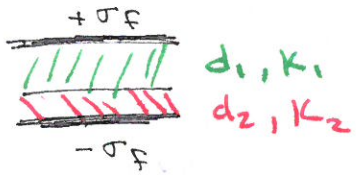
341 – class 18 – Ch 6 – 6-13 & old exam #2 & #3

6-13) – like previous problem 4-16

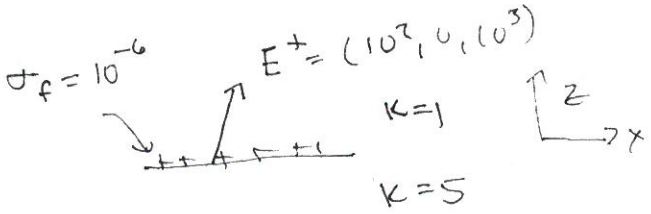


$D = \sigma_f$; $E = \frac{\sigma_f}{\varepsilon_0 K}$ ← for which ever material

$$\Delta\psi = E_1 d_1 + E_2 d_2 = \frac{\sigma_f}{\varepsilon_0}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)$$

$$= \frac{Q}{\varepsilon_0 A}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right) = \frac{Q}{C}$$

$$C = \frac{\varepsilon_0 A}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$$



#2
$\Delta D_n = \sigma_f$
$\Delta E_t = 0 \rightarrow E_x^- = 10^2$

$D_z^+ - D_z^- = 10^{-6} \Rightarrow D_z^+ - 10^{-6} = \varepsilon_0 10^3 - 10^{-6} = D_z^-$ (D_z^+)

$D_z^- = (8.85\times10^{-12})(10^3) - 10^{-6}$

$D_z^- = 8.85\times10^{-9} - 10^{-6} = -.991\times10^{-6}\ \frac{C}{m^2}$

$= 5\varepsilon_0 E_z^- \rightarrow E_z^- = \frac{-.991\times10^{-6}}{5\cdot 8.85\times10^{-12}} = -2.24\times10^4\ \frac{V}{m}$

$D_x^+ = \varepsilon_0 E_x^+ = 8.85\times10^{-12}\cdot 10^2 = 8.85\times10^{-10}\ \frac{C}{m^2}$

$D_x^- = 5\varepsilon_0 E_x^- = 5\cdot 8.85\times10^{-12}\cdot 10^2 = 4.43\times10^{-9}\ \frac{C}{m^2}$

$\vec{D}^+ = (8.85\times10^{-10},\ 0,\ 8.85\times10^{-9})$

$\vec{D}^- = (4.43\times10^{-9},\ 0,\ -.991\times10^{-6})$ } $\frac{C}{m^2}$

$\vec{E}^- = (10^2,\ 0,\ -2.24\times10^4)\ V/m$

#3

$r<a$, $r>b$ } no enclosed charge (free or bound) $E=D=0$

between: $4\pi r^2 D = Q$

$$D = \frac{Q}{4\pi r^2} \qquad E \to \frac{Q}{4\pi K\varepsilon_0 r^2}$$

$$V(a) - V(b) = \int_a^b E\,dr = \frac{Q}{4\pi\varepsilon_0 K}\int_a^b \frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0 K}\left[-r^{-1}\right]_a^b = \frac{Q}{4\pi\varepsilon_0 K}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$V(a) - V(b) = \Delta V$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0 K}\left[\frac{1}{a}-\frac{1}{b}\right] \Rightarrow \quad C = \frac{4\pi\varepsilon_0 K}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$U = \frac{1}{2} Q\,\Delta V = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 K}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$OR = \frac{1}{2}\int_a^b \vec{E}\cdot\vec{D}\; 4\pi r^2 dr = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 K}\,\frac{Q}{4\pi}\int_a^b \frac{1}{r^2}\,4\pi\,dr$$

$$= \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 K}\left[-r^{-1}\right]_a^b = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 K}\left(\frac{1}{a}-\frac{1}{b}\right) \quad \checkmark \text{ same}$$