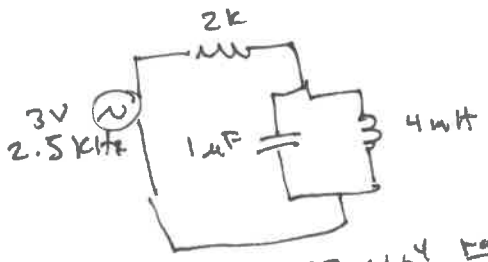
Phys 341 Class 36 Ch 13: Problems18.pdf : 32 & 24

32)



$\omega = 2\pi f = 1.57\times10^4$ rad/sec

$$Z_{eq} = R + \frac{1}{i\omega C + \frac{1}{i\omega L}}$$

$$= R + \frac{i}{\frac{1}{\omega L} - \omega C} \quad \} \; 2.075\times10^{-4}$$

$$= (2 + i\,4.819)\,k\Omega$$

$$= 5217 \angle 67.5^\circ$$

$$V = \frac{i\,4.819}{2 + i\,4.819}\cdot 3 = (.853 + i\,.354)\,3 \quad \uparrow \text{volt lead}$$

$$= (.924 \angle 22.5^\circ)\,3 = 2.77\,V \angle 22.5^\circ$$

$$A = \frac{V}{\omega L} = \frac{2.77}{\omega L} = 49.1\ mA$$

24) Mathematica would be a fast solution — but lets do this "by hand"

$$\frac{1}{Z_T} = \frac{1}{\frac{1}{i\omega C} + R} + \frac{1}{i\omega L + R} = \frac{R + \frac{i}{\omega C}}{R^2 + \left(\frac{1}{\omega C}\right)^2} + \frac{R - i\omega L}{R^2 + (\omega L)^2}$$

$$= \frac{\left(R + \frac{i}{\omega C}\right)\left(R^2 + (\omega L)^2\right) + \left(R - i\omega L\right)\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)\left(R^2 + (\omega L)^2\right)}$$

start by looking at imaginary part of numerator —

$$\frac{i}{\omega C}\left(R^2 + (\omega L)^2\right) - i\omega L\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right) = i\left[R^2\left(\frac{1}{\omega C} - \omega L\right) + \frac{1}{C}\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$= i\left[\left(R^2 - \frac{1}{C}\right)\left(\frac{1}{\omega C} - \omega L\right)\right] \quad \text{so..} = 0 \text{ if } R^2 = L/C$$

now look at real part: $R\left[2R^2 + (\omega L)^2 + \left(\frac{1}{\omega C}\right)^2\right]$

compare to denominator: $R^2\left[R + \frac{1}{R}\frac{1}{(\omega L)^2}\right]\left[R + \frac{(\omega L)^2}{R}\right]$

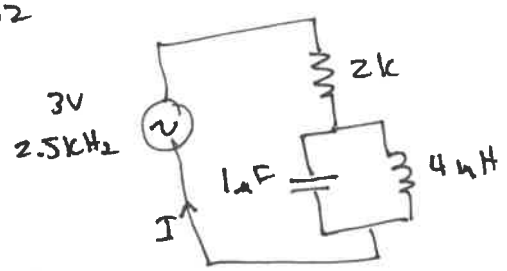
$$= R^2\left[R^2 + \frac{1}{(\omega C)^2} + (\omega L)^2 + \frac{(L/C)^2}{R^2}\right]$$

$\to R^2$

so: $\frac{1}{Z_T} = \frac{R}{R^2} \Rightarrow Z_T = R$ ✓

phys 341 – class 36 – problem 18.pdf #32 & #24

#32



$\omega = 2\pi \cdot 2.5\times10^3 = 1.57\times10^4$ rad/sec

$X_L = \omega L = 62.8\,\Omega$

$X_C = \frac{1}{\omega C} = 63.7\,\Omega$

$Z_{eq} = R + \dfrac{1}{\frac{1}{i\omega L} + i\omega C}$

$= R + \dfrac{i}{\frac{1}{\omega L} - \omega C}$

$= 2000 + i\,4819\ \Omega$

$I = \dfrac{3V}{2000 + i4819}$

$= (.220 - i.531)\times10^{-3}A$

$= 5.75\times10^{-4}\,A \angle -67^\circ$

voltage leads

voltage across  from $Z = i4819$ & $I = 5.75\times10^{-4}$ A

$V = 4819 \cdot 5.75\times10^{-4} = 2.77$ V

current thru $= \dfrac{V}{\omega L} = \dfrac{2.77}{62.8} = 44.1$ mA (complex values not requested or wanted)