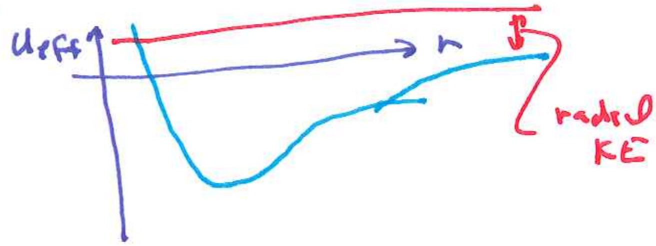
Gravitional "sling shot" Effect

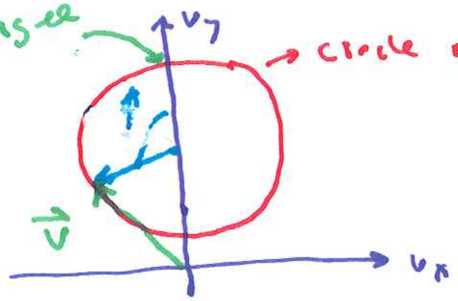
Argument 1: no such thing – if you drop a ball it does not bounce back higher than it fell – conservation of energy!

In terms of diff eqs: $\underbrace{\frac{1}{2}\mu \dot{r}^2}_{\text{radial KE}} + \underbrace{\left(\frac{\ell^2}{2\mu r^2} - \frac{\alpha}{r}\right)}_{U_{eff}} = E$

particle approaching from ∞ has a fixed Energy that is 100% KE cuz $U_{eff} \to 0$ as $r \to \infty$. when particle bounces at turning point and again goes to infinity → same E & same KE



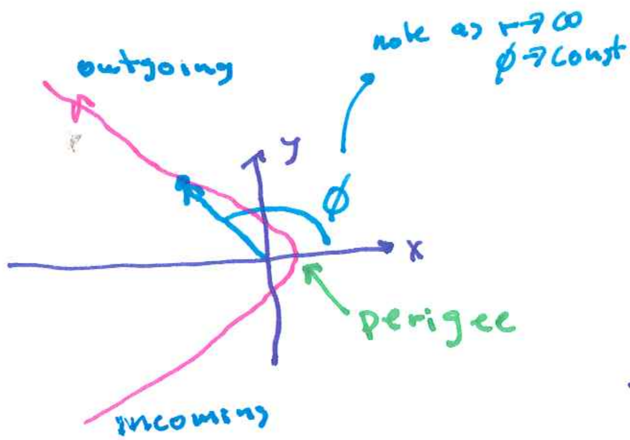
For orbital mechanics,



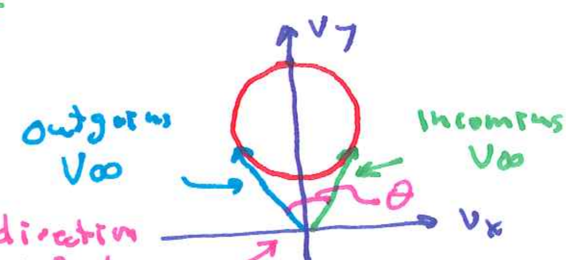
→ circle radius $\equiv v_0 = \frac{\alpha}{L}$

$L = \mu V_\infty b$ ← impact parameter

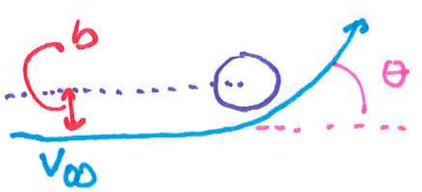
speed far from planet → V_∞

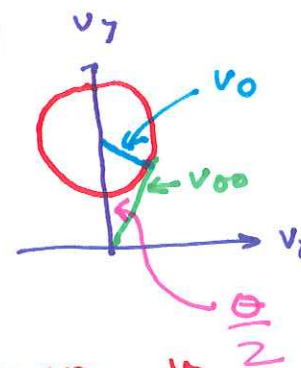


hyperbola – not a closed orbit



Seek how direction has changed post "collision" → "scattering angle"





$b \to \infty \quad \theta \simeq 0$

$b \to 0 \quad \theta \simeq \pi$

$$\tan\left(\frac{\theta}{2}\right) = \frac{v_0}{V_\infty} = \frac{\alpha}{L V_\infty} = \frac{\alpha}{\mu V_\infty^2 b}$$

alternate form: $\tan\frac{\theta}{2} = \frac{\alpha/b}{2T_\infty} = \frac{\text{grav PE}}{T_\infty}$

$T_\infty = \frac{1}{2}\mu V_\infty^2$

$T_\infty \gg$ grav PE $\to \theta \simeq 0$

Argument 2: True: if I throw a ball at a wall it bounces back with equal/opposite speed — BUT if I throw a ball at a moving wall (eg bat) its speed can be increased — Recall 1d elastic collision results: relative velocity reversed —

$$(v_1 - v_2)_{initial} = (v_2 - v_1)_{final}$$

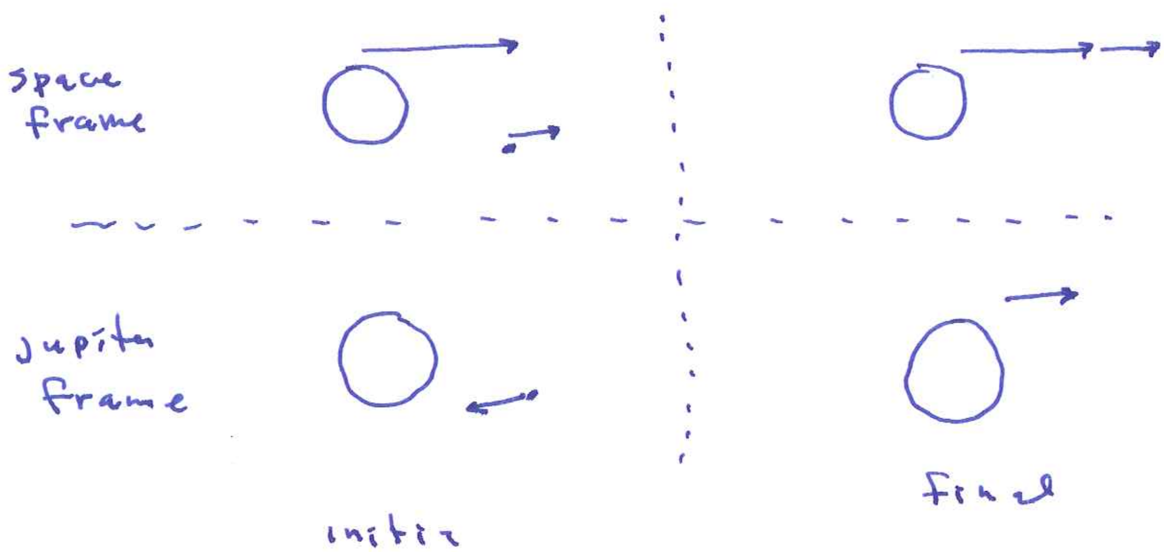
V_1 initial $= -V_0$

V_2 initial $= V_2$ final (massive walls speed largely unchanged)

$$(-V_0 - V_2) = V_2 - V_1 \text{ final}$$

$$V_1 \text{ final} = \underbrace{2V_2}_{\text{speed increased by } 2\times \text{ wall's speed}} + V_0$$

In 3d this result: $|\vec{v_1} - \vec{v_2}|_{initial} = |\vec{v_1} - \vec{v_2}|_{final}$

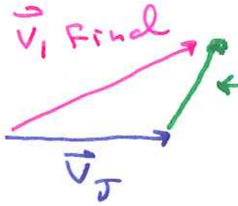


Like a tennis racket — jupiter shoots the spacecraft foward.

Note: In the space frame the final velocity is the sum

$$\vec{V_1} \text{ final} = \vec{V_J} + \underbrace{(\vec{V_1} - \vec{V_J})_{final}}_{\text{in magnitude this equals } (\vec{V_1} - \vec{V_J})_{initial}}$$

By careful adjustment in flight we can adjust b so the scattering angle makes $\vec{v}_{1\,final}$ point in the direction we want to go (eg to next planet)



← a vector of length $|\overrightarrow{v_1 - v_J}|$ initial at the angle needed to get the desired $\overrightarrow{v_1}$ final

Remark: the result: $\tan\frac{\theta}{2} = \frac{\alpha/b}{2T_\infty}$ allows calculation of Rutherford scattering cross section.

Remark: we have ignored the Sun's force in all of this because both the spacecraft & Jupiter are in free fall — recall Galileo's expt: heavy object & light object fall together .... tidal forces will change this result abit in chapter 9.

Remark: Moon-Sun force > Moon-Earth force