View in a non-inertial coordinates system (= S_1)

S_0 = inertial coordinate system

→ a moving fly: $\dot{\vec{r}}_0 = \vec{V} + \dot{\vec{r}}$ ← velocity in S_1

velocity in S_0

$\left.\begin{matrix}\vec{V} \\ \vec{A}\end{matrix}\right\}$ The current velocity & acceleration of S_1 as seen from S_0

$$\ddot{\vec{r}}_0 = \vec{A} + \ddot{\vec{r}}$$

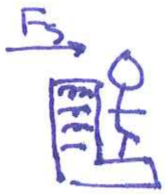
← acceleration in S_1

← acceleration in S_0

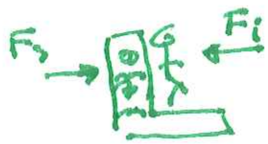
Now: $m\,\ddot{\vec{r}}_0 = F$ ← $\vec{A} + \ddot{\vec{r}}$

$$m\ddot{\vec{r}} = \vec{F} \underbrace{- m\vec{A}}$$

a pseudo force "inertial force" that like gravity is proportional to m



So view: the springs in the car seat are compressed because they are supplying the force that accelerates me



S_1 view

Force balance: inertial force back, Spring force forward No motion in my frame.

Special Case: uniformly rotating frame - Ω $\frac{\text{rads}}{\text{sec}}$

Remark: ω & Ω denote the rotational velocity - $\frac{\text{rad}}{\text{sec}}$ or $\frac{\text{deg}}{\text{sec}}$ or rpm

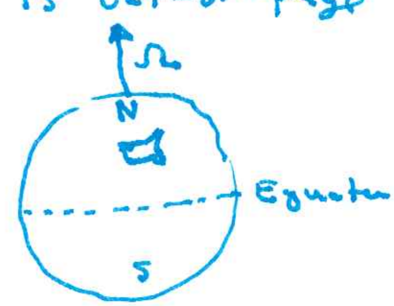
The magnitude of the vector is that rate

The direction is given by right hand rule.

Eg

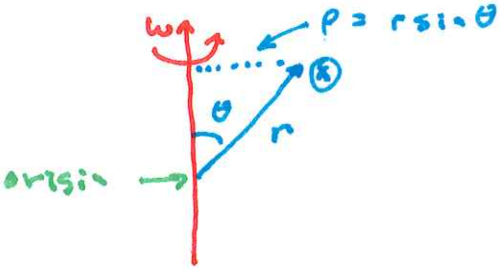
→ For this spinning disk $\vec{\omega}$ is out-of-page

Often the rotating frame is Earth



Here in MN $\vec{\Omega}$ points toward North Star

Consider a point & a vector $\vec{r}$ pointing from origin (on spin axis) to that point:



Speed of point $= \dfrac{2\pi\rho}{T} = \omega\rho = \omega r \sin\theta$

T ← period

direction of velocity = into page

$\vec{v} = \vec{\omega} \times \vec{r}$ gives correct magnitude/direction

The above derivation applies to any rotating vector Q

$$\frac{d}{dt}\vec{Q} = \overline{\omega \times Q}$$

Question: what if in the rotating frame S_1 $\vec{Q}$ shows change?

Let $\hat{e}_i$ be the coordinate axes in the rotating frame S_1

Note: $\underbrace{\frac{d}{dt}\hat{e}_i}_{\text{time rate of change of } \hat{e}_i \text{ in frame } S_0} = \Omega \times e_i$ ← assumed constant rotation speed of frame S_1

$\vec{Q} = \sum Q_i \hat{e}_i$ ← the components of $\vec{Q}$ in the rotating frame (I'm assuming these are changing)

$$\underset{\text{in } S_0}{\left(\frac{d\vec{Q}}{dt}\right)_0} = \sum \frac{dQ_i}{dt}\hat{e}_i + \sum Q_i \overline{\Omega \times e_i} = \underset{\text{in } S_1}{\left(\frac{d\vec{Q}}{dt}\right)_1} + \overline{\Omega \times Q}$$

Apply twice to $\vec{r}$ to get acceleration:

$$\left(\frac{d}{dt}\vec{v}\right)_0 = \left(\frac{d\vec{v}}{dt}\right)_1 + \overline{\Omega \times v} = \left(\frac{d^2\vec{r}}{dt^2}\right)_1 + \vec{\Omega} \times \left(\frac{d\vec{r}}{dt}\right)_1 + \Omega \times \left(\frac{d\vec{r}}{dt}\right)_1 + \overline{\Omega \times (\Omega \times r)}$$

$$\left(\frac{d}{dt}\vec{r}\right)_0 = \left(\frac{d\vec{r}}{dt}\right)_1 + \vec{\Omega} \times \vec{r}$$

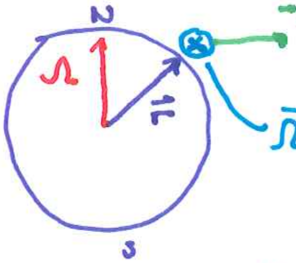
$$= \left(\frac{d^2\vec{r}}{dt^2}\right)_1 + 2\,\overline{\Omega \times v} + \overline{\Omega \times \Omega \times r}$$

(v in S_1)

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_1 = \vec{F} - m\Big(\underbrace{2\overline{\Omega \times v}}_{\text{Coriolis force.}} + \overline{\Omega \times \Omega \times r}\Big)$$

↑ centrifugal force

Centrifugal Force $-m\,\overrightarrow{\Omega\times\Omega\times r}$

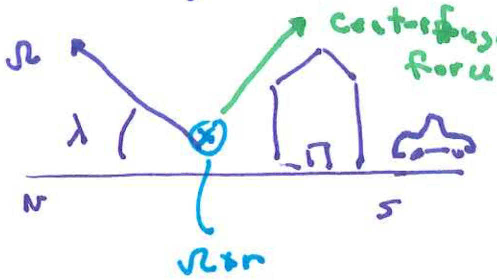


$\overrightarrow{-\Omega\times\Omega\times r}$ } cylindrically outward

magnitude $=\Omega^2\rho$

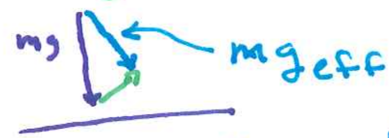
$\overrightarrow{\Omega\times r}$

magnitude $=\Omega r\sin\theta$
$=\Omega\rho$



On the equator the centrifugal force is straight up; at the north pole it is zero.

Note: The centrifugal force like $\vec{g}$ is proportional to m — what we call $\vec{g}$ is really the combo of gravity & centrifugal

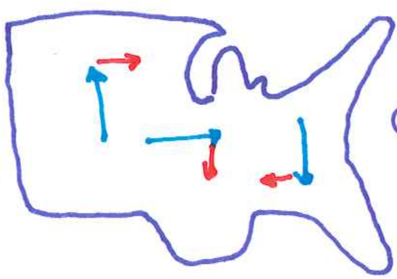


we declare things "level" if they are $\perp$ to geff ie balls don't roll off as no net "force"

Coriolis Force. $-2m\,\overrightarrow{\Omega\times V}$ ← like a magnetic field $\vec{B}=-2m\vec{\Omega}$

The vertical component of Ω is $\Omega\sin\lambda$ ← latitude

For the following we ignore the horizontal component of Ω — in the end it does not do much.



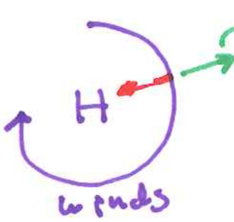
⊙ Ω assumed straight up.

velocity vectors

Coriolis force vectors

Coriolis force deflects speeding bullets to right

High Pressure systems in US



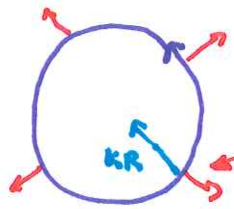
pressure force balanced by Coriolis (in fact a bit less than Coriolis to supply centripetal force for circular motion)

Remark: its an urban legend that Coriolis controls water flow down drains — needs a large scale to have noticeable effect

Foucault Pendulum I – consider conical motion

(for notational ease spring force $-k\vec{r}$ replaces gravity $mg\sin\theta$)

Ω ⟳



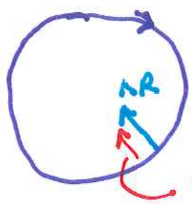
$2m\Omega v$ outward

ωr where ω = angular velocity of conic pendulum

$$KR - 2m\Omega\omega R = m\omega^2 R$$

$$\frac{K}{m} = \omega_0^2 = \omega^2 + 2\omega\Omega \rightarrow \omega_1 = \omega_0 - \Omega$$

approx



$2m\Omega\omega R$

$$KR + 2m\omega\Omega R = m\omega^2 R$$

$$\frac{K}{m} = \omega_0^2 = \omega^2 - 2\omega\Omega \rightarrow \omega_2 = \omega_0 + \Omega$$

approx

Note: $\left.\begin{array}{l}\omega_0 \rightarrow \text{period of a few seconds} \\ \Omega \rightarrow \text{period of a day}\end{array}\right\} \Omega \ll \omega_0$

Given the very small difference in periods – how detect?

Note the system is linear so superposition applies –

Consider superposition of oppositely rotating solutions –

$$\underset{\substack{\uparrow \\ \text{rotates}}}{e^{i\omega_1 t}} + \underset{\substack{\uparrow \\ \text{rotate}}}{e^{-i\omega_2 t}} = e^{i\left(\frac{\omega_1-\omega_2}{2}\right)t}\left[e^{i\left(\frac{\omega_1+\omega_2}{2}\right)t} + e^{-i\left(\frac{\omega_1+\omega_2}{2}\right)t}\right]$$

$$= e^{i\left(\frac{\omega_1-\omega_2}{2}\right)t}\left[\cos(\omega_0 t)\, 2\right]$$

$$\underset{\substack{\uparrow \\ \text{slow rotation}}}{e^{-i\Omega t}}\; \underset{\substack{\uparrow \\ \text{normal back \& forth}}}{2\cos(\omega_0 t)}$$