Foucault the hard way:

$$L = \frac{1}{2}(\dot x^2 + \dot y^2) + \Omega(\dot y x - \dot x y) - \frac{1}{2}\omega^2(x^2+y^2)$$

$$\boxed{x} \quad \frac{d}{dt}(\underbrace{\dot x - \Omega y}_{p_x}) = \ddot x - \Omega \dot y = -\omega^2 x + \Omega \dot y \Rightarrow \ddot x = -\omega^2 x + 2\Omega \dot y$$

$$\boxed{y} \quad \frac{d}{dt}(\underbrace{\dot y + \Omega x}_{p_y}) = \ddot y + \Omega \dot x = -\omega^2 y - \Omega \dot x \Rightarrow \ddot y = -\omega^2 y - 2\Omega \dot x$$

$$-\frac{1}{2}(\dot x^2 + \dot y^2)$$

$$H = \sum p_i \dot q_i - L = (\dot x - \Omega y)\dot x + (\dot y + \Omega x)\dot y - \Omega(\dot y x - \dot x y) + \frac{1}{2}\omega^2(x^2+y^2)$$

$$= \frac{1}{2}(\dot x^2 + \dot y^2) + \frac{1}{2}\omega^2(x^2+y^2)$$

$$= \frac{1}{2}\left[(p_x + \Omega y)^2 + (p_y - \Omega x)^2\right] + \frac{1}{2}\omega^2(x^2+y^2)$$

$$\dot x = \frac{\partial H}{\partial p_x} = p_x + \Omega y \qquad \dot p_x = -\frac{\partial H}{\partial x} = (p_y - \Omega x)\Omega - \omega^2 x$$

$$\dot y = \frac{\partial H}{\partial p_y} = p_y - \Omega x \qquad \dot p_y = -\frac{\partial H}{\partial y} = -(p_x + \Omega y)\Omega - \omega^2 y$$

in pure 2d SHO $A_{ij} = \omega^2 x_i x_j + p_i p_j$ is conserved

Trace $\to 2E$; det $\to \omega^2 L^2$

$$\dot A_{xy} = \omega^2\left[(p_x + \Omega y)y + x(p_y - \Omega x)\right] + \{(p_y - \Omega x)\Omega - \omega^2 x\}p_y + p_x\{-(p_x + \Omega y)\Omega - \omega^2 y\}$$

$$= \Omega\left[\omega^2(y^2 - x^2) + p_y^2 - p_x^2\right] + O(\Omega^2)$$

$$= \Omega\left[A_{yy} - A_{xx}\right]$$

$$\dot A_{xx} = 2\omega^2 x(p_x + \Omega y) + 2p_x\{(p_y - \Omega x)\Omega - \omega^2 x\}$$

$$= 2\Omega(\omega^2 xy + p_x p_y) = 2\Omega A_{xy} + O(\Omega^2)$$

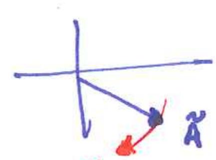
$$\dot A_{yy} = 2\omega^2 y(p_y - \Omega x) + 2p_y\{-(p_x + \Omega y)\Omega - \omega^2 y\}$$

$$= -2\Omega(\omega^2 xy + p_x p_y) = -2\Omega A_{xy}$$

$$2\dot A_{xy} = 2\Omega[A_{yy} - A_{xx}] \qquad \tilde A = (A_{xx} - A_{yy}, 2A_{xy})$$

$$[\dot A_{yy} - \dot A_{xx}] = -4\Omega A_{xy}$$

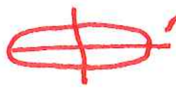
note: Tr & det still conserved $O(\Omega^2)$



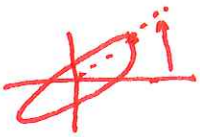
rate $= 2\Omega$

$$L = \tfrac{1}{2}(\dot{x}^2+\dot{y}^2) + \Omega(\dot{y}x - y\dot{x}) - \tfrac{1}{2}\omega^2(x^2+y^2)$$

$$E_1 = \tfrac{1}{2}\dot{x}^2 + \tfrac{1}{2}\omega^2 x^2 \rightarrow x_{max} = a \; ; \; \dot{x} = \pm\omega a = p_x$$



$$\vec{r} = (a,0)$$
$$\vec{p} = \dot{\vec{r}} = (0, \pm\omega b)$$



$$\tilde{r} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} ac \\ as \end{pmatrix}$$

$$\tilde{p} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} 0 \\ \pm\omega b \end{pmatrix} = \begin{pmatrix} \mp\omega b s \\ \pm\omega b c \end{pmatrix}$$

$$A_{ij} = \omega^2 x_i x_j + p_i p_j$$

$$A_{11} = \omega^2 a^2 c^2 + \omega^2 b^2 s^2$$

$$- \; A_{22} = \omega^2 a^2 s^2 + \omega^2 b^2 c^2$$

$$A_{11} - A_{22} = \omega^2\left[a^2(c^2-s^2) - b^2(c^2-s^2)\right] = \omega^2(a^2-b^2)(c^2-s^2)$$

$$A_{12} = \omega^2 a^2 cs - \omega^2 b^2 cs = (a^2-b^2)\omega^2 cs$$

$$\frac{2A_{12}}{A_{11}-A_{22}} = \frac{2cs}{c^2-s^2} = \tan(2\theta)$$

$$\tilde{A} = ((A_{11}-A_{22}), 2A_{12})$$

$$\left.\begin{aligned} 2\dot{A}_{12} &= -2\Omega(A_{11}-A_{22}) \\ (A_{11}-A_{22})^{\cdot} &= +2\Omega(2A_{12}) \end{aligned}\right\} \Rightarrow$$

$$\dot{\phi} = 2\Omega$$

$$2\theta$$

$$\rightarrow \underline{\underline{\dot{\theta} = \Omega}}$$

$$2A_{12} = (a^2-b^2)\omega^2 \sin 2\theta$$

$$A_{11} - A_{22} = (a^2-b^2)\omega^2 \cos 2\theta$$

$$b = a(1-e^2)^{1/2}$$

$$(2A_{12})^2 + (A_{11}-A_{22})^2 = \Big[\underbrace{(a^2-b^2)}_{=a^2e^2}\omega^2\Big]^2 = \left[a^2e^2\omega^2\right]^2$$

$$|\tilde{A}| = a^2 e^2 \omega^2$$

since $\tilde{A}$ rotates $\Rightarrow$ same magnitude $\Rightarrow$ same e