Find $\vec{B}$ by integration: $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

Note: $I\,d\vec{l}$, $\vec{K}\,dA$, $\vec{J}\,dV$ depending...

$\vec{r}$: location at which $\vec{B}$ is being found

$\vec{r}'$: location of a source - ie current

$d\vec{l}$, $\vec{K}$, $\vec{J}$: direction current is going

Long straight wire: $\vec{r} = (x,0,0)$ $\vec{r}' = (0,0,z')$

$d\vec{l} = (0,0,dz')$

$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ in cylinders from z axis where wire is

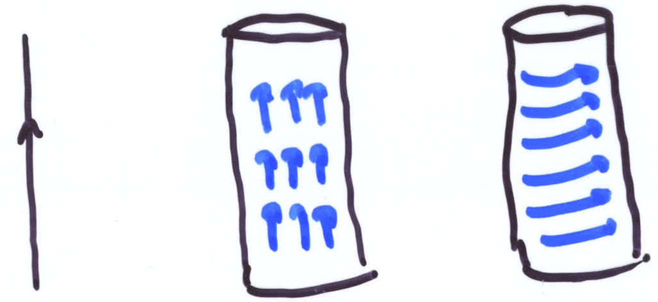
Cylindrical Shell: $\vec{K}\,dA = (0,0,\underbrace{K}_{I/2\pi R})\,R\,d\theta\,dz'$

$\vec{r}' = (R\cos\theta, R\sin\theta, z')$

outside: $\vec{B}$ as above; inside $\vec{B} = 0$

Solenoid: $\vec{K}\,dA = K(-\sin\theta, \cos\theta, 0)\,R\,d\theta\,dz'$

outside: $\vec{B} = 0$; inside $\vec{B} = \mu_0 \underbrace{K}_{\frac{N}{\ell}I}\hat{k}$



Note use of odd symmetry to show integral $= 0$

Dwight 200.03

$$\int \frac{dx}{r^3} = \frac{1}{a^2}\frac{x}{r}$$

$$r = \sqrt{a^2+x^2}$$

859.124

$$\int_{-\pi}^{\pi} \frac{a - b\cos x}{a^2 - 2ab\cos x + b^2}\,dx = \begin{cases} \frac{2\pi}{a} \\ 0 \end{cases} \text{if} \begin{cases} a > b \\ a < b \end{cases}$$

$$\vec{E} = -\vec{\nabla}\underset{\uparrow}{\phi} \leftarrow +C \qquad \vec{B} = \vec{\nabla}\times\underset{\substack{\uparrow \\ \text{vector pot}}}{\vec{A}} \leftarrow \vec{\nabla}f$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dv' \qquad \nabla'$$

$$-\nabla\frac{1}{|\vec{r}-\vec{r}'|}$$

p20 1-1-9

$$\nabla\times(\phi\vec{F}) = \underline{\nabla\phi\times\vec{F}} + \phi\,\nabla\times F$$

dV

$$= \frac{\mu_0}{4\pi}\int \nabla\times\left(\frac{J(\vec{r}')}{|r-r'|}\right) - \underbrace{\phi\,\nabla\times J(r')}_{0}$$

$$B = \frac{\mu_0}{4\pi}\nabla\times\underbrace{\int\frac{\vec{J}(r)}{|r-r'|}\,dv'}_{\vec{A}}$$

Gauge Transform

↑

Gauss Forc

$$\phi = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\, dv}{|r-r'|}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(r')}{|r-r'|}\,dv'$$

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0} \quad \ldots\ldots \quad \nabla^2 \frac{1}{|r-r'|} = -4\pi\, \delta(\overrightarrow{r-r'})$$

$$\nabla^2 A_x = \underbrace{\frac{\mu_0}{4\pi} \int J_x(r')\, \underbrace{\nabla^2 \frac{1}{|r-r'|}}_{-4\pi\delta}\, dV'}_{-4\pi J(r)}$$

$$= -\mu_0 J_x$$

$$\rightarrow \underset{\uparrow}{\nabla}{}^2 \vec{A} = -\mu_0 \vec{J}$$

$$A\times(B\times C) = B\cdot A\,\vec{C} - C\cdot A\,\vec{B}$$

$$\nabla\times(\nabla\times C) = \nabla^2 C \underset{\nabla}{-} (\nabla\cdot C)$$

$$\nabla^2 \vec{C} = \nabla\times(\nabla\times C) + \nabla(\nabla\cdot C)$$

$$\rightarrow \nabla\cdot \underset{\nwarrow}{A} = 0$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r-r'|}\, dV'$$

$$\nabla \cdot A = \frac{\mu_0}{4\pi} \int \nabla \cdot \left(\frac{J(r')}{|r-r'|} \right) dv'$$

$$\nabla \cdot (\phi \vec{F}) = \overbrace{\vec{\nabla} \cdot \phi \cdot \vec{F}} + \phi \cdot \underline{\underline{\vec{\nabla} \cdot \vec{F}}}$$

$$= \frac{\mu_0}{4\pi} \int \left(\underset{\uparrow \; -\nabla' \frac{1}{|r-r'|}}{\nabla \frac{1}{|r-r'|}} \right) \cdot J(r') \, dv'$$

$$= \frac{-\mu_0}{4\pi} \int \underbrace{\nabla' \frac{1}{|r-r'|} \cdot J(r')} \, dv'$$

$$\nabla' \cdot \left(\frac{1}{|r-r'|} J(r') \right) - \frac{1}{|r-r'|} \underbrace{\nabla' \cdot \vec{J}(r')}_{0}$$

$$= \frac{-\mu_0}{4\pi} \oint \frac{1}{|r-r'|} \vec{J}(r') \cdot \hat{n} \, dA'$$

$$\begin{cases} \nabla \cdot A = 0 \\ \nabla^2 \vec{A} = -\mu_0 J \end{cases}$$

$$\nabla \times B = \nabla \times (\nabla \times A) = \vec{\nabla} (\overbrace{\nabla \cdot A}^{0}) - \nabla^2 \vec{A}$$

$$= \mu_0 \vec{J}$$

$\vec\nabla \times (\vec\nabla f) = 0$

$\nabla \cdot (\nabla \times F) = 0$

$$\nabla \cdot \left|\begin{matrix} \partial_x & \partial_y & \partial_z \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & \bar{F}_z \end{matrix}\right) = 0$$

$\nabla \cdot E = \rho / \varepsilon_0$ ✓

$\nabla \times E = 0$ induct

↳ $\vec{E} = -\nabla \phi$

$\nabla \cdot B = 0$

$\nabla \times B = \mu_0 J$

Maxwell's Eq

"proof"

Coulomb

Biot-Savart ✗

static

$\oint \vec{E} \cdot \hat{n}\, dA = \frac{Q_{enc}}{\varepsilon_0}$

$\oint \vec{B} \cdot \vec{dl} = I_{enc}\, \mu_0$

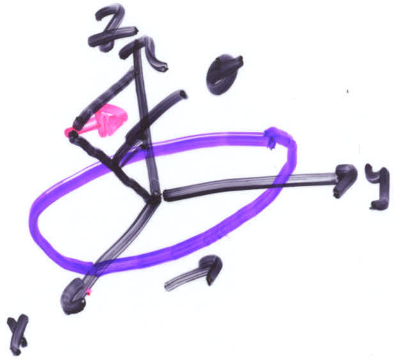
$\oint$ Ampere Law

$\int \vec{J} \cdot \hat{n}\, dA = I = \frac{1}{\mu_0} \int \overline{\nabla \times B} \cdot d\vec{A}$

$\frac{1}{\mu_0} \nabla \times B$

$= \frac{1}{\mu} \oint \vec{B} \cdot \vec{dl}$

$\overline{A}$ is not a friend.



$$\overline{A} = \frac{\mu_0}{4\pi}\int \frac{\overline{J}(r')}{|r-r'|}\, dV'$$

$$\overline{J}\, dV \rightarrow \overline{K}\, dA \rightarrow I\underset{\uparrow}{\overline{dl}}$$

$$(-\sin\phi, co\phi, 0)$$

$$\overline{r} = (x, 0, z)$$

$$\overline{r}' = (R\cos\phi, R\sin\phi, 0)$$

$$\overline{r-r}' = (x - Rc, -Rs, z)$$

$$A \quad |\overline{r-r}'| = \sqrt{\underset{\uparrow}{x^2} + R^2 + \underset{\uparrow}{z^2} - 2xRc}$$

$$\overline{A} = \frac{I\mu_0}{4\pi}\int_{-\pi}^{\pi} \frac{(-\sin\phi, \cos\phi, 0)}{|r-r'|}\, R\, d\phi$$

$$A_\phi = \frac{I\mu_0 R}{4\pi}\int_{-\pi}^{\pi} \frac{\cos\phi \; d\phi}{\sqrt{\underbrace{r^2+R^2} - 2\underset{\substack{\uparrow \\ r\sin\theta}}{x} R\cos\phi}}$$

K E

$$(r^2+R^2)^{1/2}\left[1 - \frac{2rR\sin\theta}{r^2+R^2}\cos\phi\right]^{1/2}$$

$$[1-x]^{-1/2} = 1 + \frac{1}{2}x \;\dots\; \frac{1}{n!}\left(\frac{1}{2}\right)_n x^n + \dots$$

$$A_\phi = \frac{I\mu_0 R}{4\pi(r^2+R^2)^{1/2}} \int_{-\pi}^{\pi} \overset{\cos\phi}{\left[\frac{1}{\uparrow} + \frac{t}{2} \smile\right]} d\phi$$

$$\int \cos^2\phi = \frac{1}{2}\cdot 2\pi \qquad r\frac{R\sin\theta}{r^2+R^2}\cos\phi$$

$$= \frac{\check{I}\mu_0 \check{R}\; r\check{R}\sin\theta}{4\pi(r^2+R^2)^{3/2}}\; \check{\pi}$$

$\pi R^2 I$

m
μ $= AI$

$\bar{p} = \pm\uparrow$

qd

magnetic dipole moment

$$= \frac{\mu_0 m}{4\pi}\frac{r\sin\theta}{(r^2+R^2)^{3/2}} \qquad r^3 \sim$$

$r \gg R$ $\qquad r \ll R$

$$= \frac{\mu_0 m}{4\pi}\frac{\check{\sin\theta}}{r^2} \;\Big|\; \frac{\mu_0 m}{4\pi}\frac{r\sin\theta}{R^3}$$

$\hat{\phi}\sin\theta$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$\hat{k} = \underset{\cos\theta \leftarrow \hat{k}\cdot\hat{r}}{A}\,\hat{r} + B\hat{\theta} \qquad \hat{k}\cdot\hat{\theta} = -\sin\theta$$