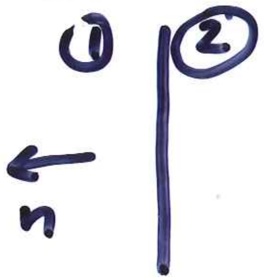
Fresnel Eqs for reflection: in theory simple just apply BC for each polarization

BC



$\nabla \cdot B = 0 \longrightarrow B_{1n} = B_{2n}$

$\nabla \times E = -\partial_t B \longrightarrow E_{1t} = E_{2t}$

$$\left.\begin{array}{l}\nabla \cdot D = \rho \\ \nabla \cdot J = -\partial_t \rho\end{array}\right) \longrightarrow \left(\varepsilon_1 + i\frac{g_1}{\omega}\right)E_{1n} = \left(\varepsilon_2 + \frac{i g_2}{\omega}\right)E_{2n}$$

$\nabla \times H = J + \partial_t D \longrightarrow H_{1t} = H_{2t}$

unless $g \to \infty$ in which case $E, H \to 0$

Complex g

$\dot{v} + \frac{1}{\tau} v = \frac{q}{m} E$

$\hookrightarrow g = \dfrac{\frac{Nq^2\tau}{m}}{1 - i\omega\tau}$ define $\omega_p^2 = \dfrac{Nq^2}{m\varepsilon_0}$

$$0 = \left(\nabla^2 - \varepsilon\mu\partial_t^2 - g\mu\partial_t\right)\begin{matrix}H \\ E\end{matrix} \qquad e^{i(\hat{k}z - \omega t)}$$

$\uparrow$ complex

$\varepsilon\mu + \dfrac{i g\mu}{\omega} = \dfrac{\hat{k}^2}{\omega^2} = \dfrac{\hat{n}^2}{c^2}$ $\qquad \dfrac{1}{k_1} = \delta =$ skin depth

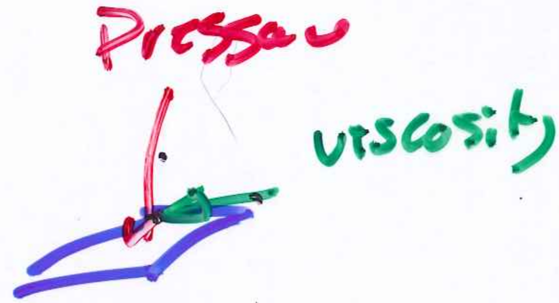
$\omega \ll \frac{1}{\tau}$ $\quad g = \dfrac{Nq^2\tau}{m}$ $\qquad \hat{k} = \sqrt{i}\sqrt{g\mu\omega}$

$\delta = \sqrt{\dfrac{2}{g\mu\omega}}$

$\omega \gg \frac{1}{\tau}$ $\quad g = \dfrac{Nq^2/m}{-i\omega}$ $\qquad \sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$

$n^2 = 1 - \dfrac{\omega_p^2}{\omega^2}$

$F = pA$



$\int$

$\vec{T}\cdot\hat{n}$

$$\overleftrightarrow{T} = \begin{pmatrix} p & \sigma & \sigma \\ \sigma & p & \sigma \\ \sigma & \sigma & p \end{pmatrix}$$

Stress Tensor

F/Area

$\nabla \cdot E = \rho$

$\overleftarrow{T}$

$= T_{\mu\nu}$

$\int \vec{T} \cdot n\, dA = \vec{F}$

$\frac{\text{momentum}}{\text{volume}}$ of Light

$p = \frac{E}{c}$

The Lorentz force per unit volume **f** is

$$\mathbf{f} = \rho \mathbf{E} + \mathbf{J} \times \mathbf{B} \tag{8–94}$$

We begin by eliminating the sources ρ and **J**, using Maxwell I and Maxwell IV in empty space, giving

$$\mathbf{f} = \varepsilon_0 \mathbf{E}\, \nabla \cdot \mathbf{E} + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} \tag{8–95}$$

Using the relation

$$\frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}) = \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} + \mathbf{E} \times \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} - \mathbf{E} \times (\nabla \times \mathbf{E}) \tag{8–96}$$

we can rewrite Equation (8–95) as

$$\mathbf{f} = \varepsilon_0[\mathbf{E}\, \nabla \cdot \mathbf{E} + (\nabla \times \mathbf{E}) \times \mathbf{E}] + \frac{1}{\mu_0}[\overset{0}{\underline{\mathbf{B}\, \nabla \cdot \mathbf{B}}} + (\nabla \times \mathbf{B}) \times \mathbf{B}]$$
$$- \varepsilon_0 \frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}) \tag{8–97}$$

where $\mathbf{B}\, \nabla \cdot \mathbf{B}/\mu_0 = 0$ is added for reasons of symmetry.

The identity

$$[(\nabla \times \mathbf{E}) \times \mathbf{E}]_i = E_j \left(\frac{\partial E_i}{\partial x_j} - \frac{\partial E_j}{\partial x_i} \right) \tag{8–98}$$

follows directly from expansion in Cartesian coordinates. Here and in the rest of this section a sum is understood over the repeated index j. With this identity and a similar one involving **B**, Equation (8–97) becomes

$$f_i = \varepsilon_0 \left(E_i \frac{\partial E_j}{\partial x_j} + E_j \frac{\partial E_i}{\partial x_j} - E_j \frac{\partial E_j}{\partial x_i} \right) + \frac{1}{\mu_0} \left(B_j \frac{\partial B_i}{\partial x_j} + B_j \frac{\partial B_i}{\partial x_j} \right.$$
$$\left. - B_j \frac{\partial B_j}{\partial x_i} \right) - \varepsilon_0 \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B})_i \tag{8–99a}$$

This relation can be reexpressed as

$$f_i = \varepsilon_0 \frac{\partial}{\partial x_j} \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \frac{\partial}{\partial x_j} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$$
$$- \varepsilon_0 \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B})_i \tag{8–99b}$$

where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$).

The Maxwell stress tensor is defined as follows:

Maxwell Stress Tensor

$$T_{ij} \equiv \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2} \delta_{ij} \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \tag{8–100}$$

The field momentum volume density is defined as follows:

Field Momentum Density

$$\mathbf{p}_{\text{field}} = \varepsilon_0 \mathbf{E} \times \mathbf{B} \tag{8–101}$$

In terms of these quantities Equation (8–99b) is

$$f_i = \frac{\partial T_{ij}}{\partial x_j} - \frac{\partial (p_{\text{field}})_i}{\partial t} \tag{8–102}$$

When Equation (8–102) is integrated over a finite volume, the momentum conservation law is obtained. With only electromagnetic forces acting, the momentum change of the charges is related by Newton's second law to the volume force by

$$\frac{d(\mathbf{P}_{\text{charges}})_i}{dt} = \mathbf{F}_i = \int \mathbf{f}_i \, dV \tag{8–103}$$

From the divergence theorem the volume integral of the gradient of the Maxwell stress tensor in Equation (8–102) can be converted to a surface integral:

$$\int \frac{\partial T_{ij}}{\partial x_j} dV = \int \nabla \cdot \mathbf{T}_i \, dV = \oint \mathbf{T}_i \cdot d\mathbf{S} = \oint T_{ij} \, dS_j \tag{8–104}$$

Here we applied the usual divergence theorem by thinking of T_{ij} as three separate vectors $\mathbf{T}_i$ having components $(\mathbf{T}_i)_j = T_{ij}$. The volume integral o Equation (8–102) now becomes

Momentum Conservation

$$\frac{d}{dt}(\mathbf{P}_{\text{charges}} + \mathbf{P}_{\text{field}})_i = \oint T_{ij} \, dS_j \tag{8–105}$$

The capitalized momenta are the *volume-integrated momentum densities* We see that the total rate of change of momentum has been expressed as a *stress force acting over the bounding surface.*

$$\nabla \times H = J + \partial_t D$$

$$J = \nabla \times \frac{1}{\mu_0} B - \partial_t \epsilon_0 E$$

$$\nabla \times E = -\partial_t B$$

$$\left[(\nabla \times E) \times E \right]_i = \epsilon_{jab}\, \partial_a E_b\, \epsilon_{ijk} E_k$$

$$(A \times B)_i = \epsilon_{ijk} A_j B_k$$

$$\epsilon_{jab}\, \partial_a E_b$$

$$* \sum_j \underset{1\,2\,3}{\epsilon_{jab}}\, \underset{3\,2}{\epsilon_{jki}} = \delta_{ak}\delta_{bi} - \delta_{ai}\delta_{bk}$$

$$\partial_a E_b E_k$$

$$E_b \partial_i E_b$$

$$E_a \partial_a E_i$$

$$= E \cdot \nabla \bar{E}$$

$$\frac{1}{2} \partial_i E_b^2$$

$$\sum \delta_{al} E_l = E_a$$

$$\delta_{12} = 0$$

$$\delta_{22} = 1$$

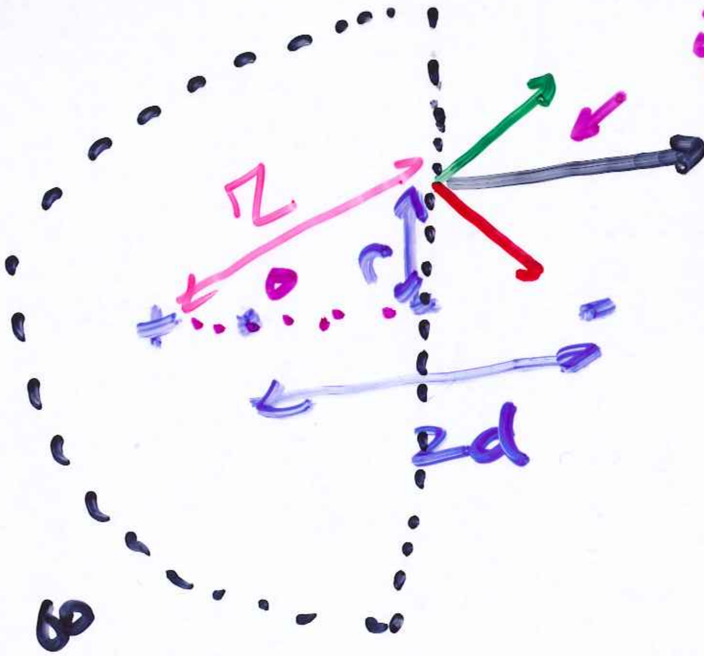
$$\nabla \frac{1}{2} E^2$$

$$\delta_{ij} = \begin{cases} 0 & \\ 1 & i = j \end{cases}$$

$$= E \cdot \nabla \bar{E} - \nabla \frac{1}{2} E^2$$

$$\vec{f} = \nabla \cdot \overleftrightarrow{T} - \epsilon_0 \partial_t (\vec{E} \times \vec{B})$$

$$F = \int f \, dV \rightarrow \oint \overleftrightarrow{T} \cdot \hat{n} \, dA$$



$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}\cos\theta \sim \frac{d}{r} \times 2$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2d)^2}$$

$$E \sim \frac{1}{r^2}$$

$$E^2 \sim \frac{1}{r^4}$$

$$E^2\, 2\pi r^2 \sim \frac{1}{r^2}$$

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2 \right)$$

$$\begin{pmatrix} \frac{1}{2}E^2 & & \\ & -\frac{1}{2}E^2 & \\ & & -\frac{1}{2}E^2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} 12 = \begin{pmatrix} \frac{1}{2}E^2 \\ 0 \\ 0 \end{pmatrix}$$

$$F_x = \epsilon_0 \int \frac{1}{2} E^2 \, dA \leftarrow 2\pi r\, dr$$

$$E \uparrow \quad \frac{1}{4\pi\epsilon_0}\quad \frac{q}{r^3}2d$$

$$r^2 = r^2 + d^2$$

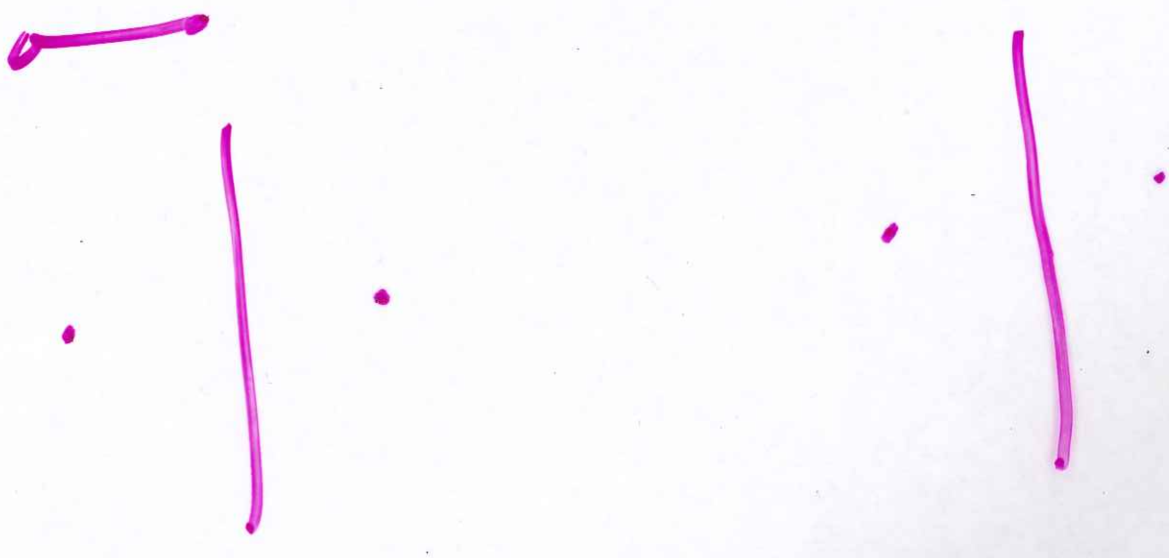
$$= \epsilon_0 \int_0^\infty \frac{1}{2}\left[\frac{1}{4\pi\epsilon_0}\frac{q\,2d}{(r^2+d^2)^{3/2}}\right]^2 2\pi r\, dr$$

$$= \frac{q^2 d^2}{4\pi\epsilon_0} \frac{1}{2} \int_{\infty}^{0} \frac{2r\,dr}{(\underset{\substack{\uparrow\\ u}}{r^2}+d^2)^3}$$

$$= \frac{q^2 d^2}{4\pi\epsilon_0} \frac{1}{2} \left. \frac{(r^2+d^2)^{-2}}{-2} \right|_0^{\infty}$$

$$= \frac{q^2 d^2}{4\pi\epsilon_0} \frac{1}{2^2} \frac{1}{d^4}$$

$$= \frac{q^2}{4\pi\epsilon_0 (2d)^2}$$



Radiation & Relativity

arrow

Power rad accelr

$$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2 a^2}{c^3} \qquad \frac{q}{m}E$$

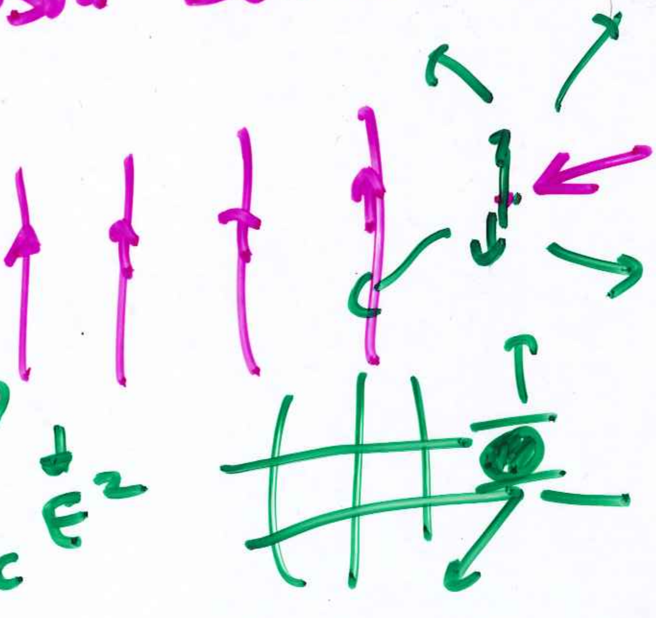
NR

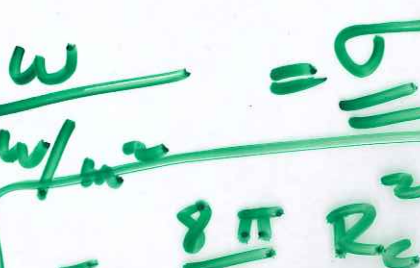
Thompson Scattering

$$\frac{q^2}{4\pi\epsilon_0 R} = mc^2$$

$\sim 10^{-15}$ m

~ nucleus





$$\frac{h}{mc}$$

$$\alpha \approx \frac{1}{137}$$

$$S = \frac{1}{\mu_0 c}E^2$$

W/m^2

$$\frac{W}{W/m^2} = \sigma \boxed{= \frac{8\pi}{3}R_c^2} = \frac{8\pi}{3}\alpha^2 R_C^2$$