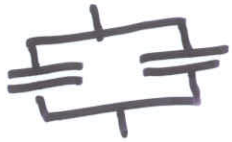
Capacitors: $Q = CV$ $\quad U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$

ideal: $C = \frac{K \varepsilon_0 A}{d}$

parallel:  $C = C_1 + C_2$

V same; Qs add

series:  $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Q same; Vs add

Energy $\quad U = \frac{1}{2} \sum_i {\sum_j}' \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$

$= \frac{1}{2} \int \rho \phi \, dV$ "position"

$= \frac{1}{2} \int \vec{E} \cdot \vec{D} \, dV$ "field"

energy density

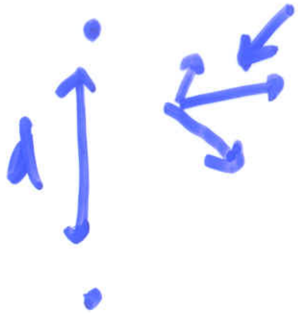
Example: uniformly charged sphere 2 ways

$$U = \frac{3}{5} \frac{Q^2}{4\pi\varepsilon_0 R}$$

$F = \frac{-dU}{dx}$; $\tau = \frac{-dU}{d\theta}$

Note problem: point charges have infinite self-energy ① not actual points so not actual infinite ② position independent so no effect on $-\vec{\nabla} U$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2} = \frac{-dU}{dd}$$

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 d}$$

$$\varepsilon_0 \int \frac{E^2}{2}\, dV$$

$$(\vec{E}_1 + \vec{E}_2)\cdot(\vec{E}_1 + \vec{E}_2)$$

$$= \underbrace{E_1^2 + E_2^2}_{\text{indep of } d} + \underline{2\vec{E}_1\cdot\vec{E}_2}$$

$$u = \frac{1}{2}\vec{E}\cdot\vec{D} \quad \frac{\text{energy}}{\text{volume}}.$$

$U\downarrow \qquad F = -\frac{dU}{dx}$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$C\downarrow$

move in $\Rightarrow$ $C\uparrow$

$Q = CV$

$$W = -\Delta PE + W_{battery}$$

$$W = F\,dx \qquad W_{battery} = \sum_i dQ_i V_i$$

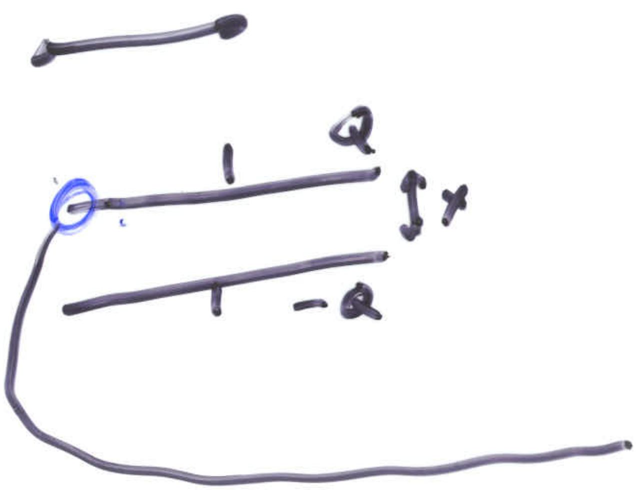
$$\left\{ \begin{aligned} F &= -\frac{dU}{dx}\bigg|_Q \ \checkmark \\ &= \frac{dU}{dx}\bigg|_V \ \checkmark \end{aligned} \right.$$

$$\frac{1}{2}\sum Q_i V_i$$

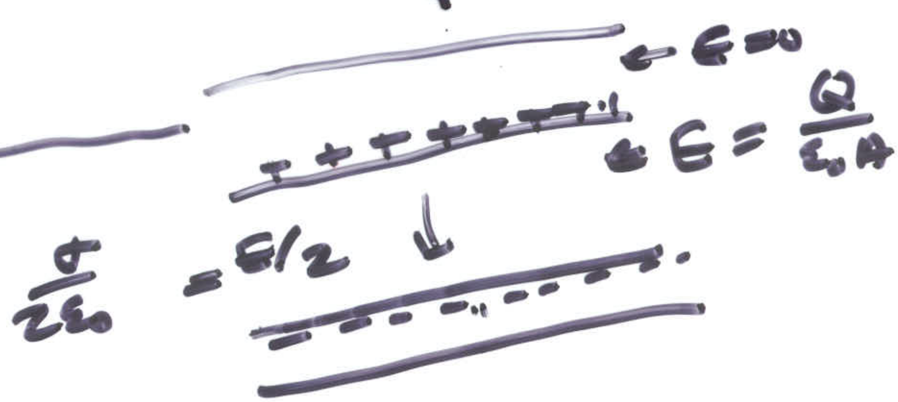
$$\frac{1}{2}\sum dQ_i V$$

$$= +\frac{1}{2}\sum dQ_i V_i = +\Delta PE$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$



$$F = \frac{1}{2}\int \sigma \cdot E_{outside}$$



$$F = \frac{1}{2} Q E = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A} \ \checkmark$$

$$F = -\frac{dU}{dx}\bigg|_{Q = \text{const}}$$

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$C = \frac{\varepsilon_0 A}{x}$$

$$F = -\frac{d}{dx}\left(\frac{1}{2} x \frac{Q^2}{\varepsilon_0 A}\right)$$

$$= -\frac{1}{2}\frac{Q^2}{\varepsilon_0 A}$$

$$U = \frac{1}{2} C V^2$$

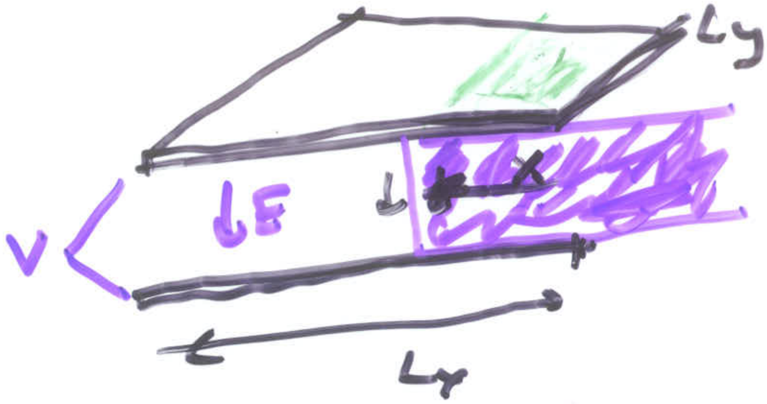
$$C = \frac{\varepsilon_0 A}{x}$$

$$F = +\frac{dU}{dx}$$

$$= \frac{d}{dx}\left(\frac{\varepsilon_0 A V^2}{2x}\right)$$

$$= -\left(\frac{\varepsilon_0 A V^2}{2x^2}\right) \qquad \frac{\varepsilon_0 A}{\varepsilon_0 A}$$

$$= -\left(\frac{C^2 V^2}{2\varepsilon_0 A}\right) = -\frac{Q^2}{2\varepsilon_0 A}$$



$V = Ax + C$

$\uparrow$

E

$\sigma = D = K\varepsilon_0 E$

$= K\varepsilon_0 \dfrac{V}{d}$

$\square \quad \sigma = D = \varepsilon_0 E$

$= \varepsilon_0 \dfrac{V}{d}$

$Q = \quad \square$

$= x L_y K\varepsilon_0 \dfrac{V}{d} +$

$(L_x - x) L_y \dfrac{\varepsilon_0 V}{d}$

$= \dfrac{L_y \varepsilon_0}{d}\left(Kx + (L_x - x)\right)V$

$= \dfrac{L_y \varepsilon_0}{d}\left(\underbrace{(K-1)}_{\chi}x + L_x\right)V$

$C_1 = \dfrac{\varepsilon_0 A}{(d-t)}$

$C_2 = \dfrac{K\varepsilon_0 A}{t}$

$\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

$C_1 = \dfrac{\varepsilon_0 L_y (L_x - x)}{d}$

$C_2 = \dfrac{K\varepsilon_0 L_y x}{d}$

$C = C_1 + C_2$

$$C = \frac{L_y \varepsilon_0}{d}\left(\chi x + L_x\right)$$

$$F = -\frac{dU}{dx}\bigg|_Q \qquad U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C V^2$$

$$= -\frac{Q^2}{2}\frac{d}{dx} C^{-1} = \frac{Q^2}{2} C^{-2} \frac{L_y \varepsilon_0}{d} \chi$$

$$= \frac{V^2}{2}\frac{L_y \varepsilon_0}{d} \chi \quad ✓$$

$$F = +\frac{dU}{dx}\bigg|_{V = \mathrm{const}}$$

$$U = \frac{q_1 q_1}{4\pi\varepsilon_0 d}$$

$$-\frac{dU}{dx} = +\frac{q_1 q_1}{4\pi\varepsilon_0 d^2}$$

$$= \frac{V^2}{2}\frac{d}{dx} C$$

$$= \frac{V^2}{2}\frac{L_y \varepsilon_0 \chi}{d} \quad ✓$$

$$\tau = -\frac{dU}{d\theta}$$

Magnetic Fields (B) $\leftarrow I$

Currents: $\vec{J}\,dV$

$\vec{K}\,dA$

$I\,d\vec{l}$

charge

$\rho\,dV$

$\sigma\,dA$

q

current density

$$J = \frac{I}{A} = \frac{\text{charge}/\text{time}}{\text{Area}}$$

$$N \leftarrow \frac{\#e^-}{\text{volume}}$$

v_D

v_D

$v_\perp\, t\, A\, N\, q$

$$J = \frac{v_\perp t A N q / t}{A} = v_\perp N q$$

$$\vec{J} = \sum \vec{v}_i N_i q_i$$

$\langle v \rangle$

v_D ~ walking speed $\ll \frac{2}{3}c$ signal

$\uparrow$ drift

$v \sim \sqrt{\frac{kT}{m}}$

drift $\ll$ speed

$\vec{E}$

$F \propto q\underline{\underline{\;}}$

e^-

warning: $e^{-t/\tau}$ $\Leftarrow$

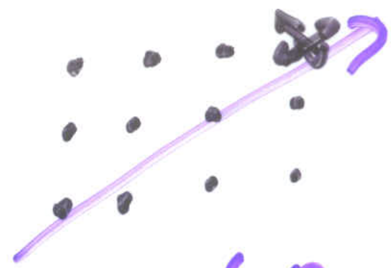
$\langle v_D \rangle \sim a\,\frac{\tau}{2}$

$a = F/m = \frac{qE}{m}$

$$\langle v_D \rangle = \underbrace{\frac{q\,\tau}{2m}}_{\mu} E$$

$$J = \underbrace{\sum \frac{q_i \tau_i}{2m_i} q_i N_i}_{\sigma} E$$

Crystal



Zero hits! (QM says)

Low Temp $\downarrow$ $\tau \uparrow$ $\sigma \uparrow \infty$

$\sigma \uparrow$

(Superconductivity $R = 0$

0