Damped SHO: $\Sigma F = -kx - \underbrace{bv}_{} = ma$

quadratic would be more realistic but more difficult

Solve using $x = e^{rt}$

$\dot{x} = r e^{rt}$

$\ddot{x} = r^2 e^{rt}$

Note: Like Laplace Transform where used e^{st}

$0 = ma + bv + kx$

$0 = \ddot{x} + \underbrace{\frac{b}{m}}_{2\beta}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_0^2}x$

$0 = \ddot{x} + 2\beta\dot{x} + \omega_0^2 x$

$0 = \underbrace{(r^2 + 2\beta r + \omega_0^2)}_{\text{quadratic equation}} e^{rt}$

$r = \frac{-2\beta \pm \sqrt{(2\beta)^2 - 4\omega_0^2}}{2}$

$= -\beta \pm \underbrace{\sqrt{\beta^2 - \omega_0^2}}$

if >0 "over damped"

if <0 "under damped"

if $=0$ "critically damped"

$r = -\beta \pm i\underbrace{\sqrt{\omega_0^2 - \beta^2}}_{\text{define } \omega_1}$

2 adjustible constants that could be matched with x_0 & v_0

over damped $x(t) = Ae^{r_+ t} + Be^{r_- t}$

these are both negative so expo decay with time

critical damped: $x(t) = Ae^{-\beta t} + Bte^{-\beta t}$

under damped: $x(t) = \left[Ae^{+i\omega_1 t} + Be^{-i\omega_1 t}\right]e^{-\beta t}$

$= \left[\tilde{A}\cos\omega_1 t + \tilde{B}\sin\omega_1 t\right]e^{-\beta t}$

$= \tilde{\tilde{A}}\cos(\omega_1 t - \delta)\, e^{-\beta t}$

equivalent forms all with 2 adjustable constants

The adjustible constants could be expressed in terms of x_0 & v_0 but its a mess.

Driven Damped SHO: $\sum F = F_0 \cos(\omega t) - kx - bv = ma$

$$\underbrace{\frac{F_0}{m}}_{f_0} \cos(\omega t) = a + \underbrace{\frac{b}{m}}_{2\beta} v + \underbrace{\frac{k}{m}}_{\omega_0^2} x$$

Note: this is an adjustable freq Not ω_0

Homogeneous solutions are damped SHO found previously eg $\tilde{A}\cos(\omega_1 t - \delta)$

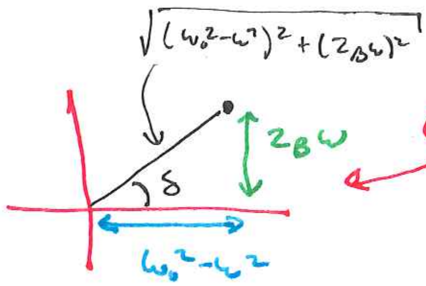
$$f_0 \cos(\omega t) = \ddot{x} + 2\beta \dot{x} + \omega_0^2 x$$

seek a particular solution to

$$f_0 e^{i\omega t} = \ddot{x} + 2\beta\dot{x} + \omega_0^2 x$$

Response must be at driving freq [Bloch, Floquet]

$x = C e^{i\omega t}$ $\longrightarrow$ $f_0 e^{i\omega t} = \left[(i\omega)^2 + 2\beta i\omega + \omega_0^2\right] C e^{i\omega t}$



$$\frac{f_0}{(\omega_0^2-\omega^2) + 2i\beta\omega} = C = |C| e^{-i\delta}$$

$$(\omega_0^2-\omega^2) + 2i\beta\omega = \sqrt{(\omega_0^2-\omega^2)^2+(2\beta\omega)^2}\, e^{i\delta}$$

any complex # can be written as magnitude angle format

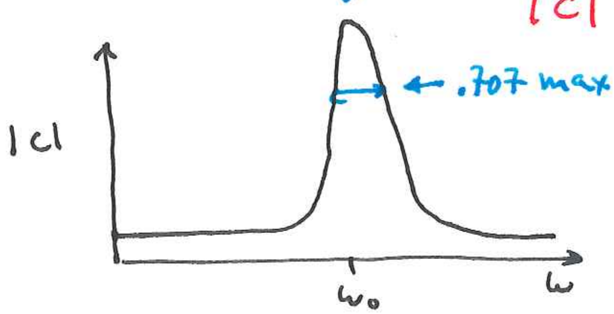
$$\underbrace{\frac{f_0}{\sqrt{(\omega_0^2-\omega^2)^2+(2\beta\omega)^2}}}_{|C|} e^{-i\delta} = C$$

phase

Peak near ω_0

$\omega_2 = \sqrt{\omega_0^2 - 2\beta^2}$

The difference between ω_0 & ω_2 is almost never important



Describe peak as

$\underbrace{\text{Full width}}_{\approx 2\beta}$ at $\underbrace{\text{half max}}_{\text{really } \frac{1}{\sqrt{2}} \text{ max}}$

$$Q = \frac{\omega_0}{2\beta}$$