Class 8

pictures in 2017 folder —

Mercury — the Earth rotates under the fixed orbital plane

period for low Earth orbit: $a = R_e$; $\omega = \frac{2\pi}{T} = \sqrt{\frac{GM}{a^3}}$

$$T = \frac{2\pi}{\sqrt{\frac{GM}{R^3}}} = 5069 \text{ sec} = 84.5 \text{ min}$$

during this time Earth rotates $\frac{360^\circ}{1 \text{ day}} \cdot 84.5 \text{ min} = 21^\circ$

iss_track — orbital plane is at a greater angle than Mercury so Russians can use it

$$T = \sqrt{\frac{2\pi}{\frac{GM}{R_e + 370e3}}} = 5516 \text{ sec} = 92 \text{ min}$$

IRN — these have $T = 24$ hours but high inclination

Halley: $r = a$ if $\cos u = 0$ i.e. $u = \pm \pi/2$

$u = +\pi/2 \rightarrow \quad \frac{\pi}{2} - e \sin u = \frac{\pi}{2} - e = 2\pi\left(\frac{t}{T}\right)$

$u = -\pi/2 \rightarrow \quad \frac{-\pi}{2} - e \sin u = -\frac{\pi}{2} + e = 2\pi\left(\frac{t}{T}\right)$

so inside $r = a$ for $2t = \left(\frac{1}{2} - \frac{e}{\pi}\right) T$

so outside $r = a$ for $T - \left(\frac{1}{2} - \frac{e}{\pi}\right) T = \underbrace{\left(\frac{1}{2} + \frac{e}{\pi}\right)}_{81\%} T$

Quantum

$$V_\infty = \sqrt{\frac{\mu/m}{a}} = \sqrt{\frac{GM}{a}} = 26.3 \text{ km/s}$$

$\sim e^{4/2}$

$\omega = \sqrt{\frac{\mu}{m a^3}}$; ωt is huge so $e \sinh u \sim \omega t$

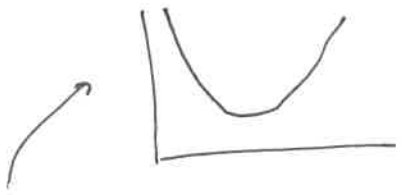
$u \sim \ln\left(\frac{2\omega t}{e}\right)$ ← need approx

root $\quad e \sinh(x) - x = \omega t$

$a(e \cosh(x) - 1) \rightarrow 563 \text{ AU}$

8.13, 8.19

8.13 (was done in class)



$\mu \ddot{r} = -\frac{\partial}{\partial r} U_{eff} = f(r)$ $\quad U_{eff} = \frac{1}{2} k r^2 + \frac{l^2}{2\mu r^2}$

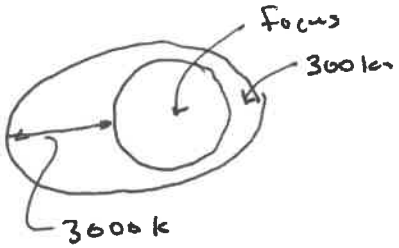
$$U' = kr - \frac{l^2}{\mu r^3} = 0 \leftrightarrow r_0^4 = \frac{l^2}{\mu k}$$

$$-f'(r) = U'' = k + \frac{3l^2}{\mu r^4}$$

$$= k + 3k = 4k$$

So near r_0: $\mu \ddot{r} = -(4k)(r - r_0)$ & $\omega^2 = \frac{4k}{\mu}$

8.19



$a = (3300\text{km} + 2R_E)/2$

$a - ae = a(1-e) = R_E + 3000\text{km}$

$$1 - e = \frac{R_E + 300}{R_E + \frac{3300}{2}}$$

$$\therefore 1 - \frac{R_E + 300}{R_E + \frac{3300}{2}} = e = .168$$

Crossing y axis $\rightarrow \phi = 90^\circ$

$r = a(1-e^2)$

height $= a(1-e^2) - R_E = 1423\text{ km}$