dipole: $\ell \updownarrow {+q \atop -q} \uparrow p = q\ell$ $\qquad \phi = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

Polarization: net dipole per volume $\vec{P} = \frac{\Sigma \vec{p}}{\text{Volume}}$

$$\phi = \frac{1}{4\pi\varepsilon_0} \int \vec{P}(\vec{r}\,')dv' \cdot \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

equivalent to $\sigma_B = \vec{P}\cdot\hat{n}$ $\quad \rho_B = -\nabla\cdot P$

For normal size $\vec{E}$ most materials are linear dielectrics: $\vec{P} = \varepsilon_0 \chi \vec{E}$

complications: large $\vec{E}$, nonlinear susceptibility

in crystals $\vec{P}$ not alligned with $\vec{E}$

$$\vec{D} \equiv \varepsilon_0\vec{E} + \vec{P} \overset{\text{linear}}{=} \varepsilon_0\underbrace{(1+\chi)}_{\text{dielectric constant}}\vec{E} = \varepsilon\vec{E}$$

$$\vec{\nabla}\cdot\vec{D} = \varepsilon_0\nabla\cdot E + \nabla\cdot P = \underbrace{(\rho - \rho_B)}_{\rho_f}$$

Gauss: $\int \vec{D}\cdot\hat{n}\,dA = Q_{free}$

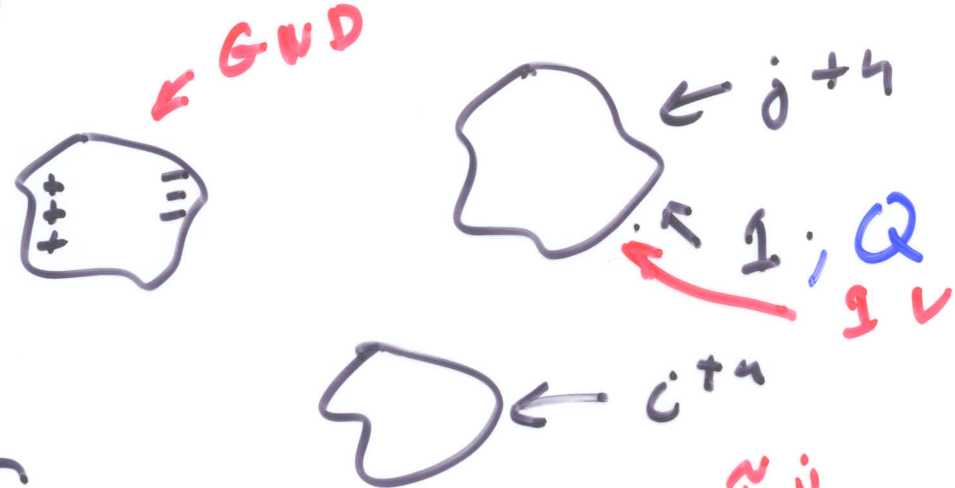
Point charge: $D = \frac{Q_f}{4\pi r^2} \Rightarrow E = \frac{Q_f}{4\pi\varepsilon_0 K r^2}$

$$\rho_f = \nabla\cdot D = \varepsilon_0 K \nabla\cdot \overset{\frac{1}{\varepsilon_0\chi}P}{E} = \frac{K}{\chi}\nabla\cdot P = -\frac{K}{\chi}\rho_B$$

$\therefore \rho_B = -\frac{\chi}{K}\rho_f$; $\rho_f = 0 \Rightarrow \rho_B = 0 \Rightarrow$ Laplace still valid

Note: in some sense a conductor is a $K=\infty$ dielectric — provides useful check

Superposition   N conduct



$Q\,\tilde{\phi}^{j}(\vec{r}_i)$ $\qquad V_j\,\tilde{\phi}^{j}$

$=$

$$\sum Q_j\,\phi^{j}(\vec{r})$$

$$\sum Q_j \underbrace{\phi^{j}(\vec{r}_i)}_{P_{ij}} = V_i$$

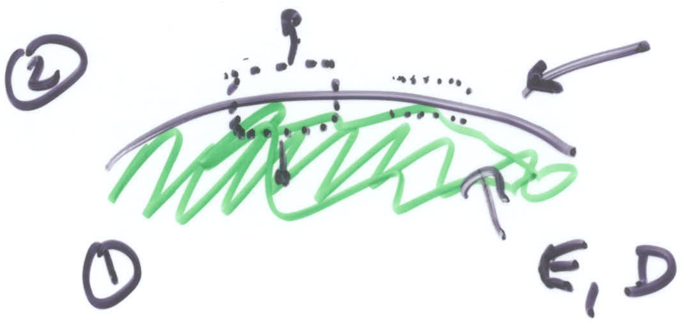
$$(P)(Q) = (V)$$

$$\sum P_{ij} Q_j = V_i$$

$$\sum r_{ij} V_j = Q_i$$

$$(\ \ )(V) = (Q)$$

B.C. = boundary Conditions



$$\oint \vec{D}\cdot\hat{n}\, dA = Q$$
$$= \sigma_f A$$

$$\vec{D}_2\, \hat{n}_2\, A + \vec{D}_1\, \hat{n}_1\, A$$

$$\hat{n}_1 = -\hat{n}_2$$

$$(\vec{D}_2 - \vec{D}_1)\cdot\hat{n}_2 = \sigma_f$$

$$\Delta D_n = \sigma_f \leftarrow 0$$

$$K\varepsilon_0 E$$

$$K_2E_2 - K_1E_1 = 0$$

$$\Delta E_t = 0$$

$$\oint_c E\cdot d\vec{\ell} = \int \underbrace{\nabla\times E}_{0}\cdot n\, dA$$

$$\vec{E} = -\nabla\phi$$

$$\nabla\times(\underline{\underline{\nabla\phi}}) = 0$$

$$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \partial_x\phi & \partial_y\phi & \partial_z\phi \end{vmatrix}$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

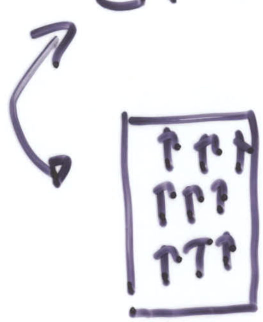
$$\vec{E}_2\cdot\hat{\epsilon}\, l + \vec{E}_1\cdot(-\hat{\epsilon})\, l$$

$$\Delta E_t = 0$$

$$\frac{E}{K}$$

$$\epsilon \leftarrow 99\,\epsilon_0 \quad \cancel{*}$$

Linear Dielectrics

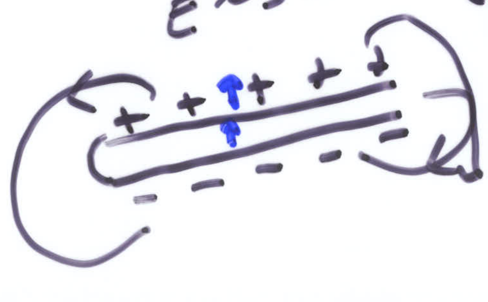


← fixed, const $\vec{P}$

$$\vec{P} \not\propto E$$

E ~ small

$$\sigma_B = P$$



$$E = \frac{\sigma}{\epsilon_0} \leq \frac{P}{\epsilon_0}$$

$D = \epsilon_0 E + P.$ $\quad$ $D = \epsilon_0 E$ (small)

outside $P = 0$

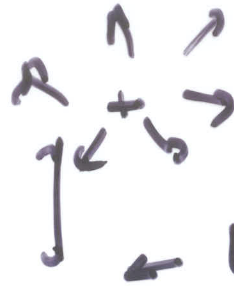
inside $\uparrow\downarrow \leftarrow \epsilon_0 E$ small

P

$\Phi_B = PA$

$D = P - \frac{2Q_B}{4\pi d^2}$

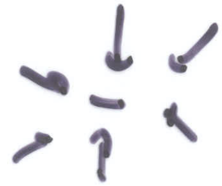
$-Q_B$



A

$E = \frac{2Q_B}{4\pi\varepsilon_0 d^2}$

~small



$\nabla^2\phi = 0$

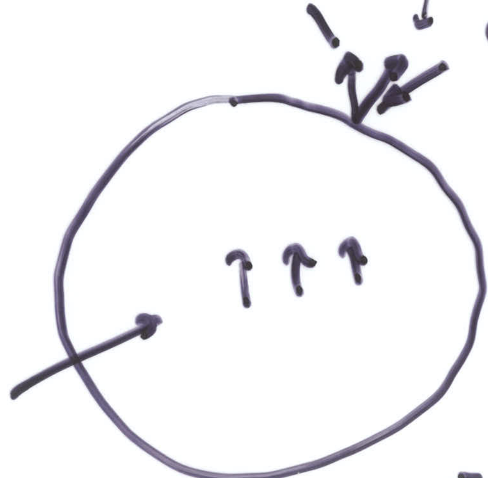
$D = \varepsilon_0 E + P$

$\nabla^2\phi = \frac{-\rho}{\varepsilon_0}$

$\hat{n}$ $= \vec{P}\cdot\hat{n}$

p

$\sigma_B = P\cos\theta$



$\bar{P}$

$\phi_{out} = \sum \frac{C_n}{r^{n+1}} P_n$

Laplace OK inside

$\rho_B = -\vec{\nabla}\cdot\vec{P} = 0$

$\phi_{in} = \sum A_n r^n P_n$

ϕ continuous $\longrightarrow \Delta E_t = 0$

$\sigma = \varepsilon_0 \left(\partial_r \phi_{in} - \partial_r \phi_{out} \right)$

polarization $\qquad P_1$

$$\sigma = P\cos\theta = \varepsilon_0\left(\partial_r \phi_{in} - \partial_r \phi_{out}\right)$$

$$\sum n A_n r^{n-1} P_n \Big|_{r=R} \qquad \sum \frac{-(n+1)C_n}{r^{n+2}} P_n$$

$$= \varepsilon_0 \sum \left(n A_n R^{n-1} + \frac{(n+1)C_n}{R^{n+2}} \right) P_n$$

PP₁

$$= \varepsilon_0 \left((2n+1) A_n R^{n-1} \right) P_n$$

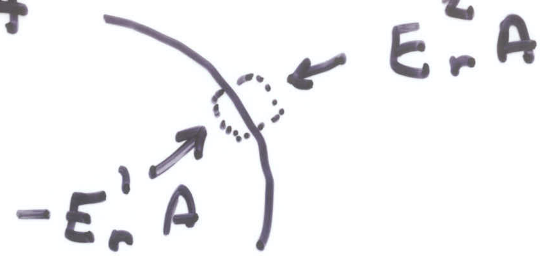
$$P = \varepsilon_0 \, 3 \, A_1 \qquad \frac{C_1}{R^3}$$

$$C_1 = \frac{P \, R^3 \, 4\pi}{3 \, 4\pi\varepsilon_0} = \frac{P \cdot \mathrm{Volu}}{4\pi\varepsilon_0}$$

$$\phi = \frac{C_1}{r^2} P_1 = \frac{P \cdot \mathrm{Vol}}{4\pi\varepsilon_0 r^2} \cos\theta$$

$$A_1 = \frac{P}{\varepsilon_0 3} \qquad \phi = \frac{P}{3\varepsilon_0} \overbrace{r\cos\theta}^{z}$$

$$(\underset{-\partial_r\phi}{E_r^2} - \underset{\partial_r\phi}{E_r^1})A = \frac{\sigma}{\varepsilon_0}A$$



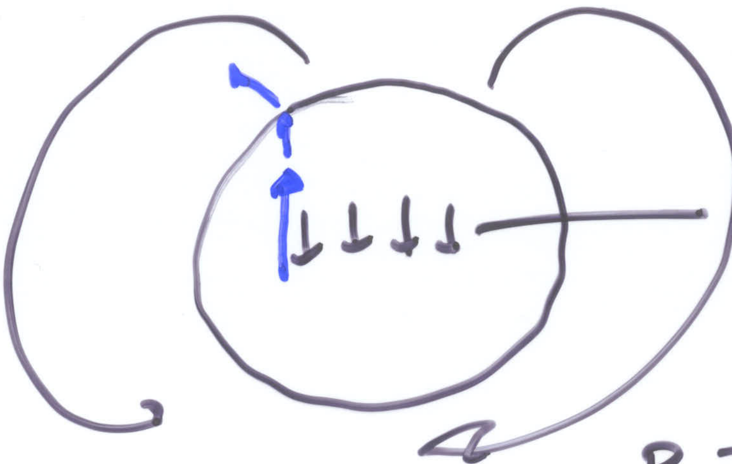
at $r=R$ $\quad \phi_{in} = \phi_{out}$

$$\sum A_n R^n P_n = \sum \frac{C_n}{R^{n+1}} P_n$$

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\int_{-1}^{1} * P_m \, dc \qquad \int_{-1}^{1} P_m P_n \, dc = \delta_{nm} \frac{2}{2n+1}$$

$$A_m R^m = \frac{C_m}{R^{m+1}}$$ * BC #1

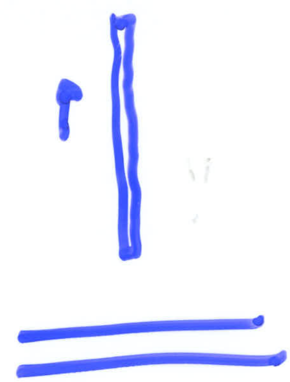


$$E = \frac{P}{\varepsilon_0 3}$$

$$p = P \cdot \frac{4}{3}\pi R^3$$

$$D = \varepsilon_0 E + P$$

$$= -\frac{P}{3} + P = \frac{2}{3}P$$



normal

$$\Delta D_n = \sigma_f$$

$$\Delta E_t = 0 \quad \leftarrow \quad \nabla \times E = 0$$

tangent