341 – Class 19 – capsl.txt ; Figure 6-4, 6-17, 6-26

Figure 6-4 – width (into/out page) w

view as parallel combo of a cap filled with dielectric $A = w \cdot x$ & a cap filled with vacuum $A = w(\ell - x)$

$$C = \frac{K \varepsilon_0 w x}{d} + \frac{\varepsilon_0 w (\ell - x)}{d} = \frac{\varepsilon_0 w}{d}(\ell + (K-1)x)$$

$$\frac{dC}{dx} = \frac{\varepsilon_0 w}{d}(K-1)$$

6-17 → working as suggested – ϕ must satisfy Laplace in spherical coordinates ⟹ $\phi = \frac{A}{r} + C$; the constant C can have no effect so: $\Delta\phi = A\left(\frac{1}{a} - \frac{1}{b}\right)$

$E_r = \frac{A}{r^2}$, $D_r = \frac{K\varepsilon_0 A}{r^2}$ } zero elsewhere

on inner shell: $\sigma = D\big|_{r=a} = \frac{K\varepsilon_0 A}{a^2}$

$$Q = \sigma \cdot 4\pi a^2 = 4\pi K \varepsilon_0 A$$

$$U = \frac{1}{2}\int_a^b E D\, 4\pi r^2 dr = \frac{1}{2} A^2 K \varepsilon_0 \int_a^b \frac{1}{r^2} 4\pi\, dr$$

$$= \frac{1}{2} 4\pi A^2 K \varepsilon_0 \left(-r^{-1}\right)_a^b = \frac{1}{2} 4\pi A^2 K \varepsilon_0 \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{1}{2} \underbrace{\frac{4\pi K \varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}}_{C} \left[A\left(\frac{1}{a} - \frac{1}{b}\right)\right]^2$$

compare: $C = \frac{Q}{\Delta\phi} = \frac{4\pi K \varepsilon_0 A}{A\left(\frac{1}{a} - \frac{1}{b}\right)}$ ✓

Alt: assume a spherically symmetric Q

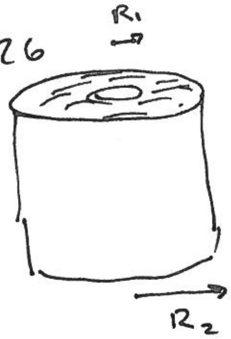
$\rightarrow D = \frac{Q}{4\pi r^2} \rightarrow E = \frac{Q}{4\pi K \varepsilon_0 r^2} \rightarrow \phi(a) - \phi(b) = \int_a^b E\, dr$

$$= \frac{Q}{4\pi K \varepsilon_0}\int_a^b \frac{1}{r^2} dr = \frac{Q}{4\pi \varepsilon_0 K}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$U = \frac{1}{2} Q \Delta\phi = \frac{1}{2}\left(\frac{\Delta\phi\, 4\pi K \varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}\right)\Delta\phi$$

$$= \frac{1}{2} \frac{4\pi K \varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \Delta\phi^2$$ ✓

6-26



must satisfy cylindrical Laplace w symmetry

$$\phi = A \ln(r) + C \rightarrow \Delta\phi = \phi(R_1) - \phi(R_2)$$

$$= A \ln(R_1/R_2)$$

$$E = \frac{-A}{r}$$

$$D = \frac{-A K \varepsilon_0}{r}$$

on R_1: $\sigma_f = D\Big|_{r=R_1} = \frac{-A K \varepsilon_0}{R_1}$

total charge in length $l = \overbrace{2\pi R_1 l}^{\text{Area}} \sigma = -2\pi A K \varepsilon_0 l$

$$C = \frac{Q}{\Delta\phi} = \frac{-2\pi A K \varepsilon_0 l}{A \ln(R_1/R_2)} = \underbrace{\frac{+2\pi K \varepsilon_0}{\ln(R_2/R_1)}}_{\text{capacitance per length}} l$$

Note: as in 17 an alternative would be to calculate energy density & get C from that.