Magnetic Materials

$\vec{M} = \frac{\sum \vec{m}_i}{\text{Volume}}$

magnetization

compare: $\vec{P} = \frac{\sum \vec{p}_i}{\text{Volume}}$

Polarization

equivalent currents

$\vec{J}_M = \vec{\nabla} \times \vec{M}$ ← volume current

$\vec{j}_M = \vec{K}_M = \vec{M} \times \hat{n}$ ← surface current

Note: uniform $\vec{M}$ rod = solenoid

$\rho_B = -\vec{\nabla} \cdot \vec{P}$

$\sigma_B = \vec{P} \cdot \hat{n}$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\vec{\nabla} \cdot \vec{D} = \rho_f$

$\vec{P} = \varepsilon_0 \chi \vec{E}$

$\vec{D} = \varepsilon_0 (1+\chi) \vec{E} = \varepsilon_0 K \vec{E}$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_f + \vec{J}_M) = \mu_0 (\vec{J}_f + \vec{\nabla} \times \vec{M})$$

$$\vec{\nabla} \times \underbrace{\left(\frac{1}{\mu_0} B - M \right)}_{H} = \vec{J}_f \rightarrow \oint \vec{H} \cdot d\vec{l} = I_f$$

$\nabla \cdot H = \underbrace{-\nabla \cdot M}_{\text{pole density } \rho_M}$

$M = \chi H$

$\sigma_M = M \cdot n$

$B = \mu_0 (H + M)$

$= \mu_0 \underbrace{(1+\chi)}_{K_m} H$

if $\vec{J}_f = 0$, $\vec{H} = -\nabla \phi$

$\phi = \frac{1}{4\pi} \int \frac{\rho_M \, dV}{|\vec{r} - \vec{r}'|}$ → or $\sigma_M \cdot dA$

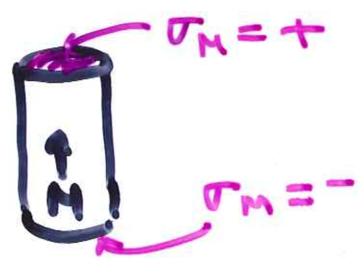
diamagnetic $\chi < 0$ } small

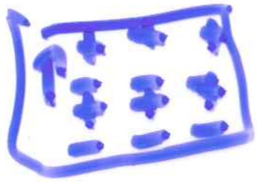
paramagnetic $\chi > 0$ } small

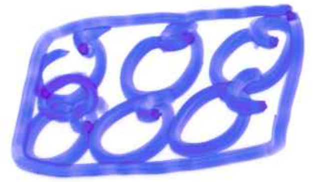
ferromagnetic χ huge

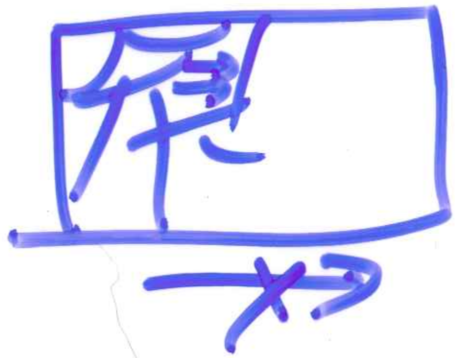
domains, non linear, hysteresis

uniform M → capacitor









$\nabla \times H = 0 \rightarrow H = -\nabla\phi$ Laplace

$\nabla \cdot H = \sigma_M \rightarrow M \cdot n$

$= 0$

$\nabla(-\nabla\phi) = \nabla^2\phi = 0$

ϕ_{in}

ϕ_{out}

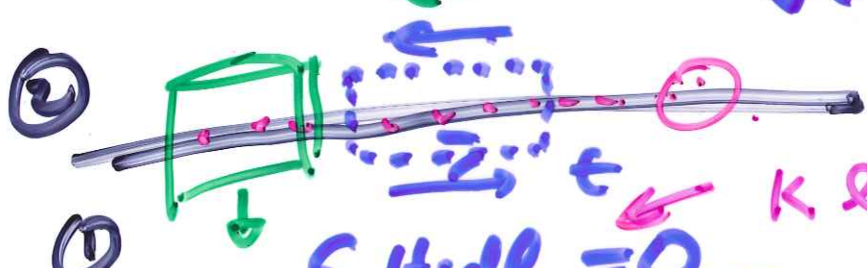
B.C.

$K_c = 0$

$\nabla \cdot B = 0$

$\nabla \times H = 0$

n



$\Delta B_n = 0$

$K\,l$

$\oint H \cdot dl = 0$

$H_t^{(1)} l - H_t^{(2)} l = 0$

$H_t^{(1)} - H_t^{(2)} = K$

$H_t^{(1)} = H_t^{(2)}$

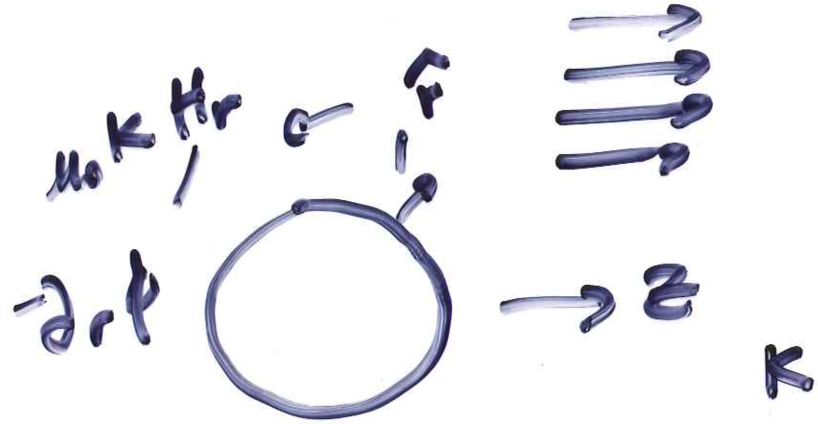
$\oint \bar{B} \cdot \hat{n}\, dA = 0$

$B_n^{(2)} A - B_n^{(1)} A = 0$

$H = -\nabla\phi$

$\nabla\cdot H = 0$

$\nabla^2\phi = 0$



$$\phi_{in} = \sum A_n r^n P_n$$

$$\phi_{out} = \sum C_n r^{-(n+1)} P_n - H_0 z$$

$r\cos\theta$

$r P_1 \ldots$

A_1

$$\phi_{in}\Big|_{r=R} = \phi_{out}\Big|_{r=R}$$

$$-\mu_0 K \partial_r \phi_{in} = -\mu_0 \partial_r \phi_{out}\Big|_{r=R}$$

$n \neq 1$ $\quad A_n R^n = \dfrac{C_n}{R^{n+1}}$

$n = 1$ $\quad A_1 R^1 = \dfrac{C_1}{R^2} - H_0 R$

$$\kappa \partial_r \phi_{in} = \partial_r \phi_{out}$$

$$\kappa \sum n A_n R^{n-1} P_n = -\sum (n+1) \frac{C_n}{R^{n+2}} P_n - H_0 P_1$$

$n \neq 1$ $$\kappa n A_n R^{n-1} = \frac{-(n+1) C_n}{R^{n+2}}$$

$n = 1$ $$\kappa A_1 = \frac{-2 C_1}{R^3} - H_0$$

$$\kappa A_1 + \frac{2 C_1}{R^3} = -H_0$$

$$\times 2 \left(A_1 - \frac{C_1}{R^3} = -H_0 \right)$$

$$(\kappa + 2) A_1 = -3 H_0$$

$$A_1 = \frac{-3}{\kappa + 2} H_0$$

$$\frac{-3}{\kappa+2} H_0 + H_0 = \frac{C_1}{R^3}$$

$$\frac{\kappa - 1}{\kappa + 2} H_0 = \frac{C_1}{R^3}$$

$B = \mu_0 \kappa H$

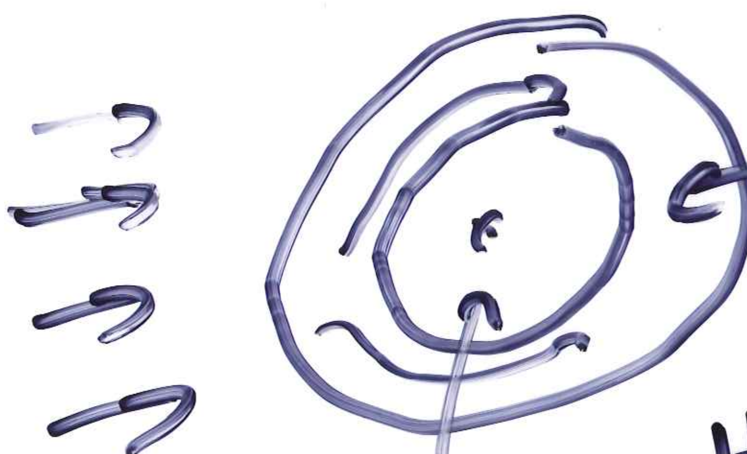
$\nabla \cdot B = 0$

$J = \sigma E$

$\nabla \cdot J = 0$

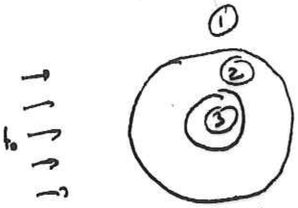
mu metal

$\kappa \sim 10^6$



reduces H, B

Magnetic Shielding Reitz 9-16 → Km pipe (outer = b, inner = a)



① $\phi = \sum C_n r^{-n} \cos(n\theta) - H_0 \underbrace{r\cos\theta}_{x}$

② $\phi = \sum (D_n r^n + E_n r^{-n}) \cos(n\theta)$

③ $\phi = \sum A_n r^n \cos(n\theta)$

even in $\theta \Rightarrow \cos$ only
non singular @ $r=0$ & $r=\infty$

H_t continuous ① → ② $r=b$ $n=1$ $C_1 b^{-1} - H_0 b = D_1 b + E_1 b^{-1}$

ϕ agrees $n \neq 1$ $C_n b^{-n} = D_n b^n + E_n b^{-n}$

② → ③ $r=a$ $D_n a^n + E_n a^{-n} = A_n a^n$

B_n continuous ① → ② $r=b$ $n=1$ $-\frac{C_1}{b^2} - H_0 = K\left(D_1 - \frac{E_1}{b^2}\right)$

$K\partial_r\phi$ continuous $n \neq 1$ $-\frac{nC_n}{b^{n+1}} = K\left(nD_n b^{n-1} - n\frac{E_n}{b^{n+1}}\right)$

② → ③ $r=a$ $K\left(nD_n a^{n-1} - n\frac{E_n}{a^{n+1}}\right) = nA_n a^{n-1}$

For $n \neq 1$: 4 homogeneous linear eqs with unknowns A_n, C_n, D_n, E_n
$\Rightarrow$ all zero

For $n=1$

$$\frac{C_1}{b^2} - D_1 - \frac{E_1}{b^2} = H_0$$

$$A_1 - D_1 - \frac{E_1}{a^2} = 0$$

$$-\frac{C_1}{b^2} - KD_1 + \frac{KE_1}{b^2} = H_0$$

$$-A_1 + KD_1 - \frac{KE_1}{a^2} = 0$$

$$-(K+1)D_1 + \frac{(K-1)}{b^2}E_1 = 2H_0$$

$$(K-1)D_1 - \frac{K+1}{a^2}E_1 = 0$$

$$D_1 = \frac{K+1}{K-1}\frac{E_1}{a^2}$$

$$\left[-\frac{(K+1)^2}{(K-1)}\frac{1}{a^2} + \frac{(K-1)}{b^2}\right]E_1 = 2H_0$$

$$\frac{E_1}{(K-1)a^2}\left[-(K+1)^2 + \frac{a^2}{b^2}(K-1)^2\right] = 2H_0$$

$$A_1 = \left[\frac{K(K+1)}{(K-1)} - K\right]\frac{E_1}{a^2}$$

$$= \frac{2K}{(K-1)}\frac{E_1}{a^2} = \frac{2K \cdot 2H_0}{\left[-(K+1)^2 + \frac{a^2}{b^2}(K-1)^2\right]}$$

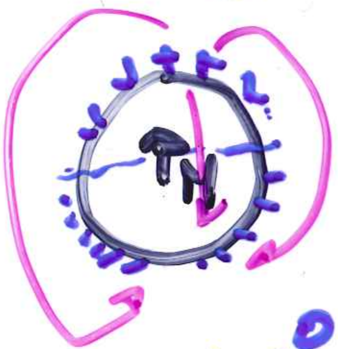
Now H inside $= -A_1$

$$\frac{H_0}{H_{inside}} = \frac{\left[(K+1)^2 - \frac{a^2}{b^2}(K-1)^2\right]}{4K} - 1 + 1$$

$$\frac{1}{4K}\left[(K-1)^2 - \frac{a^2}{b^2}(K-1)^2\right] = \frac{(K-1)^2}{4K}\left(1 - \frac{a^2}{b^2}\right)$$

$$\sim \frac{K}{4}\left(1 - \frac{a^2}{b^2}\right)$$

2



$\rho_M = 0$

$$\sigma_M = \underset{\hat{z}}{\vec{M}} \cdot \underset{\hat{r}}{\vec{n}} = M\,\hat{z}\cdot\hat{r} = M\cos\theta \checkmark$$

$\nabla^2\psi = 0$; $\quad H = -\nabla\phi$

$$\phi_{in} = \sum A_n r^n P_n$$

$$\phi_{out} = \sum \frac{C_n}{r^{n+1}} P_n$$

$$\phi_{in} = \phi_{out}\Big|_{r=R} \Rightarrow A_n R^n = \frac{C_n}{R^{n+1}} \checkmark$$

$$\Delta H_n = \sigma_M = M\cos\theta$$

$$\underset{-\partial_r \phi_{out}}{H_r^{(2)}} - H_r^{(1)} = M\cos\theta$$

$$\left(\partial_r\phi_{in} - \partial_r\phi_{out}\right)\Big|_{r=R} = M P_1$$

$$\sum\left(n A_n R^{n-1} + (n+1)\frac{C_n}{R^{n+2}}\right) P_n = M P_1$$

$n \neq 1$ <u>homo</u> linear eqns for A_n C_n

$A_n = C_n = 0$

$n=1 \quad A_1 + \frac{2C_1}{R^3} = M$

$2 \times \left(A_1 - \frac{C_1}{R^3} = 0 \right)$

$3A_1 = M \qquad A_1 = \frac{1}{3} M$

$\phi_{in} = \underbrace{\frac{1}{3} M}_{A_1} z \leftarrow r P_1$

$\vec{H}_{in} = -\frac{1}{3} M \hat{z} \leftarrow B = \mu_0 (H + M)$

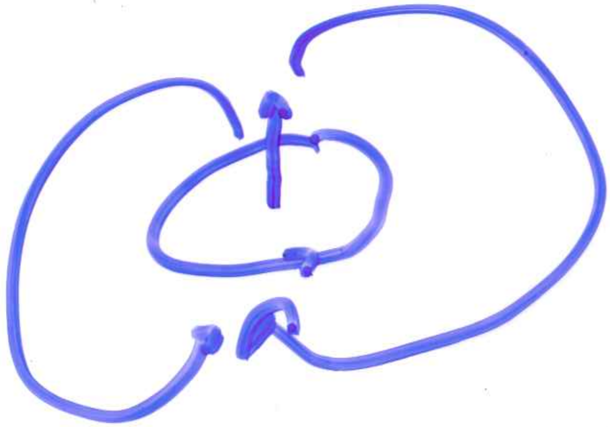
$= \mu_0 \frac{2}{3} M$

$\frac{C_1}{R^3} = \frac{1}{3} M$

$C_1 = \frac{4\pi}{3} \frac{R^3}{4\pi} M$

$= \frac{(\text{Vol}) \left(\frac{\text{dipole}}{\text{volume}} \right)}{4\pi} = \frac{m}{4\pi}$

$\phi = \frac{m \cos\theta}{4\pi r^2} \longleftarrow$ dipole $\quad B = \mu_0 H$



$\bar{K} = M \times n$

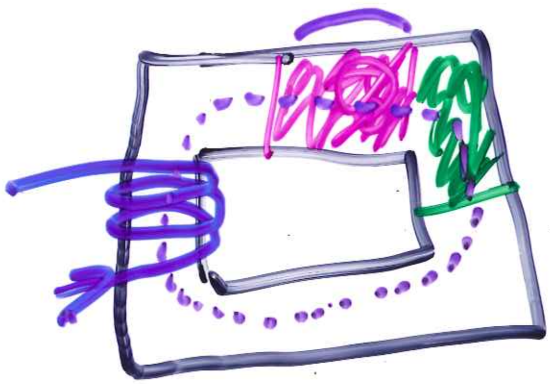
B → like to be in Fe

J → like Cu

$\underbrace{}$ in Fe always

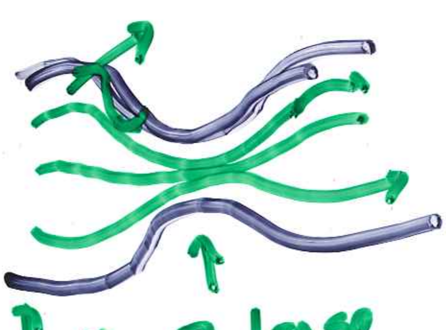
$\frac{B}{\mu_0 K}$

$\oint \vec{H} \cdot d\vec{l} = NI$



Const?

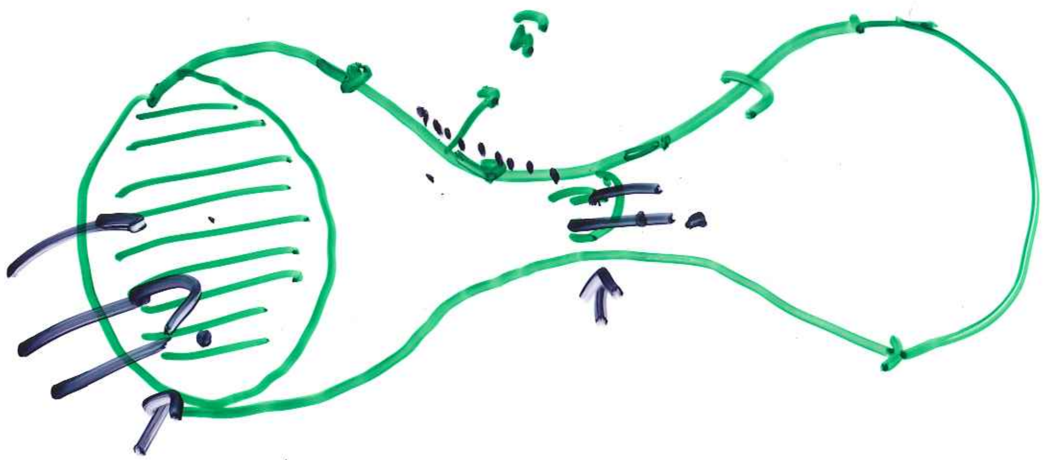
magnetic flux



$\phi_M = \int \bar{B} \cdot d\bar{A}$

$B A \cos\theta$

Flux tube



$$\oint \vec{B}\cdot \hat{n}\, d\vec{A} = 0$$

$$B = \frac{\Phi}{A}$$

const $\quad \phi$ const

$$B = \frac{\phi}{A}$$

$$\oint \vec{H} \cdot d\vec{l} = NI$$

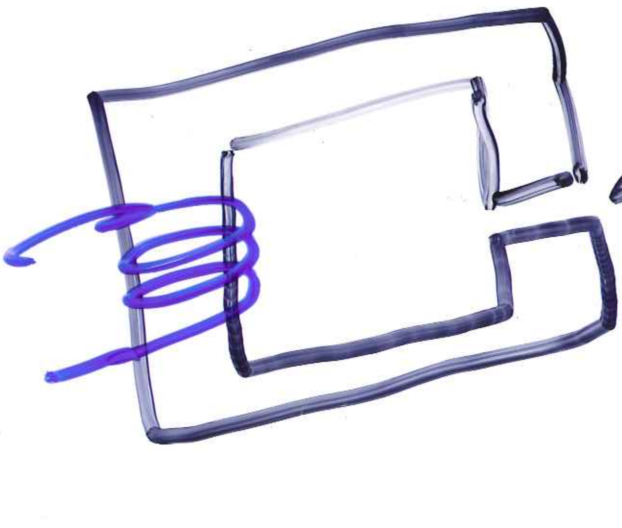
$$\frac{B}{\mu_0 K} = \frac{1}{\mu_0 K A} \phi$$

$$\frac{\phi}{\mu_0} \oint \frac{dl}{KA} = NI$$

$\frac{l}{KA}$ $\mathcal{R}$ reluctance

$R = \frac{\eta l}{A} \qquad \frac{l}{KA}$

$$\phi \sum \mathcal{R}_i = \mu_0 NI$$



gap $\mathcal{R} \gg 1$