Class 1: 1.2, 1.4, 1.10, 2.18, 2.43 (separate), canA.pdf

1.2

$$\vec{b} = (1,2,3) \qquad 5\vec{b} = (5,10,15) \qquad b\cdot c = 1\cdot3+2\cdot2+3\cdot1 = 10$$

$$\vec{c} = (3,2,1) \qquad -\ 2\vec{c} = (6,4,2)$$

$$\overrightarrow{b+c} = (4,4,4) \qquad \overrightarrow{5b-2c} = (-1,6,13)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = \hat{i}(2-6) + \hat{j}(9-1) + \hat{k}(2-6) = (-4, 8, -4)$$

1.4

$$b\cdot c = 1\cdot4 + 2\cdot2 + 4\cdot1 = 12 = |\vec{b}||\vec{c}|\cos\theta$$

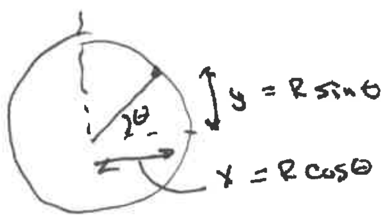
$$\sqrt{1^2+2^2+4^2} = \sqrt{21}$$

$$\sqrt{21}$$

$$\cos\theta = \frac{12}{21} \rightarrow \theta = 55^\circ$$

1.10 Clearly $x^2+y^2 = R^2(\cos^2\theta + \sin^2\theta) = R^2$ so on circle

ωt



$$\vec{r} = R(\cos\omega t, \sin\omega t)$$

$$\vec{v} = \dot{\vec{r}} = R\omega(-\sin\omega t, \cos\omega t)$$

$$\vec{a} = \dot{\vec{v}} = R\omega^2(-\cos(\omega t), -\sin(\omega t))$$

$$= -\omega^2\vec{r}$$

$$|\vec{r}| = R, \quad |\vec{v}| = R\omega, \quad |\vec{a}| = \omega^2 R$$

2.18 – Remark Mathematica Series Command

$f(x) = \ln(1+x)$ (x → δ) $\quad f(0)=0$

$$f'(x)\Big|_{x=0} = \frac{1}{1+x}\Big|_{x=0} = 1$$

$$f''(x)\Big|_{x=0} = \frac{-1}{(1+x)^2}\Big|_{x=0} = -1$$

$$f'''(x)\Big|_{x=0} = \frac{2}{(1+x)^3}\Big|_{x=0} = 2$$

$$f(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$$

$f(x) = \cos(x) \quad f(0)=1$

$$f'(x)\Big|_{x=0} = -\sin(x)\Big|_{x=0} = 0$$

$$f''(x) = -\cos(x)\Big|_{x=0} = -1$$

$$f'''(x) = +\sin(x)\Big|_{x=0} = 0$$

$$f''''(x) = \cos(x)\Big|_{x=0} = 1$$

$$f(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4$$

$f(0)=0$

$f(x) = \sin(x)$

$$f'(x) = \cos(x)\Big|_{x=0} = 1$$

$$f''(x) = -\sin(x)\Big|_{x=0} = 0$$

$$f'''(x) = -\cos(x)\Big|_{x=0} = -1$$

$$f''''(x) = \sin(x)\Big|_{x=0} = 0$$

$$f'''''(x) = \cos(x) = 1$$

$$f(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

$f = e^x \ldots$ all derivatives are $e^x\big|_{x=0} = 1$ so $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

CanA.pdf

1) $V = R\dot{\theta}$

2) $\theta = 0 \rightarrow v^2 = 0$

3) $\theta = 180^\circ \rightarrow V = 2R\dot{\theta} = 2V$

4) $v_x = v_y = R\dot{\theta} = V$

5) Start by considering two extreme cases: V "small" & V "large"

[clearly we'll need to define those]

if V small – essentially a pure drop: $R = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2R}{g}}$

$\Delta x = Vt = V\sqrt{\frac{2R}{g}}$

if V big – up & back; R adds little

$-V = V - gt \Rightarrow t = \frac{2V}{g}$ $\quad \Delta x = Vt = \frac{2V^2}{g}$

Remark: $v\uparrow \; \rightarrow v$ $\rightarrow \theta = 45^\circ$ range eq from 191: $R = \frac{V_0^2}{g}\sin(2\theta)$

where V_0 = full muzzle speed $= \sqrt{2}V$ $\rightarrow$ agree

$y = R + vt - \frac{1}{2} g t^2$

$\frac{1}{2} g t^2 - vt - R = 0$

$t = \frac{v \pm \sqrt{v^2 + 2gR}}{g}$ $\leftarrow$ Note $v^2 \gg 2gR \quad \approx \frac{2v}{g}$ ✓

$v^2 \ll 2gR \quad \approx \sqrt{\frac{2R}{g}}$ ✓

$\Delta x = \frac{v^2}{g}\left(1 \pm \sqrt{1 + \frac{2gR}{v^2}}\right)$

Remark: considering big & small limits is not at all required by this problem – But such considerations often add more insight than the exact solution

```
m=.6
c=.25*.24^2
cm=c/m
g=9.8
v0=15
theta=45.*Pi/180.
solution=NDSolve[{
x''[t]==-cm Sqrt[x'[t]^2+ y'[t]^2] x'[t]  ,
y''[t]==-g -cm Sqrt[x'[t]^2+ y'[t]^2] y'[t] ,
  x[0]==0,
  y[0]==2,
x'[0]==v0 Cos[theta],
y'[0]==v0 Sin[theta]},
{x,y},{t,0,10}]

ParametricPlot[Evaluate[{x[t],y[t]}/.First[solution]],{t,0,2.5}]
Plot[2+Tan[theta]x-.5 g x^2/(v0 Cos[theta])^2,{x,0,25}]
Show[%,%%]
```