$$\nabla \frac{1}{|\vec{r}-\vec{r}'|} = -\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \qquad \nabla^2 \frac{1}{|\vec{r}-\vec{r}'|} = 0$$

unless $\vec{r} = \vec{r}'$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dq' \rightarrow \begin{matrix} \rho\, dv \\ \sigma\, dA \\ \lambda\, dl \end{matrix}$$

r: observe   r': source

$$\frac{d}{dx} \underbrace{\int F(x,y)\, dy}_{\phi(x)} = \int \partial_x F(x,y)\, dy$$

$$\begin{tikzcd}
 & & \phi \arrow[dll] \\
\vec{E} & & \\
 & \rho \arrow[ul] \arrow[uur] &
\end{tikzcd}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int -\vec{\nabla} \frac{1}{|\vec{r}-\vec{r}'|}\, dq'$$

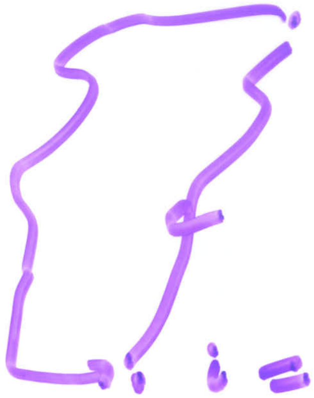
$$= -\nabla \underbrace{\frac{1}{4\pi\varepsilon_0} \int \frac{dq'}{|\vec{r}-\vec{r}'|}}_{\phi}$$

$$\int_f^i \vec{E}\cdot\vec{dl} = \int_f^i -\nabla\phi\cdot dl$$

$$= \int_i^f \nabla\phi\cdot\vec{dl}$$

$$= \underline{\underline{\phi_f}} - \phi_i$$

$$\phi_i = 0$$



f ϕ ?

i = ϕ = 0 Ground

$$\vec{r} = \langle 0, 0, z\rangle$$

$$\vec{r}' = \langle x', y', 0\rangle$$

$$\vec{r-r'} = \langle -x', -y', z\rangle$$

$$|\vec{r-r'}| = \sqrt{x'^2+y'^2+z^2}$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L}\int_{-L}^{L}\frac{1}{\sqrt{x'^2+y'^2+z^2}}\,\sigma\,dx'dy'$$

...

Multipole Expansion

$$[1-x]^{-\alpha} = 1 + \alpha x + \frac{\alpha(\alpha+1)}{2!} x^2$$

shifted factorial   Pochhammer

$5! : 1\cdot 2\cdot 3\cdot 4\cdot 5$

$$(\alpha)_n \quad \underbrace{\alpha(\alpha+1)(\alpha+2)\cdots}_{n}(\alpha+n-1)$$

$$= \sum \frac{(\alpha)_n}{n!} x^n$$

$$E_z = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]$$

$$= \frac{Q}{4\pi\epsilon_0 z^2}$$

$$[z^2+a^2]^{-1/2} = \frac{1}{z}\left[1+\frac{a^2}{z^2}\right]^{-1/2} \qquad \alpha = 1/2, \quad x = \frac{-a^2}{z^2}$$

$$= \frac{1}{z}\left[1 + \cancel{\alpha}\,\frac{1}{2}\left(\frac{-a^2}{z^2}\right) + \frac{\frac{1}{2}\cdot\frac{3}{2}}{2}\left(\frac{a^2}{z^2}\right)^2\right.$$

$$E_z = \frac{\sigma}{2\epsilon_0}\left[- \left[\frac{1}{2}\,\frac{-a^2}{z^2} + \cdots\right]\right]$$

$$E_z = \frac{\sigma}{4\varepsilon_0\pi}\frac{a^2\pi}{z^2} = \frac{Q}{4\pi\varepsilon_0 z^2} + -$$

Multipole

$r \gg r'$

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\, dv'}{|\vec{r}-\vec{r}'|}$$

$$\left[(\vec{r}-\vec{r}')\cdot(\vec{r}-\vec{r}')\right]^{1/2} = \left[r^2 - 2\vec{r}\cdot\vec{r}' + r'^2\right]^{1/2}$$

$$r\left[1 - 2\frac{\hat{r}\cdot\vec{r}'}{r} + \frac{r'^2}{r^2}\right]^{1/2}$$

$$x = \frac{2\hat{r}\cdot\vec{r}'}{r} - \frac{r'^2}{r^2}$$

$\alpha = 1/2$

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\, dv'}{r\left[1-x\right]^{1/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \int \rho\, dv' \frac{1}{r}\left[1 + \frac{1}{2}x + \frac{\frac{1}{2}\frac{3}{2}}{2}x^2 + \cdots\right]$$

$$[\;] = 1 + \frac{1}{2}\left(\frac{2\hat{r}\cdot\vec{r}'}{r} - \frac{r'^2}{r^2}\right) + \frac{3}{8}\left(\frac{2\hat{r}\cdot\vec{r}'}{r} - \right)^2$$

$$\boxed{\frac{r'}{r}} \qquad \left(\frac{r'}{r}\right)^2 \qquad \left(\frac{r'}{r}\right)^2 \quad \left(\frac{r'}{r}\right)\frac{r'^2}{r^2}$$

$$= 1 + \frac{\hat{r}\cdot\vec{r}'}{r} + \left(\frac{3}{2}\left(\frac{\hat{r}\cdot\vec{r}'}{r}\right)^2 - \frac{1}{2}\left(\frac{r'}{r}\right)^2\right)$$

$$\Phi = \frac{1}{4\pi\epsilon_0}\int \rho\, dv\, \frac{1}{r}\left[1 + \frac{\hat{r}\cdot\vec{r}'}{r} + (\ )^2\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \frac{\hat{r}}{r^2}\underbrace{\int \rho\, dv\, \vec{r}'}_{\text{electric dipole}}\right.$$

$Q \cdot L$

$\overline{P}$

$$\frac{1}{r^2}\int \rho\, dv \left(\frac{3}{2}(\hat{r}\cdot\vec{r}') - \frac{1}{2}r'^2\right)$$

electric quadrupole

tensor — matrix

$\vec{v}_i$ $\quad M_{ij}$

dyadics

$$M_{ij} = \begin{bmatrix} A_x B_x & A_x B_y & A_x B_z \\ A_y B_x & A_y B_y & A_y B_z \\ A_z B_x & A_z B_y & A_z B_z \end{bmatrix} \begin{pmatrix} c \end{pmatrix}$$

$$= A_i B_j$$

$$M \cdot C = \overline{A}\,(B \cdot C)$$

$$(\hat{r}\cdot\vec{r}')(\vec{r}'\cdot\hat{r}) = \hat{r}\cdot(\vec{r}'\,\vec{r}')\cdot\hat{r}$$

$$1 = \hat{r}\cdot\hat{r}$$

$$M_{ij} = \int \rho\, dv \left[\frac{3}{2} r'_i r'_j - \frac{1}{2} \underbrace{\delta_{ij}}_{1} r'^2 \right]$$

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

Quadrupole tensor Q_{ij}

$$\phi = \frac{1}{4\pi\epsilon_0} \left\{ \frac{Q}{r} + \frac{\hat{r} \cdot \vec{p}}{r^2} + \frac{1}{2} \frac{\hat{r} \cdot \overleftarrow{Q} \cdot \hat{r}}{r^3} + \cdots \right\}$$

$\underline{\vec{r} \ll \vec{r}'}$

$$\begin{pmatrix} 1/2 & & \\ & 1/2 & \\ & & -1 \end{pmatrix}$$

$\lambda \quad R$

$\vec{r}' = \langle R\cos\phi, R\sin\phi, 0 \rangle$

$R\,d\phi$

$$Q_{ij} = \int (3 r_i' r_j' - \delta_{ij} r'^2) \lambda \, dl$$

$$Q_{11} = \int (3(R\cos\phi)^2 - R^2) \lambda R \, d\phi$$

$\frac{1}{2} \cdot 2\pi \qquad 2\pi$

$$Q_{22} = (3 R^2 \tfrac{1}{2} 2\pi - R^2 2\pi) \lambda R$$

$$= \lambda R^3 2\pi \left(\frac{3}{2} - 1 \right)$$

$$= Q R^2 \frac{1}{2}$$

$$Q_{12} = \int_{-\pi}^{\pi} 3 \, R\cos\phi \, R\sin\phi \, \lambda R \, d\phi = 0$$

even

odd

$$Q_{13} \int 3 R\cos\phi \; 0 = 0$$

$$Q_{33} = \int (3 \cdot 0 - R^2) R \lambda \, d\phi$$

$$-R^3 \lambda \, 2\pi = -Q R^2$$

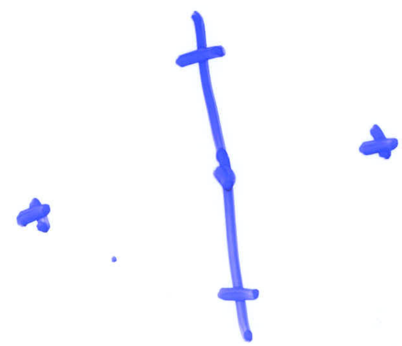
$$\hat{r} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$= (s\tilde{c}, s\tilde{s}, c)$$

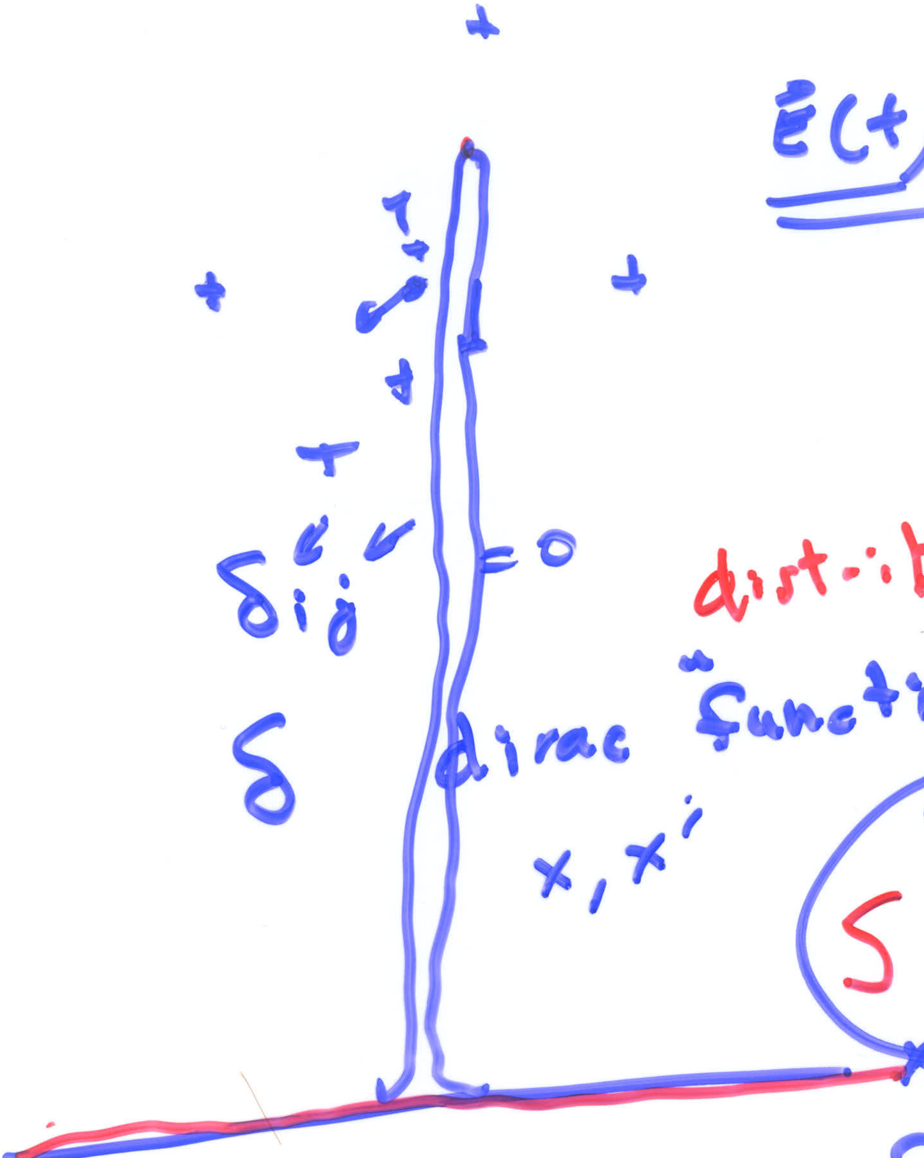
$$(s\tilde{c}, s\tilde{s}, c)\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} s\tilde{c} \\ s\tilde{s} \\ c \end{pmatrix}$$

$$= \frac{1}{2}s^2\tilde{c}^2 + \frac{1}{2}s^2\tilde{s}^2 - c^2$$

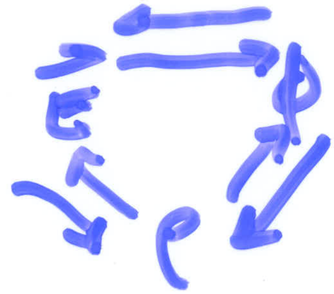
$$= \frac{1}{2}\underbrace{s^2}_{1-c^2} - c^2 = \underline{\frac{1}{2} - \frac{3}{2}c^2}$$



$$\phi = \frac{Q}{4\pi\epsilon_0 r}\left(1 + \frac{R^2}{2r^2}\left(\frac{1}{2} - \frac{3}{2}c^2\right)\right)$$



$\underline{\vec{E}(t)}$



δ_{ij} $=0$

distribution

δ dirac "function" $=0$

x, x'

$x \neq x'$

$\int \delta \, dx = 1$

$f(x)\,\delta = f(x')\,\delta$

$$\int \underline{f(x)}\,\delta\,dx = \int f(x')\,\delta\,dx$$

$$= f(x') \int \delta\,dx = \underline{f(x')}$$