi^* \psi = N^2 (1+4+1+1) = 1 \Rightarrow N = \frac{1}{\sqrt{7}}$

$E = \frac{-E_1}{n^2}$ $\langle E \rangle = \sum |c_n|^2 E_n$

$$= -E_1 \left(\underset{\substack{\uparrow \\ |100\rangle}}{\frac{1}{7}\frac{1}{1^2}} + \underset{\substack{\uparrow \\ |200\rangle}}{\frac{4}{7}\frac{1}{2^2}} + \underset{\substack{\uparrow \\ |210\rangle}}{\frac{1}{7}\frac{1}{2^2}} + \underset{\substack{\uparrow \\ |211\rangle}}{\frac{1}{7}\frac{1}{2^2}} \right)$$

$$= \underset{\substack{\uparrow \\ 13.6\,eV}}{-E_1} \left(\underset{\substack{\uparrow \\ .357}}{\frac{2.5}{7}} \right) = -4.86\,eV$$

$u(r) = r^3 e^{-r/3a}$ $n = 3$ $\ell = 2$

$u' = 3r^2 e^{-r/3a} - \frac{r^3}{3a} e^{-r/3a}$

$u'' = \underset{\substack{\uparrow \\ \text{cancels} \\ \text{from} \\ \frac{\hbar^2 \ell(\ell+1)}{2mr^2} u}}{6r\, e^{-r/3a}} \underset{\substack{\uparrow \\ \text{combine with } PE = \frac{-e^2}{4\pi\varepsilon_0 r} u}}{- \frac{2r^2}{a} e^{-r/3a}} + \frac{r^3}{9a^2} e^{-r/3a}$

$$\left(\frac{2\hbar^2}{2m\underset{\substack{\uparrow \\ \frac{4\pi\varepsilon_0 \hbar^2}{me^2}}}{a}} - \frac{e^2}{4\pi\varepsilon_0} \right) r^2 e^{-r/3a} = 0 \checkmark$$

so: $-\frac{\hbar^2}{2m} u'' + \left(\frac{\hbar^2 \ell(\ell+1)}{2mr^2} + V(r) \right) u = -\frac{\hbar^2}{2m\, 9a^2} u$ for one $a = \frac{4\pi\varepsilon_0 \hbar^2}{me^2}$

$$\underbrace{-\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 a}}_{E_1} \frac{1}{9}\, u = \underset{\substack{\uparrow \\ n=3 \\ \text{energy}}}{\frac{-E_1}{9}} u$$

For cube: $E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2)$

gs: $\vec{n} = (1,1,1) \Rightarrow 3 = 1^2+1^2+1^2$

$\vec{n} = (2,1,1) \times 3 \Rightarrow 6 = 2^2+1^2+1^2$

$\vec{n} = (2,2,1) \times 3 \Rightarrow 9$ etc

$\vec{n} = (3,1,1) \times 3 \Rightarrow 11$

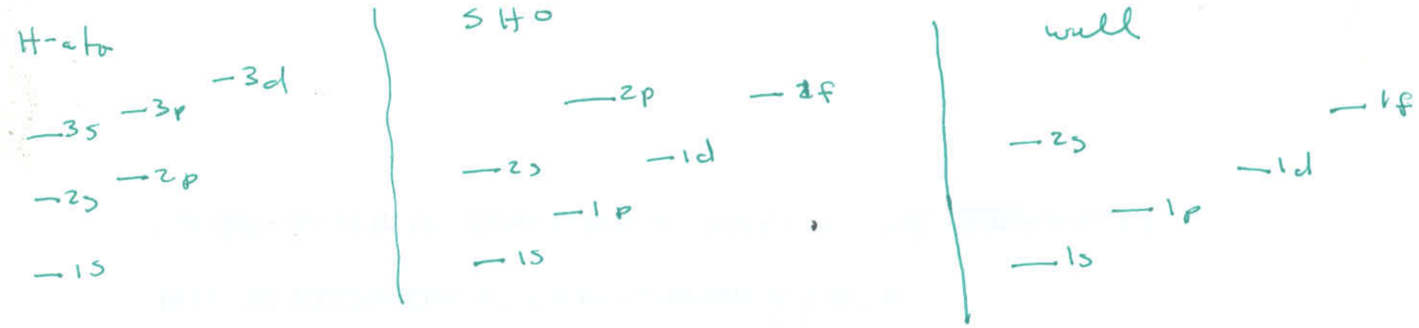
$\vec{n} = (2,2,2) \Rightarrow 12$

$\vec{n} = (3,2,1) \times \underset{\substack{\uparrow \\ \text{degeneracy}}}{6} \Rightarrow 14 = 3^2+2^2+1^2$

eg $\underset{\substack{\uparrow \\ \text{six states with the same energy} \equiv \text{degeneracy}}}{(3,2,1)}, (3,1,2), (2,3,1), (1,3,2), (2,1,3), (1 2 3)$

old exam #6 : note degeneracy $= 2l+1$ where

s	p	d	f
0	1	2	3



A	3d	5	1f	7	1f	7
B	3s	1	1d	5	2s	1
C	2p	3	2s	1	1d	5
D	2s	1	1p	3	1p	3
E	1s	1	1s	1	1s	1

Rotation.pdf − $e^{isL_z/\hbar} = e^{s\partial_\phi} = 1 + s\partial_\phi + \frac{s^2}{2}\partial_\phi^2 + \dots$

so $e^{isL_z/\hbar} f(\phi) = f(\phi) + s f'(\phi) + \frac{s^2}{2} f''(\phi) + \dots$

$= $ Taylor expans. of $f(\phi + s)$

$$\left(1 + \underset{\downarrow\, 2\phi}{s\partial_\phi} + \frac{s^2}{2}\underset{\downarrow\, 2}{\partial_\phi^2} + \frac{s^3}{3!}\underset{\downarrow\, 0}{\partial_\phi^3} + \dots\right)\phi^2 = \left(\phi^2 + s2\phi + s^2\right) = (\phi + s)^2 \checkmark$$

Eq 4.12 (or in class) $\quad L_y = \frac{\hbar}{i}\left(\cos\phi\,\partial_\theta - \sin\phi\cot\theta\,\partial_\phi\right)$

$$\frac{is}{\hbar}L_y = s\left(\cos\phi\,\partial_\theta - \sin\phi\cot\theta\,\partial_\phi\right)$$

$1 \cos\theta = \cos\theta$

$\frac{is}{\hbar}L_y\cos\theta = s\left(-\sin\theta\cos\phi + 0\right)$ look back here

$$\left(\frac{is}{\hbar}L_y\right)^2\cos\theta = s\left(\cos\phi\,\partial_\theta - \sin\phi\cot\theta\,\partial_\phi\right) s\left(-\sin\theta\cos\phi\right)$$

$$= s^2\left[-\cos^2\phi\cos\theta - \sin^2\phi\cot\theta\sin\theta\right] = s^2\left(-\cos\theta\right)$$

$\left(\frac{is}{\hbar}L_y\right)^3\cos\theta = + s^3\left(\sin\theta\cos\phi\right)$

$\left(\frac{is}{\hbar}L_y\right)^4\cos\theta = + s^4\cos\theta$

$$\text{sum} = \cos\theta\,\overbrace{\left(1 - \frac{s^2}{2} + \frac{s^4}{4!}\dots\right)}^{\cos s} - \sin\theta\cos\phi\,\overbrace{\left(s - \frac{s^3}{3!} + \dots\right)}^{\sin s} \checkmark$$

4.22

a) 0 b) $L_+ Y_{\ell\ell} = \hbar e^{i\phi}\left(\partial_\theta + i\cot\theta\, \partial_\phi\right) Y_{\ell\ell} = 0$

$\partial_\phi \to i\ell$

so $(\partial_\theta - \ell\cot\theta)\, Y_{\ell\ell} = 0$ ← notice that $\sin^\ell\theta$ works

long way: $\dfrac{dY}{Y} = \ell\cot\theta$

$\ln Y = \ell \int \cot\theta\, d\theta = \ell \ln(\sin\theta) = \ln\left[\sin^\ell\theta\right]$

$Y \sim \sin^\ell\theta$

$Y_{\ell\ell} = N \sin^\ell\theta\, \underbrace{\frac{1}{\sqrt{2\pi}}\, e^{i\ell\phi}}_{\text{normalizes } \phi \text{ integral}}$

$N^2 \int_0^\pi \sin^{2\ell+1}\theta\, d\theta = 1$ "+1" recall $d\Omega = \sin\theta\, d\theta\, d\phi$

according to Dwight 858.44 this integral is: $\dfrac{2\cdot 4\cdot 6 \cdots 2\ell}{3\cdot 5\cdot 7 \cdots 2\ell+1}\cdot 2$

eg for $\ell = 2$ $\dfrac{2\cdot 4}{3\cdot 5}\cdot 2 = \dfrac{16}{15}$

$\ell = 3$ $\dfrac{2\cdot 4\cdot 6}{3\cdot 5\cdot 7}\cdot 2 = \dfrac{32}{35}$

math doesn't seem to know this general formula

$Y = \sqrt{\dfrac{3\cdot 5\cdot 7 \cdots 2\ell+1}{2\cdot 4\cdot 6 \cdots 2\ell}}\, \dfrac{1}{\sqrt{4\pi}}\, \sin^\ell\theta\, e^{i\ell\phi}$

eg $Y_{33} \sim \sqrt{\dfrac{35}{64}}$ $Y_{44} \sim \sqrt{\dfrac{315}{512}} \leftarrow \dfrac{35\cdot 9}{64\cdot 8}$

$Y_{55} \sim \sqrt{\dfrac{693}{1024}} \leftarrow \dfrac{315\cdot 11}{512\cdot 10}$

4.22 $L_+ Y_{21} = \hbar\sqrt{2\cdot 3 - 1\cdot 2}\; Y_{22} = 2\hbar\, Y_{22}$

$$\parallel$$

$$\hbar e^{i\phi}\left(\partial_\theta + i\cot\theta\,\partial_\phi\right)\left\{-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}\right\}$$

$$\parallel$$

$$-\hbar e^{2i\phi}\sqrt{\frac{15}{8\pi}}\left(\cos^2\theta - \sin^2\theta - \cos^2\theta\right)$$

$$\parallel$$

$$+\hbar\sqrt{\frac{15}{8\pi}}\sin^2\theta\, e^{2i\phi}$$

$$\Rightarrow Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{2i\phi} \quad ✓$$

4.44 p154 $\Rightarrow R_{43} = \frac{1}{768\sqrt{35}}\, a^{-3/2}\left(\frac{r}{a}\right)^3 e^{-(r/4a)}$

in mathematica format $u_{43} = \frac{1}{4\sqrt{7!}}\left(\frac{2r}{4a}\right)^4 L_0^7\left(\frac{2r}{4a}\right) e^{-r/4a}$

$$\psi = R_{43}\, Y_{33}(\theta,\phi) \quad \text{or} \quad \frac{u_{43}}{r}\, Y_{33}(\theta,\phi)$$

$$\int_0^\infty R_{43}^2\, r^4\, dr \iint Y_{33}^*\, \cos^2\theta\, Y_{33}\, \sin\theta\, d\theta\, d\phi$$

part $r^2 dr$

part $z^2 = r^2\cos^2\theta$

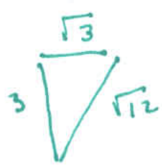
in Mathematica format units are $(\text{Bohr radius})^2 = a^2$

Note $\langle\ \rangle = \int \psi^*\ \psi\, dV$ ← $dx\,dy\,dz$ or $r^2 dr \sin\theta\, d\phi$ (sphere) or $r\,dr\,d\phi\,dz$ (cylinder)

c) $\underbrace{L_x^2 + L_y^2 = L^2 - L_z^2}$

Y_{lm} is eigenfunction with eigenvalue: $\hbar^2\left[l(l+1) - m^2\right]$

$\therefore$ with prob = 100% you would find: $\hbar^2\left[12 - 9\right] = 3\hbar^2$



27 $\quad 1 = \chi^\dagger \chi = |A|^2 (3^2 + 4^2) \Rightarrow A = \frac{1}{5}$

$$\langle S_x \rangle = \frac{\hbar}{2} \frac{1}{25} (-3i, 4) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3i \\ 4 \end{pmatrix} = 0$$

$$\hookrightarrow \begin{pmatrix} 4 \\ 3i \end{pmatrix}$$

$$\to -12i + 12i$$

$$\langle S_y \rangle = \frac{\hbar}{2} \frac{1}{25} (-3i, 4) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 3i \\ 4 \end{pmatrix} = \frac{\hbar}{2} \frac{-24}{25}$$

$$\hookrightarrow \begin{pmatrix} -4i \\ -3 \end{pmatrix}$$

$$\to (-12 \; -12)$$

$$\langle S_z \rangle = \frac{\hbar}{2} \frac{1}{25} (-3i, 4) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3i \\ 4 \end{pmatrix} = \frac{\hbar}{2} \frac{1}{25} (-7)$$

$$\hookrightarrow \begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$\to \begin{pmatrix} 9 \\ -16 \end{pmatrix}$$

Note: since $\sigma_i^2 = \overleftrightarrow{1}$

$$S_i^2 = \frac{\hbar^2}{4} \overleftrightarrow{1}$$

$$\sigma_{S_x}^2 = \langle S_x^2 \rangle - \langle S_x \rangle^2 = \frac{\hbar^2}{4} - 0 \Rightarrow \sigma_{S_x} = \frac{\hbar}{2}$$

$$\sigma_{S_y}^2 = \langle S_y^2 \rangle - \langle S_y \rangle^2 = \frac{\hbar^2}{4} \left(1 - \left(\frac{24}{25}\right)^2\right) \qquad \sigma_{S_y} = \frac{\hbar}{2} .28$$

$$\sigma_{S_z}^2 = \langle S_z^2 \rangle - \langle S_z \rangle^2 = \frac{\hbar^2}{4} \left(1 - \left(\frac{7}{25}\right)^2\right) \qquad \sigma_{S_z} = \frac{\hbar}{2} .96$$

$$\sigma_{L_x} \sigma_{L_y} = \frac{\hbar^2}{4} (1 \times .28) \; ? \; \frac{\hbar^2}{4} \frac{7}{25} \qquad .28 \geq .28 \checkmark$$

$$\sigma_{L_y} \sigma_{L_z} = \frac{\hbar^2}{4} (.28 \times .96) \; ? \; \frac{\hbar^2}{4} \cdot 0 \checkmark$$

$$\sigma_{L_z} \sigma_{L_x} = \frac{\hbar^2}{4} (.96 \times 1) \; ? \; \frac{\hbar^2}{4} \left(\frac{24}{25}\right) \qquad .96 \geq .96 \checkmark$$

Clebsch-Gordan Problem — $\overrightarrow{2 + 3/2} = \frac{7}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$

$$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{5}}\,|2\,2\rangle|\tfrac{3}{2}\,{-\tfrac{3}{2}}\rangle - \sqrt{\tfrac{3}{10}}\,|2\,1\rangle|\tfrac{3}{2}\,{-\tfrac{1}{2}}\rangle + \sqrt{\tfrac{1}{5}}\,|2\,0\rangle|\tfrac{3}{2}\,\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{10}}\,|2\,{-1}\rangle|\tfrac{3}{2}\,\tfrac{3}{2}\rangle$$

$$|2\,0\rangle|\tfrac{3}{2}\,\tfrac{1}{2}\rangle = \sqrt{\tfrac{18}{35}}\,|\tfrac{7}{2}\,\tfrac{1}{2}\rangle - \sqrt{\tfrac{3}{35}}\,|\tfrac{5}{2}\,\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{5}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{5}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

old exam #4

$S_x = \dfrac{S_+ + S_-}{2}$

$S_+ |s\,m\rangle = \hbar\sqrt{s(s+1) - m(m+1)}\;|s\;\, m{+}1\rangle$

$S_- |s\,m\rangle = \hbar\sqrt{s(s+1) - m(m-1)}\;|s\;\, m{-}1\rangle$

$S_+ \uparrow = 0 \qquad S_+ \rightarrow = \hbar\sqrt{2}\uparrow \qquad S_+ \downarrow = \hbar\sqrt{2}\rightarrow$

$S_- \uparrow = \hbar\sqrt{2}\rightarrow \qquad S_- \rightarrow = \hbar\sqrt{2}\downarrow \qquad S_- \downarrow = 0$

$S_x \uparrow = \dfrac{\hbar}{\sqrt{2}} \rightarrow$

$S_x \rightarrow = \dfrac{\hbar}{\sqrt{2}}(\uparrow + \downarrow)$

$S_x \downarrow = \dfrac{\hbar}{\sqrt{2}} \rightarrow$

$$[S_x] = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

#1 $\quad J_z \to \hbar(+\tfrac12) \qquad L^2 = \hbar^2\, 3\cdot 4 \qquad S^2 = \hbar^2\, \tfrac12 \cdot \tfrac32$

$$J^2 = \underset{\substack{\downarrow \\ 12}}{L^2} + \underset{\substack{\downarrow \\ \frac34}}{S^2} + L_+S_- + L_-S_+ + 2L_zS_z$$

$L_+ Y_{30} = \hbar\sqrt{12}\, Y_{31}$

$L_- Y_{31} = \hbar\sqrt{12}\, Y_{30}$

$S_+ \downarrow = \hbar \uparrow$

$S_- \uparrow = \hbar \downarrow$

overall factor of $\hbar^2$!

$L_+S_-\left(\sqrt{\tfrac47}\, Y_{31}\downarrow - \sqrt{\tfrac37}\, Y_{30}\uparrow\right) = -\sqrt{\tfrac37}\sqrt{12}\, Y_{31}\downarrow = -3\sqrt{\tfrac47}\, Y_{31}\downarrow$

$L_-S_+\left(\qquad\qquad\right) = \sqrt{\tfrac47}\sqrt{12}\, Y_{30}\uparrow = 4\sqrt{\tfrac37}\, Y_{30}\uparrow$

$2L_zS_z\left(\qquad\qquad\right) = \sqrt{\tfrac47}\; 2\cdot 1\cdot(-\tfrac12)\, Y_{31}\downarrow = -\sqrt{\tfrac47}\, Y_{31}\downarrow$

$-4\cdot\left(\sqrt{\tfrac47}\, Y_{31}\downarrow - \sqrt{\tfrac37}\, Y_{30}\uparrow\right)$

$\therefore\; J^2\psi = (12 + \tfrac34 - 4)\,\psi$

$= (\tfrac52 \cdot \tfrac72)\,\psi \;\Rightarrow\; j = \tfrac52$

4.49 $1 = \chi^{+}\chi = |A|^2\left[1^2+2^2+2^2\right] = 9|A|^2 \quad A = \frac{1}{3}$

$\langle S_z\rangle = \frac{1}{9}\,(1+2i,\ 2)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1-2i\\ 2\end{pmatrix}\frac{\hbar}{2} = \frac{\hbar}{2}\left(\frac{5}{9}-\frac{4}{9}\right) = \frac{\hbar}{2}\frac{1}{9}$

$\langle S_x\rangle = \frac{1}{9}\,(1+2i,\ 2)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1-2i\\ 2\end{pmatrix}\frac{\hbar}{2} = \frac{\hbar}{2}\left[\frac{4}{9}\right]$

$\langle S_y\rangle = \frac{1}{9}\,(1+2i,\ 2)\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1-2i\\ 2\end{pmatrix}\frac{\hbar}{2} = \frac{\hbar}{2}\left[\frac{8}{9}\right]$

S_z prob: $+\frac{1}{2}$: $\chi_+^{+}\cdot\chi = \frac{1}{3}(1-2i) \rightarrow \frac{5}{9}$

$-\frac{1}{2}$: $\chi_-^{+}\cdot\chi = \frac{1}{3}(2) \rightarrow \frac{4}{9}$

S_x prob $+\frac{1}{2} \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $\quad \chi_+^{+}\cdot\chi = \frac{1}{\sqrt{2}\,3}(1-2i+2) \rightarrow \frac{13}{18}$

see Eq 4.151 $\quad -\frac{1}{2} \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ $\quad \chi_-^{+}\cdot\chi = \frac{1}{\sqrt{2}\,3}(1-2i-2) \rightarrow \frac{5}{18}$

S_y prob

see Eq 4.155 $\quad \frac{1}{2} \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ $\quad \chi_+^{+}\cdot\chi = \frac{1}{\sqrt{2}\,3}(1-2i+2i) \rightarrow \frac{17}{18}$

$\theta = 90^\circ$
$\phi = 90^\circ \rightarrow \hat{y}$ $\quad -\frac{1}{2} \leftrightarrow -\frac{1}{\sqrt{2}}\begin{pmatrix}i\\1\end{pmatrix}$ $\quad \chi_-^{+}\cdot\chi = \frac{-1}{\sqrt{2}\,3}(-i-2+2) \rightarrow \frac{1}{18}$

4.55 note: This is comb $l=1$ & $s=\frac{1}{2}$ from CG table see ~~is~~ $j=3/2$, $m=1/2$

a: 100% $\hbar\, l(l+1)$ with $l=1$

(b) $\frac{1}{3}$ $0\hbar$, $\frac{2}{3}$ $1\hbar$

(c) 100% $\hbar\, s(s+1)$ with $s=1/2$

(d) $\frac{1}{3}$ $+\frac{1}{2}\hbar$, $\frac{2}{3}$ $-\frac{1}{2}\hbar$

(e) from CG table $\quad$ (j, m_j)

$Y_{10}\uparrow = \sqrt{\frac{2}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle - \sqrt{\frac{1}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$Y_{11}\downarrow = \sqrt{\frac{1}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\frac{2}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$\sqrt{\frac{1}{3}}\,Y_{10}\uparrow + \sqrt{\frac{2}{3}}\,Y_{11}\downarrow = \frac{2\sqrt{2}}{3}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \frac{1}{3}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$j = 3/2$ prob $= 8/9$ $\qquad j = \frac{1}{2}$ prob $= 1/9$

(f) 100% $J_z = +\frac{1}{2}\hbar$

(g) $|\psi|^2$ $\qquad$ (h) $\frac{1}{3}|R_{21}|^2$

5.7) (c) $\begin{vmatrix} \psi_a(x_1) & \psi_b(x_1) & \psi_c(x_1) \\ \psi_a(x_2) & \psi_b(x_2) & \psi_c(x_2) \\ \psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3) \end{vmatrix}$ $= \psi_a(1)\psi_b(2)\psi_c(3) + \psi_b(1)\psi_c(2)\psi_a(3)$

$+ \psi_c(1)\psi_a(2)\psi_b(3)$

$- \psi_c(1)\psi_b(2)\psi_a(3) - \psi_b(1)\psi_a(2)\psi_c(3)$

$- \psi_a(1)\psi_c(2)\psi_b(3)$

shorthand: $\psi_b(3)$

for (b) replace all $-$ with $+$

(a) $\psi_a(x_1)\psi_b(x_2)\psi_c(x_3)$

cm.pdf. $k_1\cdot r_1 + k_2\cdot r_2 = (\vec{k_1}+\vec{k_2})\cdot\vec{R} + \left(\frac{\vec{k_1}m_2}{M} - \frac{\vec{k_2}m_1}{M}\right)\cdot\vec{r} = \mu(\vec{v_1}-\vec{v_2})\cdot\vec{r}$

$r_1 = R + \frac{m_2}{M}r$, $r_2 = R - \frac{m_1}{M}r$

$\frac{\vec{k_1}m_2}{M} \to \underbrace{\frac{m_1 m_2}{M}}_{\mu}\vec{v_1}$, $\frac{\vec{k_2}m_1}{M} \to \frac{m_2 m_1}{M}\vec{v_2}$

define $K = k_1 + k_2$

$$k = \left(\vec{k_1}\frac{m_2}{M} - \vec{k_2}\frac{m_1}{M}\right) \xrightarrow[\frac{m_1}{M}=\frac{1}{2}]{\text{identical}} \frac{1}{2}(k_1-k_2)$$

so $e^{i(\vec{k_1}\cdot\vec{r_1}+\vec{k_2}\cdot\vec{r_2})} = e^{i(\vec{K}\cdot\vec{R}+\vec{k}\cdot\vec{r})}$

clearly if swap labels $k_1 \leftrightarrow k_2$: $K \leftrightarrow K$
& identical particles $k = \frac{1}{2}(k_1-k_2) \leftrightarrow -k$

so $e^{i(k_1\cdot r_1+k_2\cdot r_2)} \pm e^{i(k_2 r_1 + k_1 r_2)} = e^{iK\cdot R}\left(e^{ik\cdot r} \pm e^{-ikr}\right)$

$$= \begin{cases} 2e^{iKR}\cos k\cdot r \\ 2i\, e^{iKR}\sin(k\cdot r) \end{cases}$$

PE: $x_1^2 + x_2^2 = (X + \frac{m_2}{M}x)^2 + (X - \frac{m_1}{M}x)^2 = 2X^2 + \frac{1}{2}x^2$ ($\frac{m_2}{M} = 1/2$, $\frac{m_1}{M} = \frac{1}{2}$)

KE: $\frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} = \frac{P^2}{2M} + \frac{p^2}{2\mu}$ ($2M$ → $2m$, μ → $m/2$)

so $H = \left(\frac{P^2}{4m} + \frac{1}{2}m\omega^2 2X^2\right) + \left(\frac{p^2}{m} + \frac{1}{2}m\omega^2\frac{1}{2}x^2\right)$

$$= \left(\frac{P^2}{2M} + \frac{1}{2}M\omega^2X^2\right) + \left(\frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2x^2\right)$$

both are of SHO form
$\therefore E = \hbar\omega(n+1/2)$
see no ms!

$E = \hbar\omega(N+\frac{1}{2}) + \hbar\omega(n+1/2)$

using a product wavefunction $\psi = \psi_N(X)\,\psi_n(x)$

fermions require n odd (0,1) (1,1) (0,3) (2,1)
bosons require n even (0,0) (1,0) (2,0) (0,2) since $X \leftrightarrow X$ is even

nth deg poly in X

5.16 electron number density = nuclei number density $= \frac{8.96\ g}{cm^3}\ \frac{mole}{63.5\ g}$

$\times\ \frac{6\times10^{23}}{mole} \times \frac{10^6\ cm^3}{m^3}$

$= 8.497\times10^{28}\ \frac{1}{m^3}$ → Convert to nm^3: $84.97\ \frac{1}{nm^3}$

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = \frac{(\hbar c)^2}{2mc^2}(3\pi^2 n)^{2/3}$$

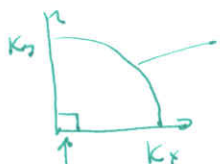
$$= \frac{(197.33)^2}{2(.511\times10^6)}(3\pi^2\, 84.97)^{2/3} = 7.05\ eV$$

using $\hbar c = 197$ eV·nm, $m_e c^2 = .511\times10^6$ eV

$= k_B T$ ← 8.18×10^4 K

$$P = \frac{(3\pi^2)^{2/3}(\hbar c)^2}{5\ mc^2}\, n^{5/3} = 239.54\ \frac{eV}{nm^3} = 3.84\times10^{10}\ \frac{N}{m^2}$$

5.34



brick → tile $= \frac{\pi^2}{A}$

$$N_{states} = \frac{\frac{1}{4}\pi k_F^2}{\pi^2/A} = \frac{N_{electrons}}{2}$$

$$\frac{k_F^2}{2\pi} = \sigma = \frac{N_{electrons}}{Area}$$

$$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}2\pi\sigma = \frac{\hbar^2\pi}{m}\sigma$$

Atomic Handout

O II $= O^+ = (1s)^2(2s)^2(2p)^3 \rightarrow {}^2P,\ {}^2D,\ {}^4S \rightarrow {}^2P_{3/2},\ {}^2P_{1/2},\ {}^2D_{5/2},\ {}^2D_{3/2},\ {}^4S_{3/2}$

O III $= O^{++} = (1s)^2(2s)^2\ (2p)^2$

N II $= N^+ = (1s)^2(2s)^2(2p)^2$

$\Big\}\ {}^1S,\ {}^1D,\ {}^3P \rightarrow {}^1S_0,\ {}^1D_0,\ {}^3P_2,\ {}^3P_1,\ {}^3P_0$

Hund correct

degeneracy of ${}^S L_j = 2j+1$

need to combine S & L to get total angular momentum j