Find forces/torques from electrostatic PE

if Qs fixed: $F = -\frac{dU}{dx}$ $\quad \tau = -\frac{dU}{d\theta}$

if Vs fixed: $F = +\frac{dU}{dx}$ $\quad \tau = +\frac{dU}{d\theta}$

Looks like opposite direction but NOT

Source of differing sign: Work done by battery maintaining V while sourcing charge : $+\sum dQ_i V_i$

important message: $\frac{1}{2}\vec{E}\cdot\vec{D}$ is $\frac{\text{energy}}{\text{volume}}$

factoid: dielectrics get sucked into regions with larger E

⟶ Chapter 7 $\quad \vec{J}\cdot\vec{dA}$ = Amps thru $\vec{dA}$

current density

$\vec{J} = \sum_{\substack{\text{ions}\\ \text{electrons}}} N_i q_i \vec{v}_i$ $\quad$ N_i: $\frac{\text{number}}{\text{volume}}$

$\vec{v}_D = \frac{q\tau}{m}\vec{E}$ $\quad$ $\frac{q\tau}{m}$: book μ, mobility

$= \sum \frac{N_i q_i^2 \tau_i}{m_i}\vec{E} = \sigma\vec{E}$

conductivity often $\sigma = \frac{1}{\rho}$

ρ: resistivity, η in text

factoids: $v_D \ll v$; QM: in perfect crystal $\sigma \to \infty$

τ due to thermal dislocations ; $T \to 0$, $\tau \uparrow$, $R \to 0$

$\vec{J}\,dV$; $\vec{K}\,dA$; $I\,\vec{dl}$ $\to$ $\rho\,dV$, $\sigma\,dA$, q

$\vec{K}$ ↳ text $\vec{j}$ (lower case)

Laplaces Eq

$$\nabla^2 \phi = 0$$

$$\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$$

$$\nabla \cdot E = 0$$

$$E = -\nabla \phi$$

$\nabla \cdot J = 0$? ← continuity

charge conserved

$$\int \frac{1}{|r - r'|} \, dV'$$

r: observer, r': source

$$\vec{J} = \sigma \vec{E}$$

$$Q = \int \rho \, dV$$

$$\oint \vec{J} \cdot d\vec{A}$$

leaves

$$\underbrace{\frac{d}{dt} Q}_{\int \dot{\rho} \, dV} + \underbrace{\oint J \cdot dA}_{\int \nabla J \, dV} = 0$$

$$\int (\underbrace{\partial_t \rho + \nabla \cdot J}_{0}) \, dV = 0$$

$$\partial_t \rho + \nabla \cdot J = 0$$

steady $\partial_t \rho = 0$:

$$\nabla \cdot J = 0$$

$$\partial_t \rho + \nabla \cdot \underset{\nwarrow\, g\vec{E}}{\vec{J}} = 0 \qquad \nabla \cdot E = \frac{\rho}{\varepsilon}$$

$$\partial_t \rho + \frac{g}{\varepsilon}\rho = 0$$

$$\rho \sim e^{-t/\tau} \qquad \frac{1}{\tau} = \frac{g}{\varepsilon}$$

$$\tau = \frac{\varepsilon}{g}$$

$$\nabla \cdot J = 0$$
$$\nabla \cdot E = 0$$
$$\nabla^2 \phi = 0$$

BC

ϕ agree

$J_n^{(1)} = J_n^{(2)}$

$n \leftarrow r$

$$\sum \left(A_n r^n + \frac{C_n}{r^{n+1}} \right) P_n$$

inside $C_n = 0$ $\qquad \phi_{in} = \sum A_n r^n P_n$

outside $A_n = 0$ $\qquad \phi_{out} = \sum \frac{C_n}{r^{n+1}} P_n$

$$\underbrace{-E_0 \, r\cos\theta}_{z}$$

$$A_1 \overbrace{r^1 \cos\theta}^{P_1}$$

$A_1 \leftarrow -E_0$

$$\phi_{in}\big|_{r=R} = \phi_{out}\big|_{r=R}$$

$$\sum A_n R^n P_n = \sum \frac{C_n}{R^{n+1}} P_n - E_0 R \underbrace{\cos\theta}_{P_1}$$

$n \neq 1$ $\quad A_n R^n = \frac{C_n}{R^{n+1}}$

$n = 1$ $\quad A_1 R^1 = \frac{C_1}{R^2} - E_0 R$

$$J_r^{(2)}\big|_{r=R} = J_r^{(1)}\big|_{r=R}$$

$$gE_r = -g\partial_r \phi$$

$$g_2 \partial_r \phi_{in} = g_1 \partial_r \phi_{out} \qquad \Delta D_n = 0$$

$$g_2 \sum n A_n R^{n-1} P_n = g_1 \left(\sum \frac{-(n+1) C_n}{R^{n+2}} P_n - E_0 P_1 \right)$$

$n \neq 1$ $\quad g_2 n A_n R^{n-1} = \frac{-(n+1) g_1 C_n}{R^{n+2}}$

$n = 1$ $\quad g_2 A_1 = g_1 \left[\frac{-2 C_1}{R^3} - E_0 \right]$

$n \neq 1$ $\quad A_n = C_n = 0$

$$\times 2 \quad A_1 - \frac{C_1}{R^3} = -E_0 \qquad \frac{g_2}{g_1} = K$$

$$K A_1 + \frac{2C_1}{R^3} = -E_0$$

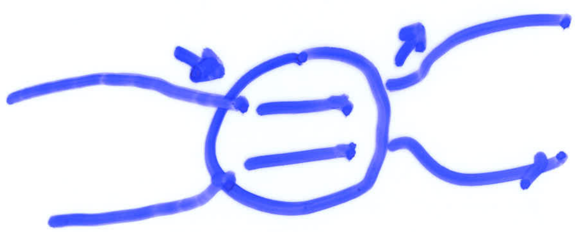
$$(K+2)A_1 = -3E_0$$

$$A_1 = \frac{-3}{K+2} E_0$$

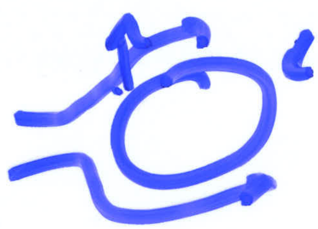
$$\frac{C_1}{R^3} = A_1 + E_0 = \left(\frac{K+2-3}{K+2}\right) E_0$$

$$= \frac{(K-1)}{(K+2)} E_0$$

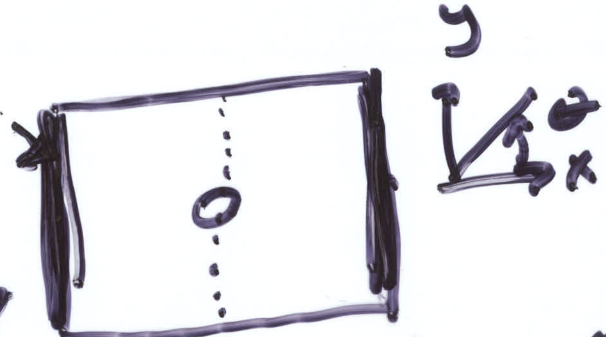
$K \to \infty$



$K \to 0$



$R = \frac{V}{I}$ ✓



$-E_0 r \cos\theta$

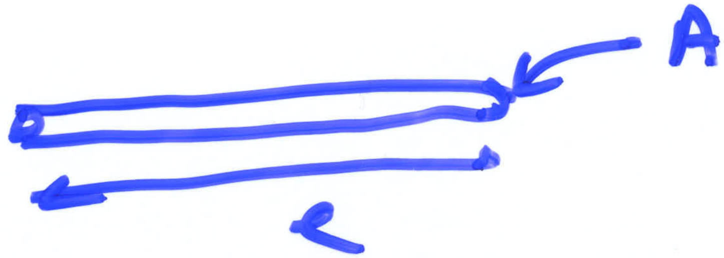
avg

Cylinder

$\Sigma A_n r^n \cos(n\theta)$

total J_x

Wires $\quad V = IR$



Assume J uniform across A ✓

skin effect: high f current avoids center

$$J = g E \qquad \frac{\Delta V}{\ell}$$

$$I = JA = gEA$$

$$= \frac{gA}{\ell} V$$

$A \sim$ gauge
big is thin $\quad$ 10, 12, 22, 30

$$R = \frac{\ell}{gA} = \frac{\eta \ell}{A}$$

$Q \Delta V$

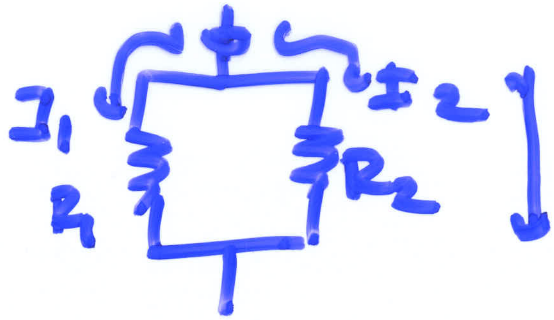
It

Power (Energy)

$$\text{Power} \quad \frac{\text{Energy}}{t} = I \Delta V = \frac{\Delta V^2}{R} = R I^2$$

Kirchhoff $\leftarrow$ AC

Parallel



V const
I add

$$I_2 = \frac{V}{R_2}$$
$$I_1 = \frac{V}{R_2}$$
$$I = I_1 + I_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V$$

$$\underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}_{R_{eff}} I = V$$

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

$$C_e = C_1 + C_2$$

$$Q = CV$$
$$RI = V$$

serial

I const
V add

$$V_2 = I R_2$$
$$V_1 = I R_1$$
$$V = V_1 + V_2 = I(R_1 + R_2)$$

$$R_e = R_1 + R_2$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$

Kirchhoff Laws

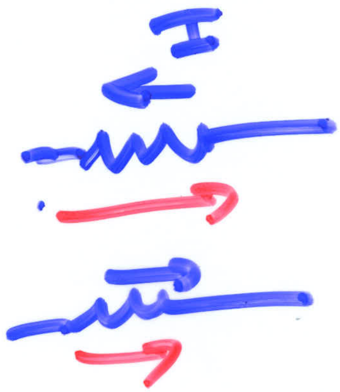
① Junction

Current in = Current out

② Loop: $\sum_{Loop} \Delta V = 0$

ref

Matrix TI-83



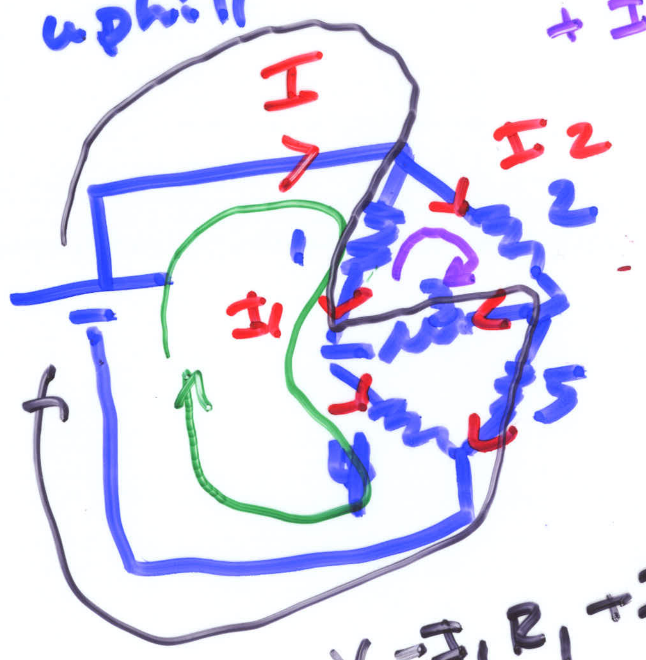
uphill

downhill

uphill

$V - I_1R_1 - I_4R_4 = 0$

$+ I_1R_1 - I_2R_2 - I_3R_3 = 0$



$V - I_1R_1 + I_3R_3 - I_5R_5 = 0$

Laplaces

r, r' stuff

$E = -\nabla\Phi$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{(\vec{r}-\vec{r}\,')\,\rho\,dv}{|r-r'|^3}$$

$\vec{J}\,dv$

$\vec{K}\,da$

$\vec{J}$

$I\,d\vec{l}$

$\vec{B}$

$q\vec{E} = \vec{F}$

$q\,\overline{v \times B} = \vec{F}$

Tesla

$I = \vec{J}\cdot\vec{A}$

$\frac{A}{m^2}\ m^3$

$=$

Am

$qC\frac{m}{s}$

$F = q\,\frac{v}{c} \times B$

$F = qE$

$4\pi \times 10^{-7}$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l} \times (\vec{r}-\vec{r}\,')}{|r-r'|^3}$$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

Ampere Law

$\vec{B} = \overline{\nabla \times A}$