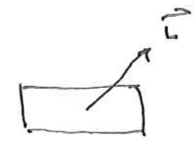
1) $\overleftrightarrow{I} = \sum m_\alpha \left(\overleftrightarrow{1} r_\alpha^2 - \vec{r}_\alpha \vec{r}_\alpha \right)$; $\quad I_{yy} = \sum m_\alpha (x_\alpha^2 + \underbrace{z_\alpha^2}_{\to 0}) = \sum m_\alpha x_\alpha^2$

$\sigma = \frac{M}{(4a)\cdot(2a)}$ $\qquad y\Big|_{-a}^{a} = 2a$ $\qquad I_{xy} = -\sum m_\alpha x_\alpha y_\alpha$

$$I_{yy} = \sigma \int_{-2a}^{2a} \int_{-a}^{a} x^2 \, dx\, dy = \frac{M}{4a} \int_{-2a}^{2a} x^2 dx = \frac{M}{4a} \frac{x^3}{3}\Big|_{-2a}^{2a} = \frac{M}{4a}\, 2\, \frac{8a^3}{3} = \frac{4}{3} M a^2$$

$$I_{xy} = \sigma \int \int xy \, dx\, dy \to \frac{y^2}{2}\Big|_{-a}^{a} = 0$$

unit vector for $\vec{\omega} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \vec{\omega} = \frac{\omega}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

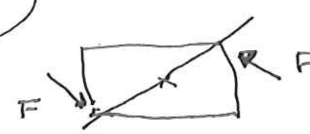


$$\vec{L} = \overleftrightarrow{I} \vec{\omega} = \frac{\omega}{\sqrt{5}} \frac{1}{3} M a^2 \begin{pmatrix} 1 & & \\ & 4 & \\ & & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \frac{\omega}{\sqrt{5}} \frac{1}{3} M a^2 \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$$

$\frac{d\vec{L}}{dt}$ must rotated with plate $\Rightarrow \overrightarrow{\Delta L}$ = out of page = $\hat{z}$

$$\frac{d\vec{L}}{dt} = \underbrace{\frac{d\vec{L}}{dt}}_{\text{body frame} = 0} + \vec{\omega} \times \vec{L} = \frac{\omega^2}{5} \frac{1}{3} M a^2 \begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 2 & 4 & 0 \end{vmatrix} \to 8 - 2 = 6\hat{k}$$

to get $\overrightarrow{r \times F} = \vec{\Gamma}$ in $\hat{z}$ direction:



2) $(\text{inertial}) = M_\phi M_\theta M_\psi (123) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\qquad$ define: $\sin\phi = s$, $\cos\phi = c$, $\sin\psi = \tilde{s}$, $\cos\psi = \tilde{c}$

$$= \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{c} & -\tilde{s} & 0 \\ \tilde{s} & \tilde{c} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} c \sin\theta \\ s \sin\theta \\ \cos\theta \end{pmatrix} \leftarrow \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix} \leftarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

3) $\frac{\partial PE}{\partial \theta_1} = + MLg\, 2 \sin\theta_1 \Rightarrow \frac{\partial^2 PE}{\partial\theta_1 \partial\theta_2} = 0 \qquad \frac{\partial^2 PE}{\partial \theta_1^2} = MLg\, 2\cos\theta_1 \Big|_{\theta_1 = 0} = MgL2$

$\frac{\partial PE}{\partial \theta_2} = MLg \sin\theta_2 \Rightarrow \frac{\partial^2 PE}{\partial \theta_2^2} = MLg \cos\theta_2 \Big|_{\theta_2 = 0} = MgL$

$$\mathcal{K} = MgL \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$\frac{\partial KE}{\partial \dot\theta_1} = ML^2 \left(2\dot\theta_1 + \dot\theta_2 \cos(\theta_2 - \theta_1) \right)$

$\frac{\partial^2 KE}{\partial \dot\theta_1^2} = 2ML^2 \qquad \frac{\partial KE}{\partial \dot\theta_1 \partial \dot\theta_2} = ML^2 \cos(\theta_2 - \theta_1) \Big|_{\theta = 0} = ML^2$

$\frac{\partial KE}{\partial \dot\theta_2} = ML^2 \left(\dot\theta_2 + \dot\theta_1 \cos(\theta_2 - \theta_1) \right) \qquad \frac{\partial^2 KE}{\partial \dot\theta_2^2} = ML^2$

$$\mathcal{M} = ML^2 \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

3 cont)
$$L = \frac{1}{2}\sum_{ij}\left(\dot\theta_i m_{ij}\dot\theta_j - \theta_i K_{ij}\theta_j\right)$$

in all of those θ_i one is θ_k

In all of these θ_j one is θ_k

$$\frac{\partial L}{\partial\theta_k} = -\frac{1}{2}\left(\sum_j K_{kj}\theta_j + \sum_i \theta_i K_{ik}\right)$$ symmetric matrix

$$= -\sum_j K_{kj}\theta_j$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot\theta_k} = \frac{d}{dt}\left[\frac{1}{2}\left(\sum_j m_{kj}\dot\theta_j + \sum_i \dot\theta_i m_{ik}\right)\right]$$

$$= \frac{d}{dt}\left[\sum_j m_{kj}\dot\theta_j\right] = \sum_j m_{kj}\ddot\theta_j$$

$$\ddot{\vec\theta} = -\omega^2\begin{pmatrix}6\\1\end{pmatrix}\underbrace{e^{i\omega t}}_{\text{cancel - is on both sides}} \Rightarrow -\omega^2\begin{bmatrix}1&0\\0&4\end{bmatrix}\begin{pmatrix}6\\1\end{pmatrix} \stackrel{?}{=} -\begin{bmatrix}3&-3\\-3&28\end{bmatrix}\begin{pmatrix}6\\1\end{pmatrix}$$

$$\omega^2\begin{pmatrix}6\\4\end{pmatrix} = \begin{pmatrix}18-3\\-18+28\end{pmatrix} = \begin{pmatrix}15\\10\end{pmatrix}$$

$\frac{5}{2}$ ✓ $\quad \frac{5}{2}\cdot 6 = 15$ & $\frac{5}{2}\cdot 4 = 10$

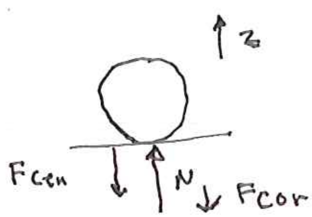
$F_{cor} = 2m\,\vec v\times\vec\omega$

4) $F_{cen} = m\omega^2\vec r = mg$

$\hookrightarrow \omega^2 R = g \Rightarrow \omega^2 = \frac{g}{R} \Rightarrow \omega = \sqrt{\frac{9.8}{10}} = .99\ \frac{\text{rad}}{\text{sec}}$

$\vec\omega$: in $-\hat y$ direction

b) $\vec v\times\vec\omega \sim \hat x\times(-\hat y) = -\hat z$

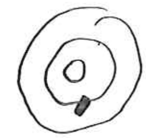


No left/right forces — ball goes straight

c) $\vec v\times\vec\omega \sim \hat y\times(-\hat y) = 0$ ⟵ No Coriolis Force

straight line motion

d) As in b) → extra force "down" due to Coriolis ⇒ dart hits "low"



e) straight line motion in any inertial frame

5) $\dot{\vec{r}}$ term is Coriolis

$$\underbrace{\frac{-\omega^2}{(2\pi)^2}}_{z^2}\vec{q} = -K\vec{q} + \underbrace{\frac{i\omega}{\pi}}_{2z}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\vec{q}$$

$$\left[K - 2iz\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - z^2 \mathbb{I} \right]\vec{q} = 0$$

$$\begin{bmatrix} -\alpha^2 - z^2 & -\frac{3}{2}\alpha(1-2e) - 2iz \\ -\frac{3}{2}\alpha(1-2e) + 2iz & -3\alpha^2 - z^2 \end{bmatrix}\vec{q} = 0$$

For non trivial solution must not be invertable so $\det[\ ] = 0$

$$(\alpha^2+z^2)(3\alpha^2+z^2) - \left[\left(\tfrac{3}{2}\alpha(1-2e)+2iz\right)\cdot\left(\tfrac{3}{2}\alpha(1-2e)-2iz\right)\right] = 0$$

This is far enough for full credit

$$z^4 + \underbrace{4\alpha^2}_{3} z^2 + 3\alpha^4 - \left[\underbrace{\left(\tfrac{3}{2}\alpha(1-2e)\right)^2}_{\frac{9}{4}\alpha^2(1-2e)^2} + \underbrace{(2z)^2}_{4z^2}\right]$$

$(1-2e)^2 = 1-4e+4e^2$

$$z^4 - z^2 + \underbrace{\left(3\alpha^4 - \tfrac{9}{4}\alpha^2\right)}_{3\cdot\left(\frac{3}{4}\right)^2 - \frac{9}{4}\frac{3}{4} = 0} - \tfrac{9}{4}\alpha^2\, 4e(e-1) = 0$$

$$\frac{9}{4}\cdot\frac{3}{4} = \frac{27}{16} \rightarrow \frac{27}{4}$$

$$z^4 - z^2 + \frac{27}{4}e(1-e) = 0$$

Note: $b^2 - 4ac = 1 - 27e(1-e) > 0$

$1 > 27e(1-e)$