How to make new vectors from existing $\mathbb{X} = (\vec{r}, ict)$

→ derivative wrt invariant $d\tau = \sqrt{dt^2 - \frac{dr^2}{c^2}} = \frac{1}{\gamma} dt$

eg 4-velocity $\mathbb{U} = \gamma(\vec{v}, ic)$ Note constant speed!

→ gradient $\square = \left(\vec{\nabla}, \frac{\partial}{\partial ict}\right) = \left(\vec{\nabla}, \frac{-i}{c}\partial_t\right)$

→ contraction (dot product)

→ dyadic (juxtaposition) $T_{ij} \sim E_i E_j - \frac{1}{2}E^2\delta_{ij}$

note 3-d's cross product is a special case in 4-d: $A_\mu B_\nu - A_\nu B_\mu$

$\mathbb{X} = (\vec{r}, ict)$ $\quad \mathbb{A} = \left(\vec{A}, i\frac{\phi}{c}\right)$ $\quad \mathbb{J} = (\vec{J}, ic\rho)$

$$F_{\mu\nu} = \partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu = \begin{pmatrix} 0 & B_3 & -B_2 & \vdots \\ & 0 & B_1 & \vdots \\ & & 0 & \vdots \\ \cdots & \cdots & \cdots & 0 \end{pmatrix}$$

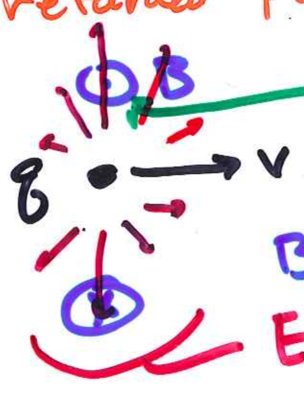
$\epsilon_{ijk} B_k$ $\quad -\frac{i}{c}\vec{E}$ $\quad \frac{i}{c}\vec{E}$

$\partial_\nu F_{\mu\nu} = \mu_0 \mathbb{J}_\mu$ ← contains: $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ & $\nabla \times B - \frac{1}{c^2}\partial_t E = \mu_0 J$

Boost? $F'_{\mu\nu} = R_{\mu\alpha} R_{\nu\beta} F_{\alpha\beta} = R \cdot F \cdot R^{-1}$

→ seek fields due to constant velocity charge

surprise! $\vec{E}$ points back to "current" position not retarded position (which is source of that $\vec{E}$)

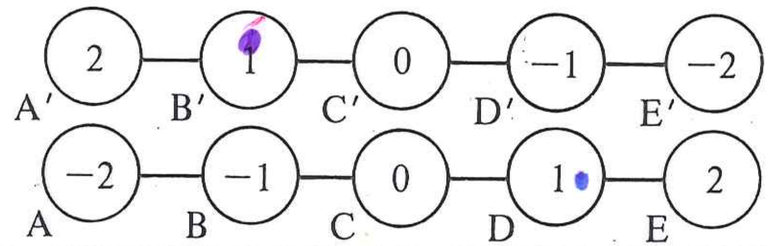


Field here look much like light with $\vec{S}$ in direction of motion $E \perp B$

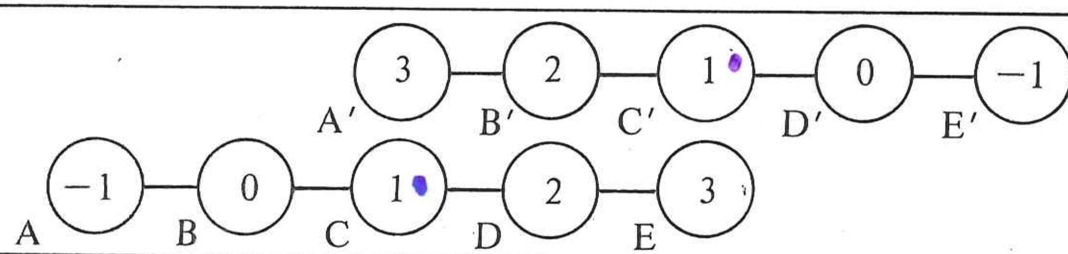
$B = \frac{1}{c^2}\vec{v} \times E$: strong in pancake

$E \sim \gamma \times$ stronger in pancake angle $\frac{1}{\gamma}$

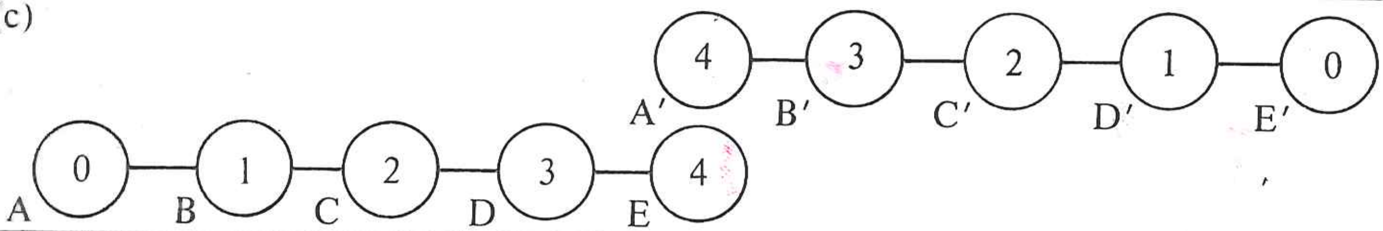
(a)


2
1
0
−1
−2
A′
B′
C′
D′
E′
−2
−1
0
1
2
A
B
C
D
E

(b)


3
2
1
0
−1
A′
B′
C′
D′
E′
−1
0
1
2
3
A
B
C
D
E

(c)


4
3
2
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A′
B′
C′
D′
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1
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3
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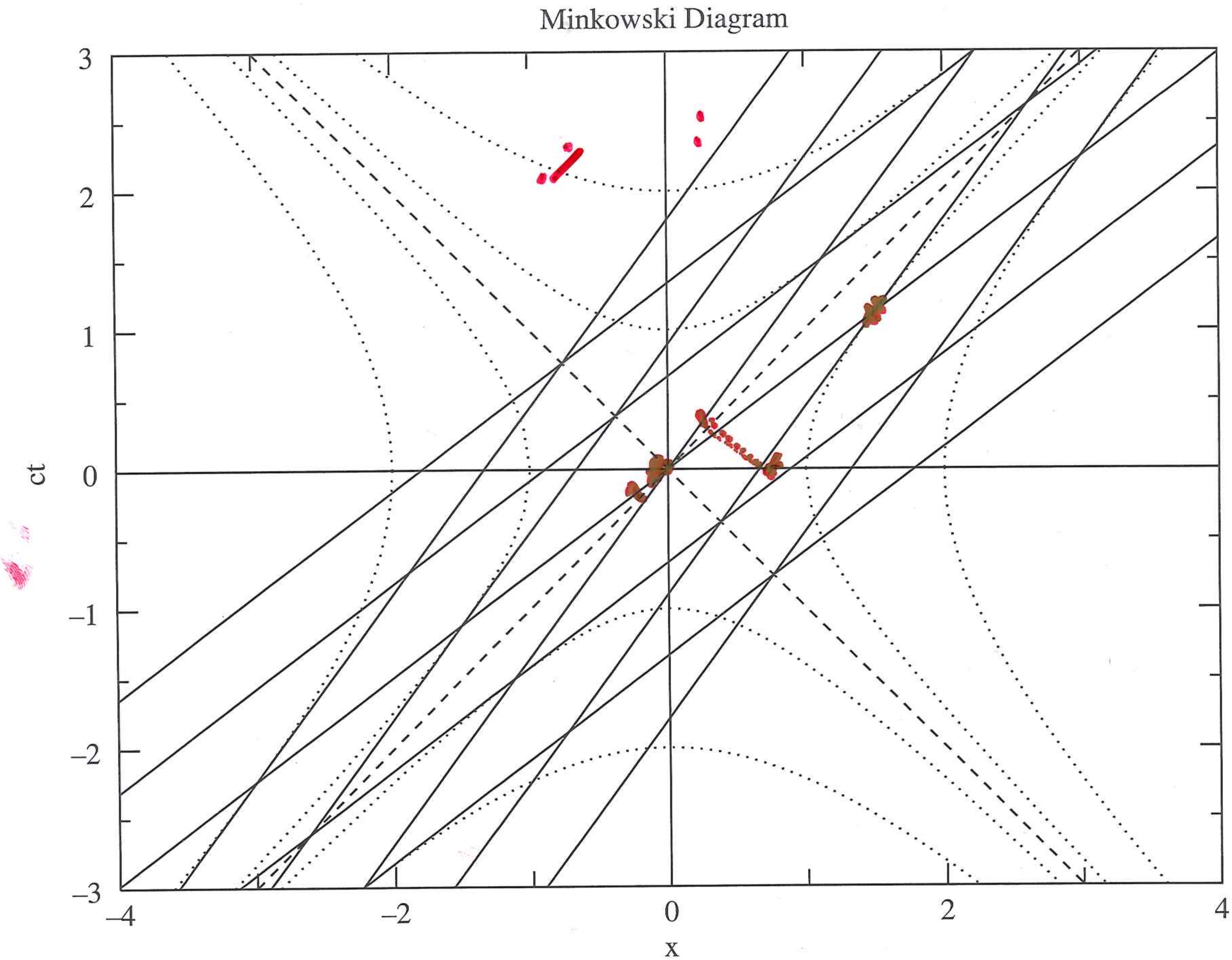
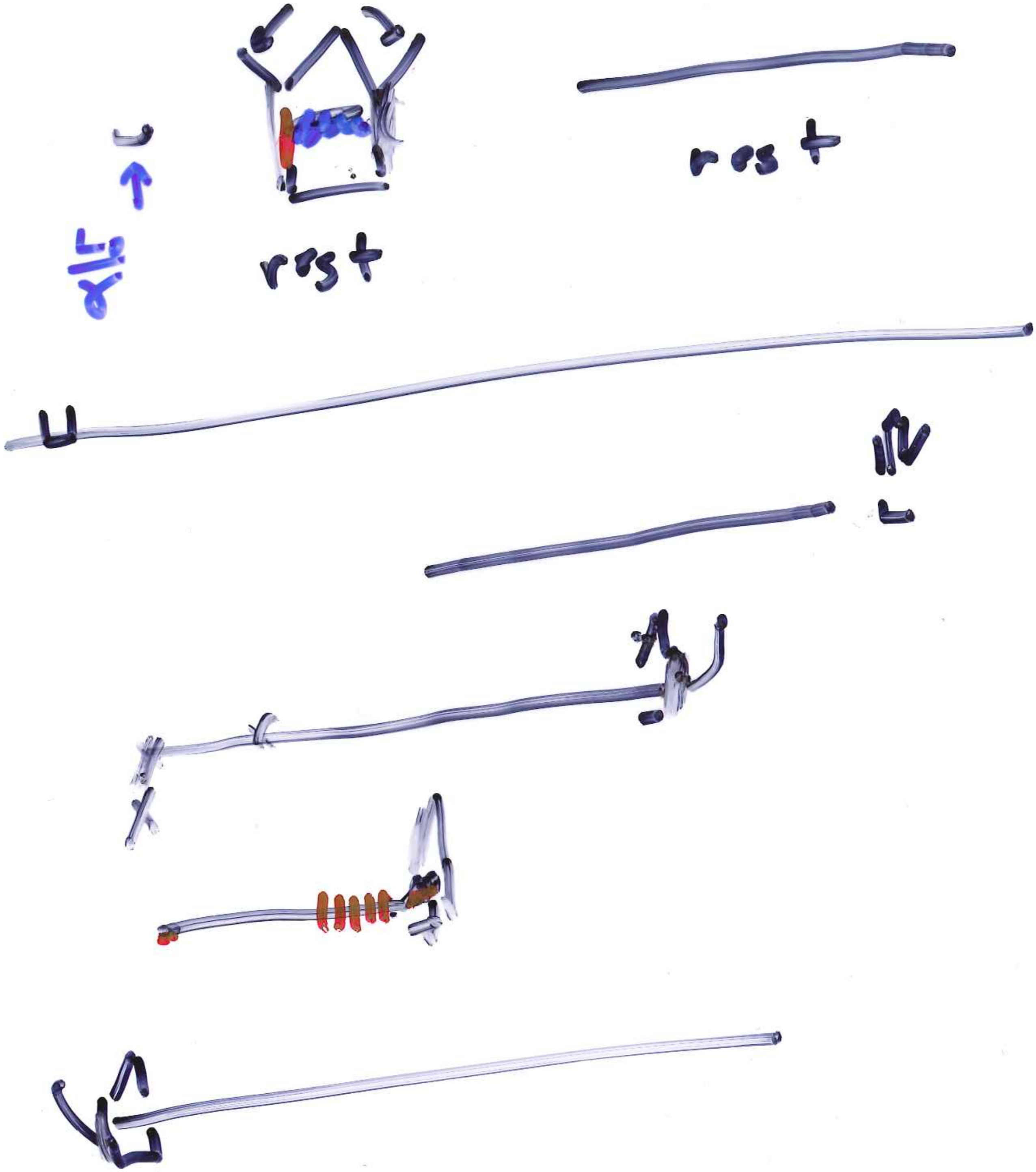
A
B
C
D
E



Minkowski Diagram
ct
3
2
1
0
−1
−2
−3
−4
−2
0
2
4
x


rest
rest

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}, \qquad \mathbf{E}'_{\perp} = \frac{1}{\sqrt{1-\beta^2}}[\mathbf{E}_{\perp} + \mathbf{u} \times \mathbf{B}]$$

$$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}, \qquad \mathbf{B}'_{\perp} = \frac{1}{\sqrt{1-\beta^2}}\left[\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{u} \times \mathbf{E}\right]$$

where $\parallel$ and $\perp$ mean components parallel to and perpenc velocity $\mathbf{u}$ of the Lorentz transformation.

The inverse transformation is obviously given by

$$\mathbf{E}_{\parallel} = \mathbf{E}'_{\parallel}, \qquad \mathbf{E}_{\perp} = \frac{1}{\sqrt{1-\beta^2}}[\mathbf{E}'_{\perp} - \mathbf{u} \times \mathbf{B}']$$

$$\mathbf{B}_{\parallel} = \mathbf{B}'_{\parallel}, \qquad \mathbf{B}_{\perp} = \frac{1}{\sqrt{1-\beta^2}}\left[\mathbf{B}'_{\perp} + \frac{1}{c^2}\mathbf{u} \times \mathbf{E}'\right]$$