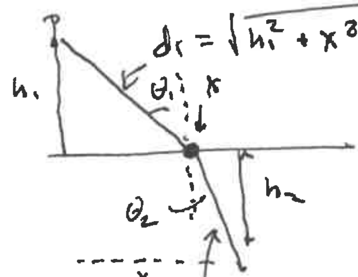
Class 6 6.4, 6.9, 6.12

6.4: $\text{time} = \dfrac{\text{distance}}{\text{speed}}$ $\quad$ speed $= \dfrac{c}{n}$



$d_1 = \sqrt{h_1^2 + x^2}$

$d_2 = \sqrt{h_2^2 + (x_2 - x)^2}$

its clear that $z \neq 0$ increases all distances so $z = 0$ [not hard to show via calculus]

$$\text{total time} = \frac{d_1}{v_1} + \frac{d_2}{v_2}$$

$$= \frac{1}{c}\Big[\underbrace{\sqrt{h_1^2+x^2}\, n_1 + \sqrt{h_2^2+(x_2-x)^2}\, n_2}_{\text{Find } x \text{ to min} \;\equiv\; f(x)}\Big]$$

$$f'(x) = \frac{n_1 x}{\underbrace{\sqrt{h_1^2+x^2}}_{d_1}} - \frac{n_2(x_2-x)}{\underbrace{\sqrt{h_2^2+(x_2-x)^2}}_{d_2}} = n_1\frac{x}{d_1} - n_2\frac{(x_2-x)}{d_2} = n_1\sin\theta_1 - n_2\sin\theta_2 = 0$$

$\Rightarrow$ Snell: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

6.9 $\quad f = y'^2 + \underbrace{yy'}_{\text{unneeded}} + y^2$

unneeded remark – this term does nothing as it does not depend on curve as $yy' = (\tfrac{1}{2}y^2)' \Leftarrow$ integral to $\oint \tfrac{1}{2}y^2\big|_0^1$ ← constant

$$\frac{\partial f}{\partial y} = y' + 2y = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = \frac{d}{dx}(2y' + y) = 2y'' + y'$$

note they cancel

$\Rightarrow y = y''$ ← solution $y = Ae^x + Be^{-x}$

$y(x=0) = A + B = 0 \Rightarrow B = -A$

$y(x=1) = 1 = A(e^1 - e^{-1}) = 2A\sinh(1)$

so $y = \underset{\uparrow\, \frac{1}{2\sinh(1)}}{A}(e^x - e^{-x}) = \dfrac{\sinh(x)}{\sinh(1)}$

6.12 $\quad f = x\sqrt{1-y'^2}$

$\dfrac{\partial f}{\partial y} = 0$ $\quad$ so $\quad \dfrac{\partial f}{\partial y'} = \text{Const} = \dfrac{xy'}{\sqrt{1-y'^2}} = a$

$$x^2 y'^2 = a^2(1-y'^2)$$

$$(x^2 + a^2)\, y'^2 = a^2$$

$$y' = \frac{a}{\sqrt{a^2+x^2}}$$

$$\int dy = \int \frac{a}{\sqrt{a^2+x^2}} = a\sinh^{-1}\left(\frac{x}{a}\right)$$

↑ Dwight 200.01