$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \Rightarrow \vec{A}\cdot\hat{k} = A_z \hat{k}\cdot\hat{k}$$

$$f(\theta) = \sum B_n \cos(n\theta) \Rightarrow \int_0^{2\pi} f(\theta)\cos(m\theta)\,d\theta$$

$$= B_m \pi$$

"Fourier Trick"

Typically we know the voltage on the boundary (e.g., $r=R$)

so we evaluate ϕ at $r=R$

Before we use Fourier Trick

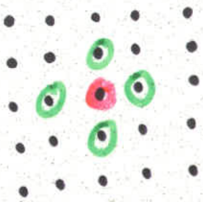
$$\Rightarrow \int_0^{2\pi} \phi(R,\theta)\cos m\theta\,d\theta$$

ie on boundary

finite differences: computer holds the voltage at a lattice of points. update

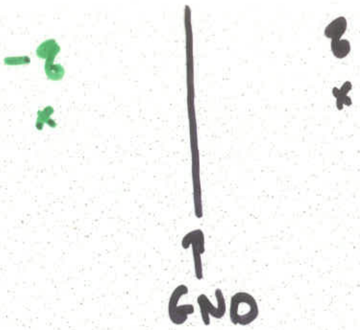
$$\phi \leftarrow \phi + \lambda(\text{avg}\,\phi - \phi)$$

$$\partial^2 f = \frac{f_1 + f_{-1} - 2f_0}{h^2}$$



$$\nabla^2 = \frac{1}{h^2}\left[(\text{sum green}) - 4\,\text{red}\right]$$

Images:



sphere:

$d' = \frac{R^2}{d}$

$q' = -\frac{R}{d} q$

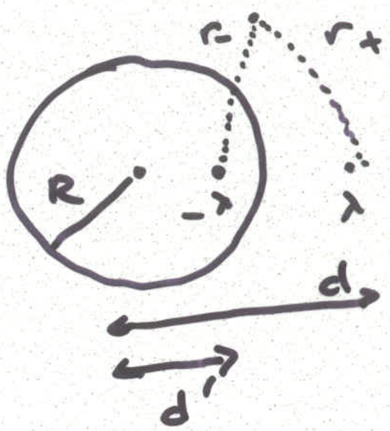
adding a charge q'' at center leaves sphere surface equipotential

Help

Tues 7:15 ~~pm~~ here.

For isolated line charge $E = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow \phi = \frac{-\lambda}{2\pi\varepsilon_0}\ln r$

For image in cylinder



$$\frac{2\pi\varepsilon_0\phi}{\lambda} = \ln r_- - \ln r_+$$

$$= \ln\left[\frac{d'^2 + r^2 - 2d'r\cos\theta}{d^2 + r^2 - 2dr\cos\theta}\right]^{1/2}$$

$$= \ln \frac{r}{d}\left[\frac{1 + \left(\frac{d'}{r}\right)^2 - 2\frac{d'}{r}\cos\theta}{1 + \left(\frac{r}{d}\right)^2 - 2\frac{r}{d}\cos\theta}\right]^{1/2}$$

<u>IF</u> $\frac{r}{d} = \frac{d'}{r}$ THEN $\frac{2\pi\varepsilon_0}{\lambda}\phi = \ln\frac{r}{d}$ ← constant!

so $r = R$ works!

note bisecting plane between $-\lambda$ & λ also const $= 0$

Twin-lead cable $\phi = \pm\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R}{d}\right)$

$$L = d + d' \Rightarrow \frac{L}{2R} = \frac{\frac{d}{R} + \frac{R}{d}}{2}$$

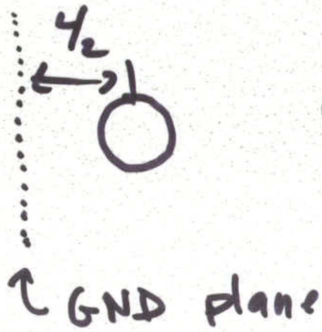
$$\cosh^{-1}\left(\frac{L}{2R}\right) = \ln\left(\frac{d}{R}\right)$$

$$D = d - d' \Rightarrow \frac{D}{2R} = \frac{\frac{d}{R} - \frac{R}{d}}{2}$$

$$\sinh^{-1}\left(\frac{D}{2R}\right) = \ln\left(\frac{d}{R}\right)$$

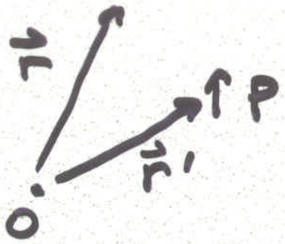
$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\frac{2\lambda}{2\pi\varepsilon_0}\ln\left(\frac{d}{R}\right)} = \frac{\pi\varepsilon_0 \ell}{\cosh^{-1}\left(\frac{L}{2R}\right)}$$

$9.85\ \frac{\text{pF}}{\text{m}}$



$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\frac{\lambda}{2\varepsilon_0\pi}\ln\left(\frac{d}{R}\right)} = \frac{2\pi\varepsilon_0 \ell}{\cosh^{-1}\left(\frac{L}{2R}\right)}$$

dipoles $\quad +q \; -q \; \uparrow \ell \qquad \vec{p} = q\vec{\ell}$



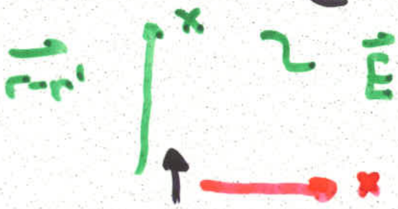
$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{\vec{p}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right)$$

$$\tilde{\phi} = \frac{1}{4\pi\epsilon_0}\frac{1}{|\vec{r}-\vec{r}'|}$$

$PE = -\vec{p}\cdot\vec{E}$

$$\phi = q\left(\tilde{\phi}(\vec{r}'+\vec{\ell}) - \tilde{\phi}(r')\right)$$
$$= q\vec{\ell}\cdot\nabla'\tilde{\phi} = \vec{p}\cdot(-\nabla\tilde{\phi})$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left\{\frac{3\,\vec{p}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^5}(\vec{r}-\vec{r}') - \frac{\vec{p}}{|\vec{r}-\vec{r}'|^3}\right\}$$



$\sim \quad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{|\vec{r}-\vec{r}'|^3}$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{-\vec{p}}{|\vec{r}-\vec{r}'|^3}$

$x \sim \quad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{|\vec{r}-\vec{r}'|^3}$

$\frac{Q}{vol} = \rho$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}')\,dv'}{|\vec{r}-\vec{r}'|}$$

$\frac{\Sigma\vec{p}}{\text{volume}} = \vec{P}$

polarization

$\vec{P}(\vec{r}')\,dv' = d\vec{p}$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{P}(\vec{r}')\,dv' \cdot (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$-\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}$$

$$\nabla'\frac{1}{|\vec{r}-\vec{r}'|}$$

$$P(\vec{r}')\,\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}'|} = \vec{\nabla}\cdot\left(\vec{P}\frac{1}{|\vec{r}-\vec{r}'|}\right) - (\nabla\cdot P)\frac{1}{|r}$$

$$\vec{\nabla}\cdot(\vec{A}\phi) = (\nabla\cdot A)\phi + \vec{A}\cdot\vec{\nabla}\phi$$

$$\partial_x(A_x\phi) = (\partial_x A_x)\phi + A_x\partial_x\phi$$

$$P\,\vec{\nabla}'\frac{1}{|r-r'|} = (\nabla'\cdot P)\frac{1}{|r-r'|} + \nabla\cdot\left(P\frac{1}{|r-r'|}\right)$$

$$\phi = \frac{1}{4\pi\varepsilon_0} \int \vec{P}(r') \cdot \underbrace{\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}_{\nabla' \frac{1}{|r-r'|}} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \int \left[\nabla' \cdot \left(\vec{P} \frac{1}{|r-r'|} \right) - \frac{\nabla' \cdot P}{|r-r'|} \right] dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \int_{\text{surface}} \frac{\vec{P} \cdot \hat{n}}{|r-r'|} dA' + \frac{1}{4\pi\varepsilon_0} \int_{\text{vol}} \frac{-\nabla' \cdot P}{|r-r'|} dV'$$

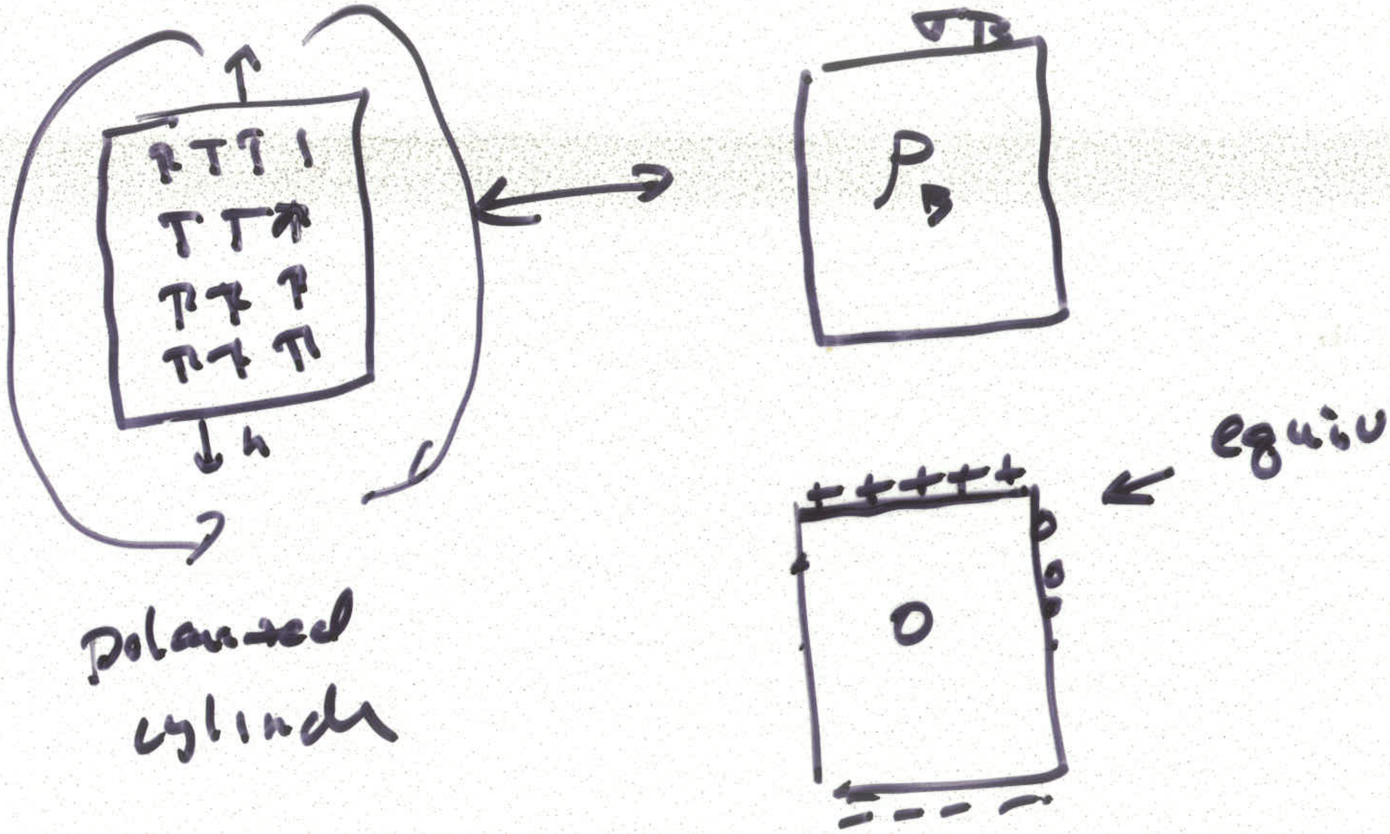
$$\vec{P} \cdot \hat{n} = \sigma_B$$

$$\frac{q\ell}{\ell^3} = \frac{q}{\ell^2}$$

$$-\nabla \cdot P = \rho_B \qquad \frac{q\ell/\ell^3}{\ell}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma_B}{|\vec{r}-\vec{r}'|}\,dA'$$

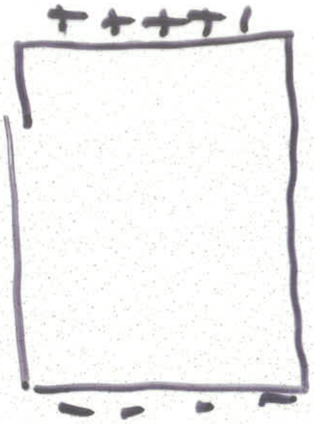
$$+\ \frac{1}{4\pi\epsilon_0}\int \frac{\rho_B\,dv'}{|\vec{r}-\vec{r}'|}$$



$$\nabla\cdot E = \frac{\rho}{\varepsilon_0} \longleftarrow \rho_f + \rho_B$$

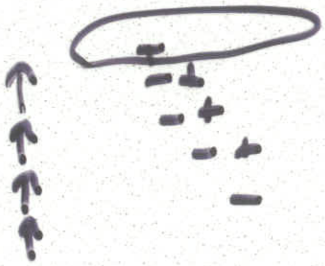
$$\varepsilon_0 \nabla\cdot E = \rho_f - \nabla\cdot P$$

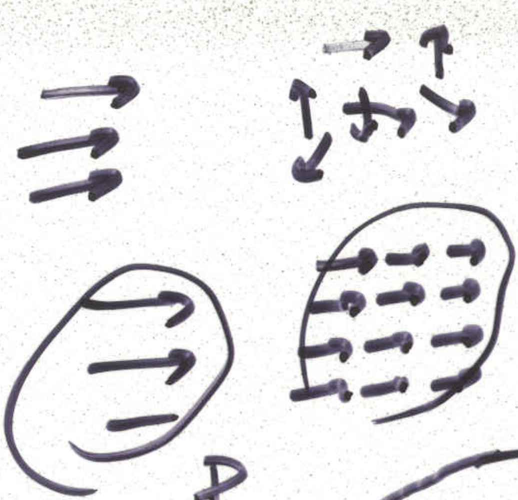
$$\nabla\cdot(\varepsilon_0 E + \vec{P}) = \rho_f$$



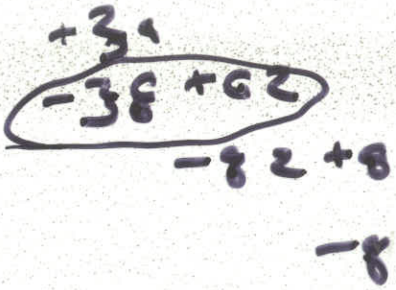
$\sigma_B = \vec{P}\cdot\hat{n}$

$\rho_B = -\nabla\cdot P$



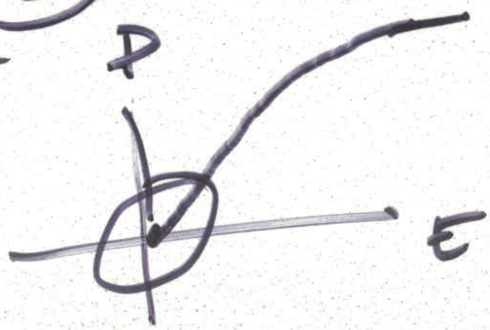


$P = 0$



$P \neq 0$

$\vec{P} \propto \vec{E}$ linear



$\frac{Q}{L^2}$ $\quad$ $\frac{Q}{\varepsilon_0 L^2}$

$\vec{F} = q\vec{E}$

$\vec{P} = \chi\varepsilon_0 E$

χ

electric sus.

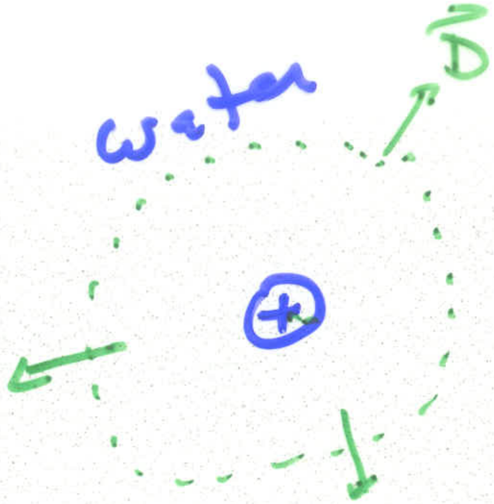
$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

$\chi\varepsilon_0\vec{E}$

$= \varepsilon_0(1+\chi)\vec{E}$

dielectric const

2-3 $\quad$ 80



$$\nabla \cdot D = \rho_e$$

$$\int \nabla \cdot D \, dV = \int \rho_e \, dV = Q$$

$$\| $$

$$\int D \cdot n \, dA$$

$$\int D \cdot n \, dA = D \, 4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2} = \varepsilon_0 \underbrace{K}_{(1+\chi)} E$$

$$\frac{V_{at}}{V}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2 K}$$