While a powerful theoretical tool, the stress tensor generally makes poor homework problems: only in simple, symmetric situations can it be easily applied, and in those cases direct calculation of the force is much easier. Consider the case of a pair of identical line charges (charge per length λ), parallel to the z axis, one piercing the xy plane at $(a, 0)$ and the other at $(-a, 0)$. It's easy to directly calculate the force on a length ℓ of the lhs line charge: The rhs line charge produces a cylindrically radially outward electric field:

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

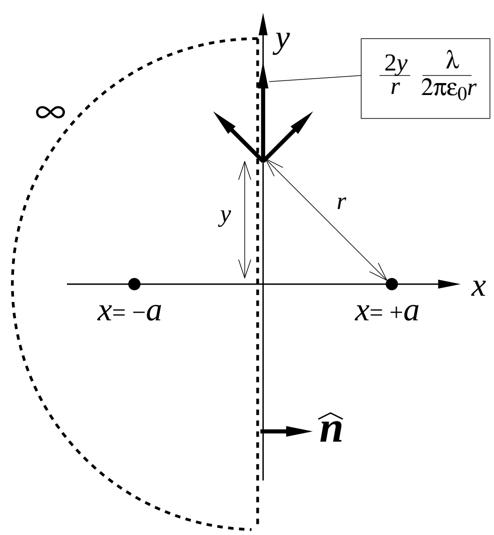
So at the location of the lhs line charge, the electric field is:

$$\mathbf{E} = -\hat{\mathbf{i}}\,\frac{\lambda}{2\pi\epsilon_0 2a}$$

A length ℓ of the lhs line charge has charge $\lambda\ell$, so the total force is:

$$\mathbf{F} = -\hat{\mathbf{i}}\,\frac{\lambda^2\ell}{2\pi\epsilon_0 2a}$$

Now the hard way: Imagine surrounding a length ℓ of the lhs line charge with the section of the yz plane from $z = 0$ to $z = \ell$. Infinitely far (so the electric field is effectively zero) we 'close' the surface with a half cylinder (that, for example, goes through $x = -\infty$). If we integrate the stress tensor over this surface, we should get the net electric force on all the enclosed charge (which is just the ℓ section of the lhs line charge). The total electric field on the yz plane is the superposition of the electric fields of both line charges. As shown in the diagram, the result is just in the y direction:



$$\mathbf{E} = \hat{\mathbf{j}}\,\frac{2y}{r}\,\frac{\lambda}{2\pi\epsilon_0 r}$$

where $r^2 = y^2 + a^2$.

Since $E_x = E_z = 0$, the stress tensor is diagonal:

$$\mathcal{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}E_y^2 & 0 & 0 \\ 0 & \frac{1}{2}E_y^2 & 0 \\ 0 & 0 & -\frac{1}{2}E_y^2 \end{bmatrix}$$

So for a segment of the yz plane:

$$d\mathbf{F} = \mathcal{T} \cdot \hat{\mathbf{n}} da = -\hat{\mathbf{i}}\, \frac{1}{2}\, \epsilon_0 E_y^2\, \ell dy = -\hat{\mathbf{i}}\, \frac{1}{2}\, \epsilon_0 \left[\frac{2y}{r}\, \frac{\lambda}{2\pi\epsilon_0 r} \right]^2 \ell dy$$

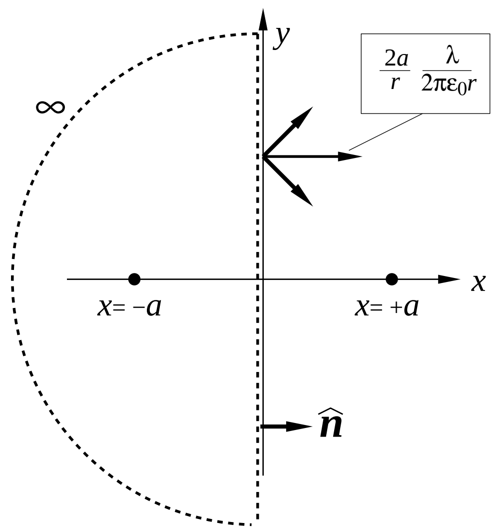
and the total force is:

$$\mathbf{F} = -\hat{\mathbf{i}}\, \frac{\lambda^2 \ell}{2\pi^2 \epsilon_0} \int_{-\infty}^{+\infty} \frac{y^2}{(y^2+a^2)^2}\, dy$$

which when evaluated, agrees with the above.

If we reverse the sign of the charge on the rhs line charge (i.e., charge per length $-\lambda$) the force between the two is equal to the above but opposite (attractive). Lets see how that works with the stress tensor.

The total electric field on the yz plane is the superposition of the electric fields of both line charges: rhs $\mathbf{E}$ points towards its line charge, lhs $\mathbf{E}$ points away from its line charge. As shown in the diagram, the result is just in the x direction:



$$\mathbf{E} = \hat{\mathbf{i}}\, \frac{2a}{r}\, \frac{\lambda}{2\pi\epsilon_0 r}$$

where $r^2 = y^2 + a^2$.
Since $E_y = E_z = 0$, the stress tensor is diagonal:

$$\mathcal{T} = \epsilon_0 \begin{bmatrix} \frac{1}{2}E_x^2 & 0 & 0 \\ 0 & -\frac{1}{2}E_x^2 & 0 \\ 0 & 0 & -\frac{1}{2}E_x^2 \end{bmatrix}$$

So for a segment of the yz plane:

$$d\mathbf{F} = \mathcal{T} \cdot \hat{\mathbf{n}} da = \hat{\mathbf{i}}\, \frac{1}{2}\, \epsilon_0 E_x^2\, \ell dy = \hat{\mathbf{i}}\, \frac{1}{2}\, \epsilon_0 \left[\frac{2a}{r}\, \frac{\lambda}{2\pi\epsilon_0 r} \right]^2 \ell dy$$

and the total force is:

$$\mathbf{F} = \hat{\mathbf{i}}\, \frac{\lambda^2 \ell}{2\pi^2 \epsilon_0} \int_{-\infty}^{+\infty} \frac{a^2}{(y^2+a^2)^2}\, dy$$

which when evaluated, agrees with the expected result.

Homework: Consider a pair of infinite wires, parallel to the z axis, carrying current I, in opposite directions. One wire pierces the xy plane at $(a, 0)$ and the other at $(-a, 0)$. Using the stress tensor, find the net magnetic force on a length ℓ of the lhs wire.