Solutions to $\nabla^2 \phi = 0$ (Laplace) in 2d

$$\phi = \sum \left(A_n r^n + \frac{C_n}{r^{n+1}} \right) P_n(\cos\theta)$$

$$\phi = \underbrace{A_0 + C_0 \ln(r)}_{A_0' + C_0 \ln(r/R)} + \sum \left(A_n r^n + C_n r^{-n} \right)\left(a_n \cos n\theta + b_n \sin n\theta \right)$$

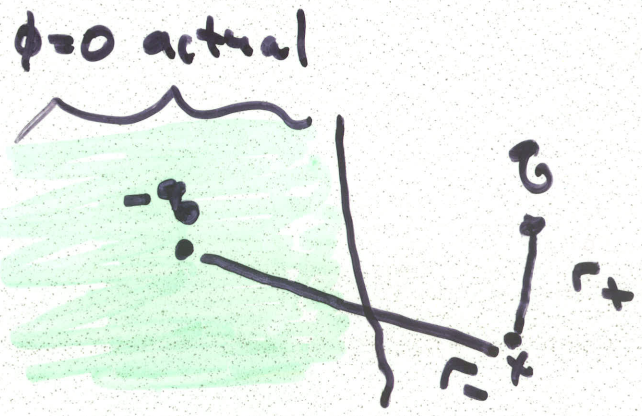
$$\phi = \sum A_n \sin\left(\frac{n\pi}{b} y\right) \frac{\sinh\left(\frac{n\pi}{b} x\right)}{\sinh\left(\frac{n\pi}{b} a\right)}$$

Expansion coef determined by Fourier

Use symmetry! $\phi(z)$ special case

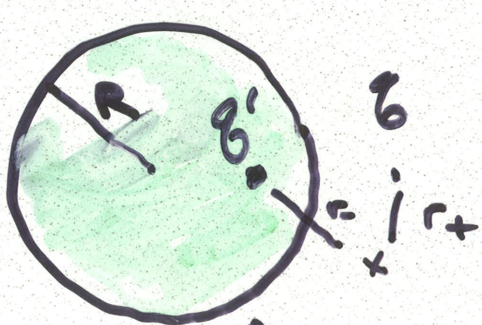
inside/outside $\vec{E} = E_0 \hat{z} \to -E_0 r^1 P_1$

Images: simple, "guessed" solutions



$$\phi = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-} \right)$$

$$F = \frac{q^2}{4\pi\epsilon_0} \frac{1}{(2d)^2}$$

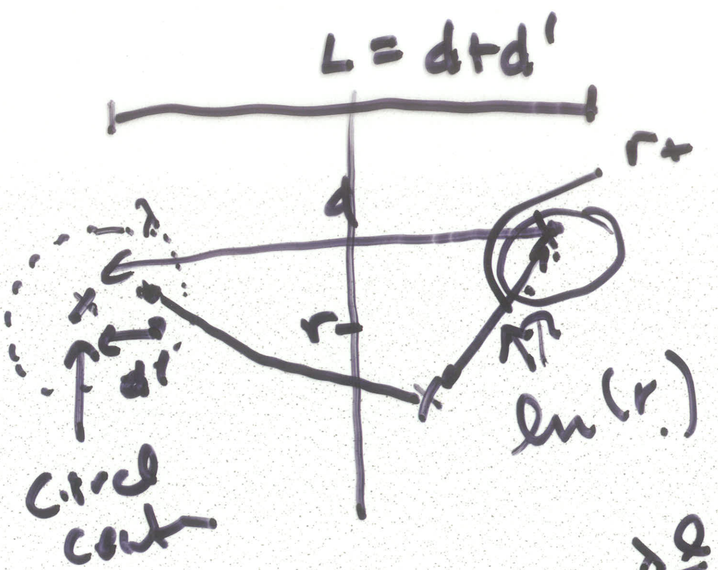


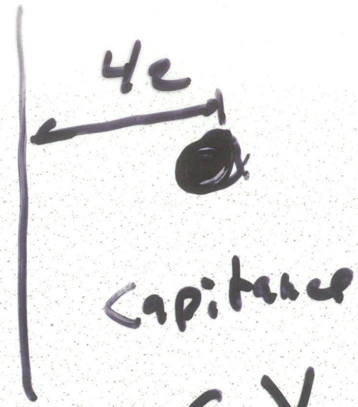
$$r' = \frac{R^2}{r}$$

$$q' = -\frac{qR}{r}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_+} + \frac{q'}{r_-} \right)$$

$\phi = 0$ or add q'' at center if desired





$Q = CV$

$$2\pi r \ell E = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0 r}$$

$$\phi = \frac{-1}{2\pi\varepsilon_0}\ln(r)$$

$$\frac{2\pi\varepsilon_0}{\lambda}\phi = \ln(r_+) - \ln(r_-)$$

$$\sqrt{d^2 + r^2 - 2dr'\cos\theta}$$

d

r

$$\ln\left[\frac{\sqrt{d^2 + r^2 - 2dr\cos\theta}}{\sqrt{d'^2 + r^2 - 2d'r\cos\theta}}\right]$$

$$\ln\left[\frac{d\sqrt{1 + \left(\frac{r}{d}\right)^2 - 2\frac{r}{d}\cos\theta}}{r\sqrt{1 + \left(\frac{d'}{r}\right)^2 - 2\frac{d'}{r}\cos\theta}}\right]$$

$$\frac{r}{d} = \frac{d'}{r}$$

$$= \ln[d/r]$$

$$L = d + d'$$

$$\frac{R^2}{d}$$

$$\frac{L}{2R} = \frac{d/R + \frac{R}{d}}{2}$$

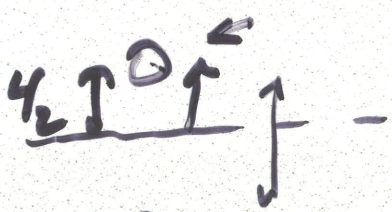
$$= \cosh\left(\underline{\ln\left(d/R\right)}\right)$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{\frac{d}{R} + \frac{R}{d}}{2}$$

$$\sinh y = \frac{e^x - e^{-x}}{2}$$

$$\cosh^{-1}\left(\frac{L}{2R}\right) = \ln\left(\frac{d}{R}\right) = \phi \frac{2\pi\varepsilon_0}{\lambda}$$

$$\frac{\lambda \ln\left(\frac{d}{R}\right)}{2\pi\varepsilon_0}$$

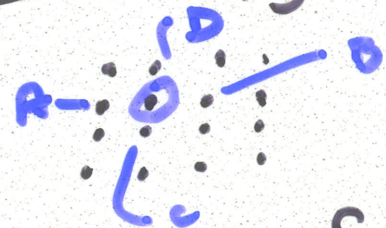


$$C = \frac{Q}{V} = \frac{\lambda l}{\frac{2\lambda \cosh^{-1}\left(\frac{L}{2R}\right)}{2\pi\varepsilon_0}}$$

$$\frac{C}{l} = \frac{\pi\varepsilon_0}{\cosh^{-1}\left(L/2R\right)}$$

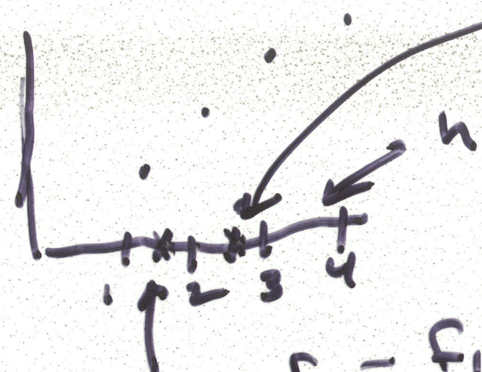
$$\nabla^2 \phi = 0$$

2-d x, y



$$\partial_x^2 \phi + \partial_y^2 \phi$$

$$f' = \frac{f_3 - f_2}{h}$$



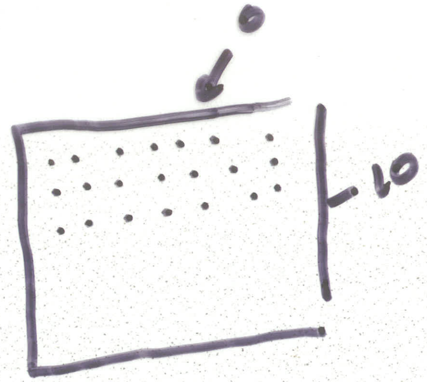
$$f' = \frac{f_2 - f_1}{h}$$

$$\frac{\left(\frac{f_3 - f_2}{h}\right) - \left(\frac{f_2 - f_1}{h}\right)}{h} = \frac{f_3 + f_1 - 2f_2}{h^2}$$

$$\frac{(\phi_B + \phi_A - 2\phi_0)}{h^2} + \frac{(\phi_D + \phi_C - 2\phi_0)}{h^2}$$

$$\frac{\phi_A + \phi_B + \phi_C + \phi_D - 4\phi_0}{h^2} = 0$$

$$\phi_0 = \mathrm{Avg}(\phi)$$



$$\vec{p} = q\vec{\ell}$$

$+q$, $-q$, ℓ

$$q\,\phi(\vec{r}, \vec{r'+\ell}) - q\,\phi(\vec{r}, \vec{r'})$$
$$= q\,\vec{\ell}\cdot\vec{\nabla}'\phi$$
$$= -q\,\vec{\ell}\cdot\vec{\nabla}\phi$$
$$q\,\vec{\ell}\cdot\vec{E}$$

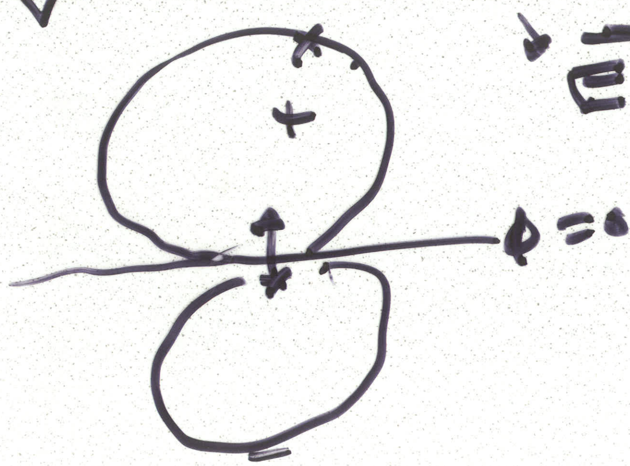
$$\phi = \frac{1}{4\pi\epsilon_0}\,\frac{1}{|\vec{r}-\vec{r'}|}$$

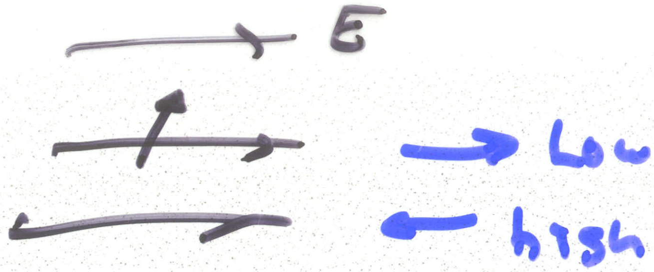
$(\vec{r}, r')$

$$\nabla' \longleftrightarrow -\nabla$$

$$\phi = \vec{p}\cdot\frac{(\vec{r-r'})}{4\pi\epsilon_0|\vec{r-r'}|^3}$$

$$\vec{E} = -\nabla\phi$$
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3\,\vec{p}\cdot(\vec{r-r'})(\vec{r-r'})}{|\vec{r-r'}|^5} - \frac{\vec{p}}{|r-r'|^3}\right]$$





PE
$\parallel$
$q\phi$

$$PE = q\,\phi(\vec{r}+\vec{\ell}) - q\,\phi(r)$$

$$= q\,\vec{\ell}\cdot\vec{\nabla}\phi = \underline{-\vec{p}\cdot\vec{E}}$$

$$\vec{F} = -\nabla PE$$

$\pm\,\varepsilon$

$\pm\,3\varepsilon$