Vectors appear trivial & insignificant but infact are powerful & required

→ Fundamental Laws in physics must be vector equations!

Scaler - like mass / charge - does not depend on coordinate system

vector - 3 tuple changes in a particular fashion with coordinate system

tensor - 3x3 matrix : elements depend on coordinate system

→ when we want to distinguish these vectors from those of a mathematical vector space we call them tensors - rank 0, rank 1, rank 2

→ we have scaler multiplication & vector addition just as in a mathematical vector space - but more

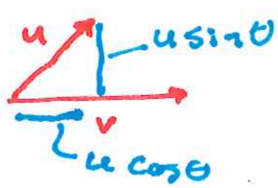
→ the dot product is a scaler : $\vec{v} \cdot \vec{u} = \sum v_i u_i$

→ the cross product is a vector $\vec{v} \times \vec{u} = \vec{w}$

$$\vec{w} = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = \hat{e}_1 (v_2 u_3 - v_3 u_2) \text{ etc}$$

→ Prove alternative formulas by using particular coordinate system

dot product: select $\hat{x}$ to be in $\vec{v}$ direction & the xy plane to include $\vec{v}$ & $\vec{u}$



Note: magnitude of v

$\vec{v} = (v, 0, 0)$

$\vec{u} = (u\cos\theta, u\sin\theta, 0)$

$\vec{v} \cdot \vec{u} = v\,u\cos\theta$ ← just geometry here

Cross product :

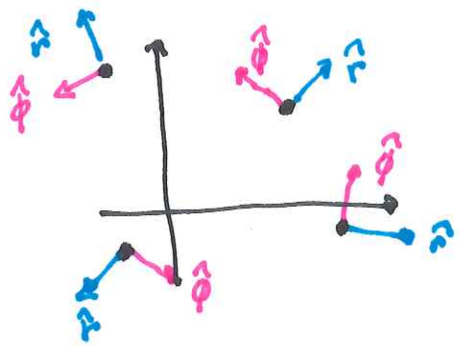
$$\vec{w} = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ v & 0 & 0 \\ u\cos\theta & u\sin\theta & 0 \end{vmatrix} = \hat{e}_3 \underbrace{v u \sin\theta}$$

re ⊥ to both

Note: $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(A \cdot C) - \vec{C}(A \cdot B)$

Alt coordinate systems (note: not Cartesian $\Rightarrow$ not normal vector)

Polar: $(x,y) \leftrightarrow (r,\phi)$



important fact: $\hat{r}$ & $\hat{\phi}$ not constant direction!

$$\vec{r} = r\hat{r} = (r\cos\phi, r\sin\phi) = \underbrace{r}_{\text{magnitude}}\underbrace{(\cos\phi, \sin\phi)}_{\hat{r}}$$

Check its unit!

$$\frac{d}{dt}\underbrace{(\cos\phi, \sin\phi)}_{\hat{r}} = (-\sin\phi, \cos\phi)\,\dot{\phi} \leftarrow \text{time derivative}$$

$\hookrightarrow$ this is a unit vector $\perp$ to $\hat{r}$ … $\hat{\phi}$

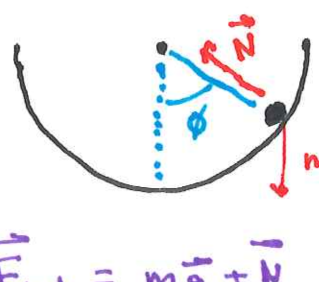
$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(r\hat{r}) = \underbrace{\dot{r}}_{\text{radial velocity}}\hat{r} + \underbrace{r\dot{\phi}}_{\text{tangential velocity}}\hat{\phi}$$

(191: $v_\perp = r\omega$; $\omega = \dot{\phi}$)

$$\frac{d}{dt}\underbrace{(-\sin\phi, \cos\phi)}_{\hat{\phi}} = (-\cos\phi, -\sin\phi)\,\dot{\phi} = -\hat{r}\,\dot{\phi}$$

$$\vec{a} = \ddot{\vec{r}} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi}\right) = \ddot{r}\hat{r} + \dot{r}\dot{\phi}\hat{\phi} + \dot{r}\dot{\phi}\hat{\phi} + r\ddot{\phi}\hat{\phi} - r\dot{\phi}^2\hat{r}$$

$$= (\ddot{r} - \underbrace{r\dot{\phi}^2}_{\text{191: centripetal acc} = r\omega^2})\hat{r} + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\phi}$$



$$\vec{F}_{net} = (mg\cos\phi - N)\hat{r} - mg\sin\phi\,\hat{\phi}$$

$$m\vec{g} = mg(\cos\phi\,\hat{r} - \sin\phi\,\hat{\phi})$$

$$\vec{F}_{net} = m\vec{g} + \vec{N} = (mg\cos\phi - N)\hat{r} \overset{\text{minus}}{-} mg\sin\phi\,\hat{\phi}$$

$$= m\vec{a} = m\left[(\underbrace{\ddot{r}}_{\to 0} - r\dot{\phi}^2)\hat{r} + (\underbrace{2\dot{r}\dot{\phi}}_{\to 0} + r\ddot{\phi})\hat{\phi}\right] \quad \text{as } r = \text{constant}$$

$mg\cos\phi - N = -mr\dot{\phi}^2 \rightarrow$ equation for N

$-mg\sin\phi = mr\ddot{\phi} \longrightarrow -\frac{g}{r}\underbrace{\sin\phi}_{\approx\phi \text{ if } \phi < .1 \text{ rad}} = \ddot{\phi} \rightarrow$ with lots of work this can be solved in terms of complete elliptic functions