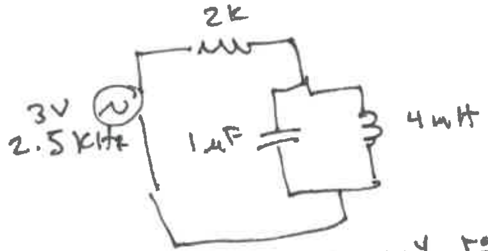
phys 341 Class 36 Ch 13 : problems18.pdf : 32 & 24

32)



$$Z_{eq} = R + \frac{1}{i\omega C + \frac{1}{i\omega L}}$$

$$= R + \frac{i}{\underbrace{\frac{1}{\omega L} - \omega C}_{2.075\times10^{-4}}}$$

$\omega = 2\pi f = 1.57\times10^{4}$ rad/sec

$$= (2 + i\,4.819)\ \text{k}\Omega$$

$$= 5217 \angle 67.5^\circ$$

↑ volt lead

$$V = \frac{i\,4.819}{2 + i\,4.819}\cdot 3 = (.853 + i\,.354)3$$

$$= (.924 \angle 22.5^\circ)3 = 2.77\text{V} \angle 22.5^\circ$$

$$A = \frac{V}{\omega L} = \frac{2.77}{\omega L} = 17.6\ \text{mA}$$

24) Mathematica would be a faster solution — but lets do this "by hand"

$$\frac{1}{Z_T} = \frac{1}{\frac{1}{i\omega C} + R} + \frac{1}{i\omega L + R} = \frac{R + \frac{i}{\omega C}}{R^2 + \left(\frac{1}{\omega C}\right)^2} + \frac{R - i\omega L}{R^2 + (\omega L)^2}$$

$$= \frac{\left(R + \frac{i}{\omega C}\right)\left(R^2 + (\omega L)^2\right) + (R - i\omega L)\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)\left(R^2 + (\omega L)^2\right)}$$

start by looking at imaginary part of numerator —

$$\frac{i}{\omega C}\left(R^2 + (\omega L)^2\right) - i\omega L\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right) = i\left[R^2\left(\frac{1}{\omega C} - \omega L\right) + \frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$= i\left[\left(R^2 - \frac{L}{C}\right)\left(\frac{1}{\omega C} - \omega L\right)\right]$$

so.. $= 0$ if $R^2 = L/C$

now look at real part: $R\left[2R^2 + (\omega L)^2 + \left(\frac{1}{\omega C}\right)^2\right]$

compare to denominator: $R^2\left[R + \frac{1}{R}\frac{1}{(\omega L)^2}\right]\left[R + \frac{(\omega L)^2}{R}\right]$

$$= R^2\left[R^2 + \frac{1}{(\omega C)^2} + (\omega L)^2 + \frac{(L/C)^2}{R^2}\right]$$

$\hookrightarrow R^2$

so : $\frac{1}{Z_T} = \frac{R}{R^2} \Rightarrow Z_T = R$ ✓