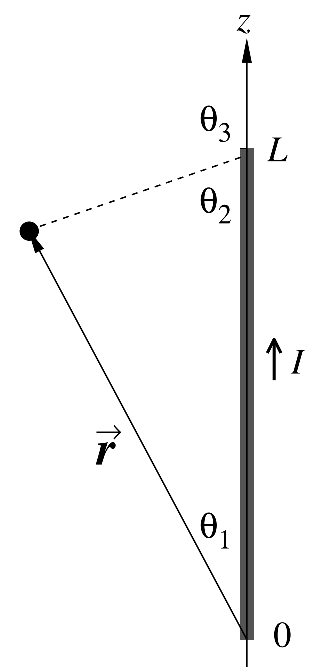
1. The standard infinite parallel plate capacitor (separation d) is charged to a potential difference V and then disconnected from the voltage source. It is then modified by inserting, in parallel between the plates, a dielectric with thickness $t < d$ and dielectric constant K.

   (a) Find $V(z), \mathbf{E}, \mathbf{D}, \sigma_f, \sigma_b, \rho_b$ (if appropriate) both before and after the dielectric is inserted.

   (b) Find the capacitance per unit area in both cases.

   (c) Find the energy per unit area stored in the capacitor before and after the dielectric is inserted (careful…what is constant?). What happened to the missing energy?

2. Consider a current I flowing along the z axis from $z = 0$ to $z = L$. (Current conservation says is is impossible for current to come from nowhere at $z = 0$ and then disappear to nowhere at $z = L$, but nevertheless that's what this problem supposes.) Describe (words, sketch?) the direction/symmetry of the resulting magnetic field.



   Report what you are using for $\mathbf{r}, \mathbf{r}', d\boldsymbol{\ell}$. Calculate the cross product and write down the integral for the magnetic field vector. Integrate your results perhaps using Dwight 200.03:

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

   Note that to make your integral look like Dwight's you may have to make a substitution and appropriately adjust the range of integration.

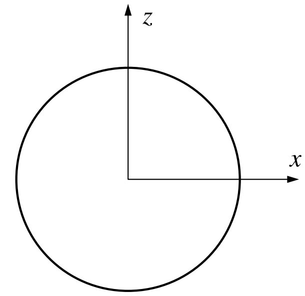
   Try to rewrite your formula in terms of $\cos\theta_1$ and $\cos\theta_2$ shown right. (Note that since $\theta_2 = \pi - \theta_3$, it follows: $\cos\theta_2 = -\cos\theta_3$.) Note that if the wire is actually infinite, $\theta_1 = \theta_2 = 0$. In that case does your formula match the well-know result for an infinite wire?

   Describe (words, sketch?) the direction/symmetry of the vector potential $\mathbf{A}$. Write down the integral for the vector $\mathbf{A}$. Integrate your results perhaps using Dwight 200.01:

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)$$

   (Is this the first time you've seen inverse hyperbolic sin? FYI: *Mathematica* will give you the same result but in terms of log.)

3. A plastic sphere of radius R with dielectric constant K is surrounded by vacuum. An azimuthally symmetric free surface charge has been placed on the surface of the sphere:



$$\sigma(\theta) = \alpha\, \epsilon_0\, \frac{1}{2}\left(3\cos^2\theta - 1\right) = \alpha\, \epsilon_0\, P_2(\cos\theta)$$

where α is a given constant. Aim: find the resulting voltage ϕ inside and outside the sphere. This is a rather long problem, so let's step through the problem together. What counts in this problem is explanations (words) not calculations.

(a) Using the provided circle that represents the pole-to-pole great circle that is the intersection of the xz plane and the sphere, mark where the free surface charge is positive (mark +), negative (mark −) or zero (mark 0). Would you call this an even, odd or neither charge distribution? Explain.

(b) I claim the following general forms for the voltage inside ($r < R$) and outside ($r > R$) the sphere:

$$\begin{aligned} \phi_{\text{in}}(r,\theta) &= \sum_{n=0}^{\infty} A_n r^n P_n(\cos\theta) \\ \phi_{\text{out}}(r,\theta) &= \sum_{n=0}^{\infty} C_n r^{-n-1} P_n(\cos\theta) \end{aligned}$$

where A_n and C_n are currently undetermined constants. Explain why ϕ must have this particular form. What assumptions have been used?

(c) At the boundary (i.e., $r = R$) the following conditions must hold:

$$\begin{aligned} \phi_{\text{in}}(R,\theta) &= \phi_{\text{out}}(R,\theta) \\ \alpha\frac{1}{2}\left(3\cos^2\theta - 1)\right) &= K\left.\frac{\partial\phi_{\text{in}}}{\partial r}\right|_{r=R} - \left.\frac{\partial\phi_{\text{out}}}{\partial r}\right|_{r=R} \end{aligned}$$

Explain why these particular conditions must hold.

(d) I conclude from the first condition that, for every n,

$$A_n R^{2n+1} = B_n$$

Explain the basis for this conclusion.

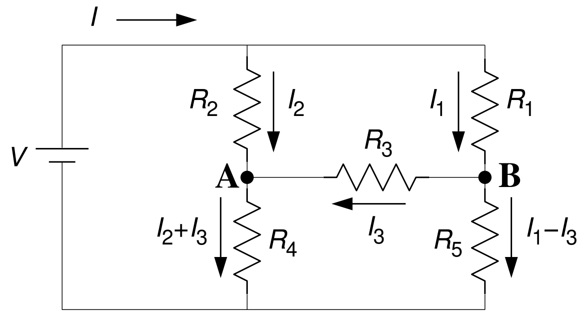
(e) The second boundary condition is more easily decomposed using the fact that the lhs is just $\alpha P_2(\cos\theta)$ from which I conclude:

$$\begin{aligned} \alpha &= K2A_2R + C_2 3R^{-4} && \text{for: } n = 2 \\ 0 &= KnA_nR^{n-1} + (n+1)C_nR^{-n-2} && \text{for: } n \neq 2 \end{aligned}$$

Explain/prove how the above follows from the second boundary condition.

(f) Explain why this means $A_n = C_n = 0$ for: $n \neq 2$.

4. Recall the circuit shown:



Applying Kirchhoff's laws, write down the linear equations needed to determine the currents $I_1, \ldots, I_5$, where V and $R_1, \ldots, R_5$ are known constants. If you solve these equations (solving is not required) you'd find $I_3 = 0$ if $R_1/R_5 = R_2/R_4$.

Write down the linear equations for the voltages V_A and V_B using the nodal version of Kirchhoff's laws for this circuit again considering V and $R_1, \ldots, R_5$ as known. If you solve these equations (not required) you'd find $V_A = V_B$ if $R_1/R_5 = R_2/R_4$.

Bonus: See if you can come up with a simple explanation for why if $I_3 = 0$, you must have $V_A = V_B$ and $R_1/R_5 = R_2/R_4$