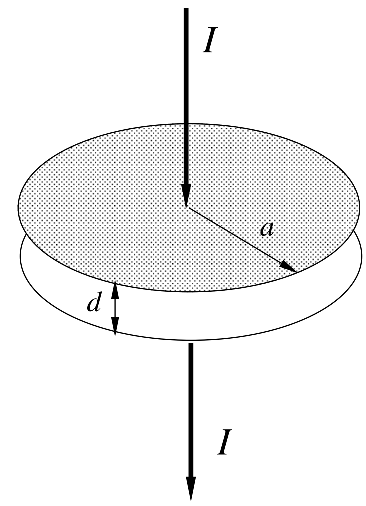
1. An ideal capacitor consists of two circular plates of radius a separated by a distance d surrounded by vacuum. Assume the E-field is uniform between the plates (i.e., neglect the fringing field at the edge of the plates). The capacitor is being charged by a constant current I.



   (a) Find the electric field between the plates as a function of time.

   (b) Find the displacement current density between the plates. Using Ampère's law find the magnetic field $\vec{\mathbf{B}}(r)$ between the plates (i.e., for any $r < a$) generated by the displacement current. Clearly state or show the direction of $\vec{\mathbf{B}}(r)$.

   (c) Find the Poynting vector on the circumference of the capacitor. Is energy entering or leaving the capacitor?

   (d) Integrate $\vec{\mathbf{S}} \cdot \hat{\mathbf{n}}$ over the cylindrical edge of the capacitor to find the energy flowing *into* the capacitor. Show that the result is equal to the *time rate of change* in the electric energy stored between the capacitor plates. (Use the electric energy density $\frac{1}{2}\,\vec{\mathbf{E}} \cdot \vec{\mathbf{D}}$ to find the total electric energy between the plates.)

2. Consider an infinite solenoid with radius R and N turns per meter filled with linear magnetic material (relative permeability $K_m \gg 1$). The current flowing around the solenoid is increasing, producing an increasing magnetic field which I name $\dot{B}$.

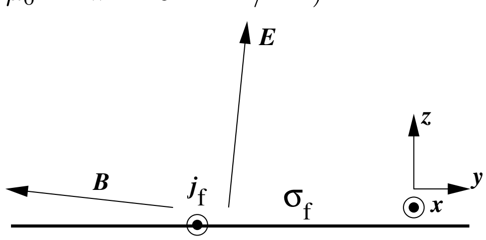
   (a) Calculate the magnetic energy stored in a length ℓ of the solenoid.

   (b) The changing magnetic field will induce an electric field. Find $\vec{\mathbf{E}}$ everywhere (inside and outside the solenoid). Provide a drawing that shows the direction of $\vec{\mathbf{E}}$.

   (c) Using $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ just inside the solenoid, calculate the Poynting vector (direction and magnitude).

   (d) Show that the rate of increase in the magnetic energy in a length ℓ of the solenoid matches the rate of energy inflow via the Poynting vector.

3. The $z = 0$ plane is the boundary between two materials: the region of space with $z > 0$ is vacuum, the region with $z < 0$ has $g = 0$, $\epsilon = 4\epsilon_0$ and $\mu = 1000\mu_0$. The boundary carries a surface charge density of $\sigma_f = 8.85 \times 10^{-8}$ C/m^2 and a surface current (flowing in the x direction) of $j_f = 10^3$ A/m. On the vacuum side of the boundary $\vec{\mathbf{E}} = 10^3\hat{\mathbf{j}} + 10^4\hat{\mathbf{k}}$ V/m, and $\vec{\mathbf{B}} = -10^{-4}\hat{\mathbf{j}} + 10^{-5}\hat{\mathbf{k}}$ T. Find $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ inside the material. ($\epsilon_0 = 8.85 \times 10^{-12}\quad \mathrm{C^2/(N \cdot m^2)}$, $\mu_0 = 4\pi \times 10^{-7}\quad \mathrm{N/A^2}$)



4. Consider the following electric and magnetic fields (in spherical coordinates):

$$\begin{aligned} \mathbf{E} &= -\frac{pk^2 \sin\theta\, e^{i(kr-\omega t)}}{4\pi\epsilon_0 r}\,\widehat{\boldsymbol{\theta}} \\ \mathbf{B} &= -\frac{p\mu_0\omega k \sin\theta\, e^{i(kr-\omega t)}}{4\pi r}\,\widehat{\boldsymbol{\phi}} \end{aligned}$$

where $\omega/k = c$ and p (electric dipole moment) is a constant. (Note that's kr not $\mathbf{k}\cdot\mathbf{r}$ and I mean to take the real part of the rhs). FYI: this is called electric dipole radiation.

(a) What are the units of p, k, and ω? Show that the resulting E, B have proper units.

(b) Plug the above into Maxwell's equations in vacuum (i.e., $\rho = 0$ and $\mathbf{J} = \mathbf{0}$). Which are satisfied? (Not all of them are, but for $k \gg 1/r$ (i.e., large r), I'd say: "close enough".)

(c) Note that these fields fall off like $1/r$ and hence the power through a distant surface $\hat{\mathbf{r}}\, dA = \hat{\mathbf{r}}\, r^2 d\Omega$ in a particular direction (θ, ϕ) is independent of r allowing us to talk about the power per solid angle going in a particular direction. Calculate the time-average Poynting vector ($\langle \mathbf{S} \rangle$) as a function of direction. Describe (words) the distribution of this light. How much total (all directions) light energy would leave a large radius sphere per second?

(d) A large distance from the origin on the positive x axis, the above E and B fields will look a bit like a plane wave:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 \exp\left(i(kx - \omega t)\right) \\ \mathbf{B} &= \mathbf{B}_0 \exp\left(i(kx - \omega t)\right) \end{aligned}$$

with $\mathbf{E}_0$, $\mathbf{B}_0$ falling off like $1/r$. Describe the directions of $\mathbf{E}_0$, $\mathbf{B}_0$ in rectangular coordinates (xyz).

5. To show: given $\mathbf{E}, \mathbf{B}$ that solve Maxwell's Equations in vacuum, you can create a new solution to Maxwell's Equations in vacuum essentially by adjusting units and swapping $\mathbf{E} \leftrightarrow \mathbf{B}$, in particular:

$$\begin{aligned} \mathbf{B}_{\text{new}} &= \mathbf{E}/c \\ \mathbf{E}_{\text{new}} &= -c\mathbf{B} \end{aligned}$$

Note that $\mathbf{S}$ is invariant under this operation and that any such solution (fields without sources) must essentially be a wave.