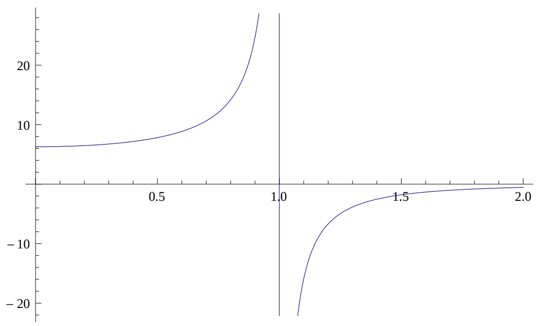
A circular loop of wire (radius R) sits in the xy plane with its center at the origin. The loop carries a current I flowing in the counter-clockwise direction as seen from above (i.e., $z > 0$). Consider an attempt to calculate the resulting magnetic field in the xy plane a distance d from the coil center using the formula:

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \int I d\vec{\boldsymbol{\ell}}' \times \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|^3}$$

Report expressions for all of the following: $d\vec{\boldsymbol{\ell}}'$, $\vec{\mathbf{r}}$, $\vec{\mathbf{r}}'$, $|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|$.

Mathematica can solve the resulting integral in terms of `EllipticE` and `EllipticK`. Below find a plot of $B_z/(\mu_0 I/4\pi R)$ verses d/R. This problem aims to find 'checks' of this result, i.e., approximations or simplifications that allow calculation (or approximation) of the full complex result in some regions.

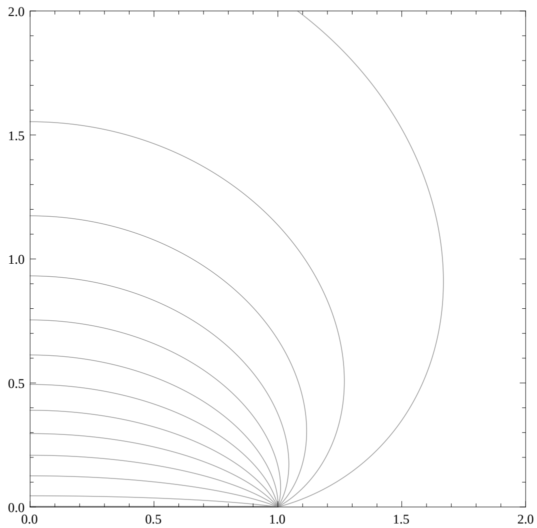


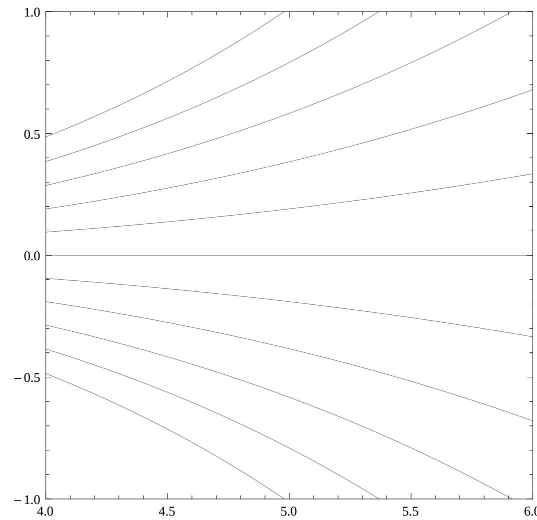
1. Calculate the magnetic field at the origin: all that is required is a simple integral. Does your result match that in the graph?

2. For $d \gg R$ (still in the xy plane) I expect a dipole should approximate the field produced by the loop. What magnetic dipole, $\vec{\mathbf{m}}$, should approximate the loop? What magnetic field would be produced by that dipole at at $d = 5R$. Mathematica finds an exact result of: $-0.0263 \times (\mu_0 I/4\pi R)$.

3. Clearly something 'special' is going on at $d = R$. Explain why B_z is changing so radically there and report how you might find an approximate value of B_z, for example at $d = 1.01 \times R$. FYI: Mathematica reports $-193 \times (\mu_0 I/4\pi R)$.

Mathematica is able to plot the solid angle (Ω) subtended by the current loop at a locations in the xz plane (which is equivalent to any plane that includes the z axis). The below left plot shows the solid angle contours $(.5, 1, 1.5, 2.0, \ldots, 6)$ steradians at scaled locations x/R and z/R; the below right plot shows sold angle contours $(-.025, -.020, .-015, \ldots, +.025)$ in the vicinity of $x = 5 \times R$. Directly on the below plots put the proper contour label on three contours on the left plot and three contours on the right plot. For both plots particularly locate/label the $\Omega = 0$ contour.





1. Derive a formula for the solid angle subtended by the loop as viewed from locations on the z axis. Reading between the lines on the left contour plot I conclude that at $z = 1 \times R$, $\Omega \approx 1.8$ sr; Mathematica gives 1.8403 sr.

2. Directly on the above left plot sketch 3 appropriately oriented and sized arrows showing the magnetic field at the location of the arrow. Label a location **A** in the plot where the magnetic field is rather large, and a location **B** where the field is small.

3. Using the data of the right plot calculate the magnetic field at $(x, z) = (5 \times R, 0)$. You may recall:

$$\vec{\mathbf{B}} = -\vec{\nabla}\phi \qquad \text{where: } \phi = \frac{\mu_0 I}{4\pi}\,\Omega$$