Stress Tensor - like pressure (F/A) but allows forces parallel & $\perp$ to surface. Surround object with gaussian surface:

$$\vec{F} = \oint \overleftrightarrow{T} \cdot \hat{n}\, dA$$

$$T_{ij} = \varepsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2 \right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2 \right)$$

Note: E $\perp$ surface $\Rightarrow$ attraction

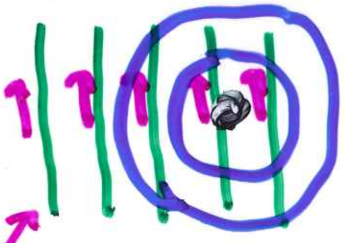
E $\parallel$ surface $\Rightarrow$ repulsion $\rightarrow$ pressure

$$\frac{\text{momentum}}{\text{volume}} = \varepsilon_0 \overline{E \times B} = \frac{1}{c} \frac{\text{energy}}{\text{volume}}$$

Power radiated from accelerated charge NR

"Larmor" $\quad P = \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{\ddot{s}^2 q^2}{c^3}$

Thompson scattering of light due to charged particle



$$\vec{E} \Rightarrow a = \frac{q}{m} E$$

$$P = \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{q^2 \left(\frac{q}{m} E\right)^2}{c^3}$$

input $\frac{W}{m^2} = \frac{1}{\mu_0 c} E^2$

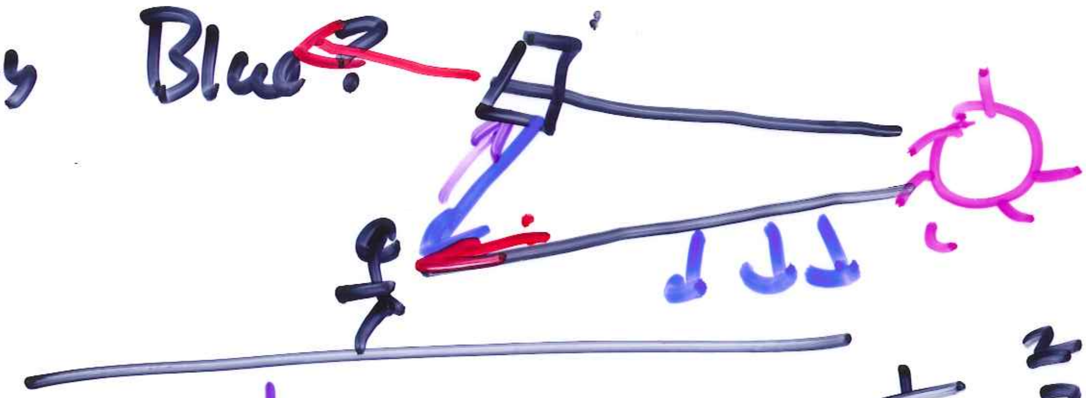
$$\frac{\text{scattered power}}{\text{input Power/Area}} = \sigma = \frac{8\pi}{3} R_c^2$$

$$\frac{1}{4\pi\varepsilon_0} \frac{q^2}{R} = mc^2$$

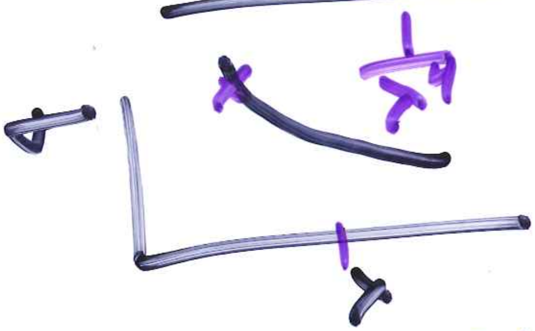
"classical electron radius" $\sim 10^{-15}$ m

Note: Compton radius $= \frac{h}{mc}$

why Blue?



$$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2 a^2}{c^3}$$



a

SHO $\quad \omega << \omega_0$

$$m\ddot{x} + \underline{b\dot{x} + kx} = qE$$

$$x = A e^{i\omega t}$$

$$\left(-\omega^2 + \frac{1}{\tau} i\omega + \omega_0^2\right) A = \frac{q}{m} E$$

$$\sqrt{\frac{k}{m}} = \omega_0$$

$$kx = F$$

$$x = \frac{F}{k}$$

$$A = \frac{\frac{q}{m} E}{\omega_0^2}$$

$$a = \ddot{x}$$
$$= \omega^2 A$$

$$P \propto q^2 \times \omega^4$$

Radiation A

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$\nabla \cdot B = 0$

$\nabla \times B = \mu_0 \left(J + \varepsilon_0 \partial_t E \right)$

6 $\nabla \times E = -\partial_t B$

4 A, ϕ

$\nabla \times A = B$

$E = -\left(\nabla \phi + \partial_t A \right)$

$\nabla \times \left(-\left[\nabla \phi + \partial_t A \right] \right) \stackrel{?}{=} -\partial_t \nabla \times A$

$-\frac{1}{c^2} \partial_t \phi$

$\nabla \cdot E = \rho$

$\nabla \cdot E = -\left(\nabla^2 \phi + \partial_t \overbrace{\nabla \cdot A} \right) = \frac{\rho}{\varepsilon_0}$

$\left(\nabla^2 - \frac{1}{c^2} \partial_t^2 \right) \phi = 0$

$\nabla \cdot A + \frac{1}{c^2} \partial_t \phi = 0$?

Gauge

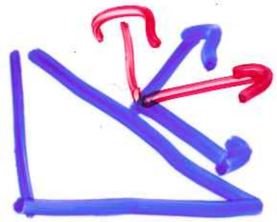
Lorentz

$\nabla \cdot A = 0$

$\left(\nabla^2 - \frac{1}{c^2} \partial_t^2 \right) \phi = -\frac{\rho}{\varepsilon_0}$

Gauge is Good

Gauge is $\boxed{\text{Bad}}$



Gauge is Truth

$$\nabla\times B = \vec{\nabla}(\nabla\cdot A) - \nabla^2\vec{A}$$

$$\nabla\times A \qquad = \mu_0\left(J + \epsilon_0\partial_t\left[-\nabla\phi - \partial_t A\right]\right)$$

$$= -\mu_0\epsilon_0\partial_t^2 A - \frac{1}{c^2}\partial_t^2 A$$

$$\vec{\nabla}(\nabla\cdot A + \mu_0\epsilon_0\partial_t\phi) = 0$$

$$-\mu_0 J = \nabla^2 A - \frac{1}{c^2}\partial_t^2 A$$

$$-c\rho/\epsilon_0 = \left(\nabla^2\phi - \frac{1}{c^2}\partial_t^2\phi\right)/c$$

$r^2 = c^2t^2$

$$\nabla^2 - \frac{1}{c^2}\partial_t^2 = \Box^2$$

x, ct $\qquad\qquad \overrightarrow{E, B}$?

$J, c\rho$

$\bar{A}, \phi/c$

$$-\mu_0 \begin{pmatrix} \bar{J} \\ c\rho \end{pmatrix} = \Box^2 \begin{pmatrix} \bar{A} \\ \phi/c \end{pmatrix}$$

$$\frac{1}{c^2 \varepsilon_0} = \mu_0$$

$$f = \Box^2 \phi$$

$$\bar{A} = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r-r'|} \, dv'$$

$$\phi = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{|r-r'|} \, dv'$$

retarded time $= t - \frac{|r-r'|}{c}$

Radiation

$\underline{l < \lambda}$

dipole antenna —

$\dot{q} = I$ $\quad$ I indep of z (experience said $E, B \Rightarrow l \ll \lambda$)

eventually: $I = I_0 \sin \omega t$ $\quad$ $I_0 = -\omega q_0$

$q = q_0 \cos \omega t$

$$|\vec{r} - \vec{r}'| = [r^2 - 2 r \cdot r' + r'^2]^{1/2}$$

$z' \hat{k}$

$$\cong r\left[1 - \frac{2\hat{r}\cdot r'}{r} + \left(\frac{r'}{r}\right)^2\right]^{1/2}$$

$$\cong r\left[1 - \frac{\hat{r}\cdot\vec{r}'}{r}\right] = r - \cos\theta \, z'$$

$$A_z = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I(z', t - |r-r'|/c)\, dz'}{|r-r'|}$$

$$\cong \frac{\mu_0}{4\pi} l \frac{I(t - r/c)}{r}$$

$\hat{z} = \cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}$

$$A_r = \frac{\mu_0 l}{4\pi}\frac{I}{r}\cos\theta$$

$$A_\theta = \frac{-\mu_0 l}{4\pi}\frac{I}{r}\sin\theta$$

$(x^2+y^2+z^2)^{1/2}$

$2z$

$\frac{1}{r^3}$

$$\nabla \cdot A + \frac{1}{c^2}\partial_t \phi = 0$$

$$\partial_t \phi = -c^2 \partial_z \frac{\mu_0}{4\pi} l \frac{I(t - r/c)}{r}$$

$$= \frac{-l}{4\pi\varepsilon_0}\left[\frac{-1}{r^2} I + \frac{1}{r} I'\right] \partial_z r \quad \frac{z}{r}$$

$-\frac{1}{c}$

$$= \frac{l}{4\pi\varepsilon_0}\left[\frac{1}{r^2} I + \frac{1}{rc} I'\right]\frac{z}{r}$$

$$\phi = \frac{l}{4\pi\varepsilon_0}\left[\frac{1}{r^2} q + \frac{1}{cr} I\right]\frac{z}{r} \qquad t_R = t - r/c$$

$$= \frac{l}{4\pi\varepsilon_0}\left[\frac{1}{r^2}\frac{I_0}{\omega}\cos\omega t_R + \frac{1}{cr} I_0 \sin\omega t_R\right]\cos\theta$$

$$\partial_r \phi = \frac{l}{4\pi\varepsilon_0}\cos\theta\left[\frac{2}{r^3}\frac{I_0}{\omega}\cos\omega t_R + \frac{1}{r^2}\frac{I_0}{\omega}\sin\omega t_R\left(\frac{-\omega}{c}\right) - \frac{1}{cr^2} I_0 \sin\omega t_R + \frac{1}{cr} I_0\cos(\omega t_R)\left(\frac{-\omega}{c}\right)\right]$$

cancel

$$\partial_t A_r = \frac{l}{4\pi\varepsilon_0 c^2 r} I_0 \omega \cos\omega t_R \cos\theta$$

$$E_r = -(\partial_r\phi + \partial_t A_r) = \frac{2l}{4\pi\varepsilon_0}\cos\theta\left[\frac{1}{r^2 c} I_0 \sin\omega t_R - \frac{I_0}{r^3\omega}\cos\omega t_R\right]$$

20^{-2} t_1 light pulse

$\omega = 10^5$

$l = 1$ ft

time for power = λ/ω

$$A_r = \frac{\mu_0 l}{4\pi r} I_0 \sin \omega t_R \cos\theta$$

$$A_\theta = -\frac{\mu_0 l}{4\pi r} I_0 \sin \omega t_R \sin\theta$$

$$B_\phi = \frac{1}{r}\partial_r (r A_\theta) - \frac{1}{r}\partial_\theta A_r$$

$$= \frac{-\mu_0 l}{4\pi r} I_0 \cos \omega t_R \left(\frac{\omega}{c}\right) \sin\theta + \frac{\mu_0 l}{4\pi r} I_0 \sin\omega t_R \frac{\sin\theta}{r}$$

$$= \frac{\mu_0 l}{4\pi r} I_0 \sin\theta \left[\frac{\omega}{c}\cos\omega t_R + \frac{1}{r}\sin\omega t_R\right]$$

$$B_r = \frac{1}{r\sin\theta}\left[\partial_\theta (\sin\theta A_\phi) - \partial_\phi A_\theta\right] = 0$$

$$B_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\partial_\phi A_r - \frac{\partial_r (r A_\phi)}{r}\right] = 0$$

$$E_\theta = -\frac{1}{r}\partial_\theta \phi - \partial_t A_\theta$$

$$= \frac{l}{4\pi\varepsilon_0}\sin\theta\left[\frac{1}{r^3}\frac{-I_0}{\omega}\cos\omega t_R + \frac{1}{cr^2} I_0 \sin\omega t_R\right]$$

$$+ \frac{l}{4\pi\varepsilon_0 c^2 r} I_0 \cos\omega t_R \,\omega \sin\theta$$

$$= \frac{l I_0 \sin\theta}{4\pi\varepsilon_0 r}\left[\frac{\omega}{c^2}\cos\omega t_R - \frac{1}{r^2\omega}\cos\omega t_R + \frac{1}{cr}\sin\omega t_R\right]$$

Long range fields:

$$S = E \times H = \frac{1}{\mu_0} E \times B$$

$$B_\phi = \frac{\mu_0 l}{4\pi r} I_0 \sin\theta \frac{\omega}{c}\cos\omega t_R$$

$$E_\theta = \frac{l I_0 \sin\theta}{4\pi\varepsilon_0 r}\frac{\omega}{c^2}\cos\omega t_R$$

← don't

$$\vec{S} = \hat{r}\left(\frac{\omega l I_0}{c 4\pi r}\right)^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos^2\omega t_R \sin^2\theta$$

$$\frac{\omega}{2\pi c} l = \frac{l}{\lambda}$$

$$= \hat{r}\left(\frac{l}{\lambda}\frac{I_0}{2}\right)^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos^2\omega t_R \frac{\sin^2\theta}{r^2} \longrightarrow dA = r^2 \overbrace{\sin\theta\, d\theta}^{dc}\, d\phi \rightarrow 2\pi$$

$$2\pi\int_{-1}^{1}(1-c^2)\,dc$$

$$2 - \frac{2}{3} = \frac{4}{3}$$

$$\Rightarrow 2\pi\left(\frac{l}{\lambda}\right)^2 \frac{I_0^2}{2}\frac{1}{3}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$R = 789\left(\frac{l}{\lambda}\right)^2$$