Circular Orbit (as in 191)

$\alpha = G m_1 m_2$

$\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

$\dfrac{\alpha}{\mu} = G(m_1 + m_2)$

$$F = \frac{-\alpha}{r^2} = -\mu \underbrace{\omega^2 r}_{\text{centripetal acceleration}}$$

both force and acceleration point toward attracting mass

$$\frac{\alpha}{\mu r^3} = \omega^2$$

$KE = \frac{1}{2}\mu(\omega r)^2 = \frac{1}{2}\mu \frac{\alpha}{\mu r}$

$PE = \frac{-\alpha}{r}$

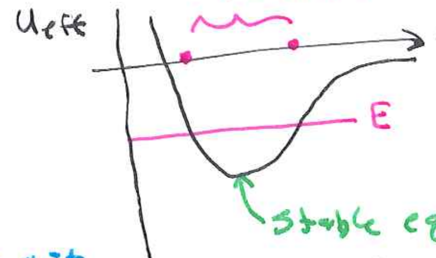
$E = KE + PE = \frac{-\alpha}{2r}$

$$L = \mu r^2 \omega = \sqrt{\mu \alpha r^3}$$

Lagrange: $U_{eff} = \underbrace{\dfrac{L^2}{2\mu r^2}}_{\substack{\text{centrifugal potential} \\ = \text{transverse KE}}} + U$ ← For small r this → ∞; For larger r this → 0^-

$U = \frac{-\alpha}{r}$

turning pts: rmin/rmax



stable equilibrium circular orbit

→ as shown in textbook: seek orbit ie $r(\phi)$, switch variables $u = \frac{1}{r}$ seek $u(\phi)$ produces a solvable diff eq → $r = \dfrac{a(1-e^2)}{1+e\cos\phi}$ ← ellipse

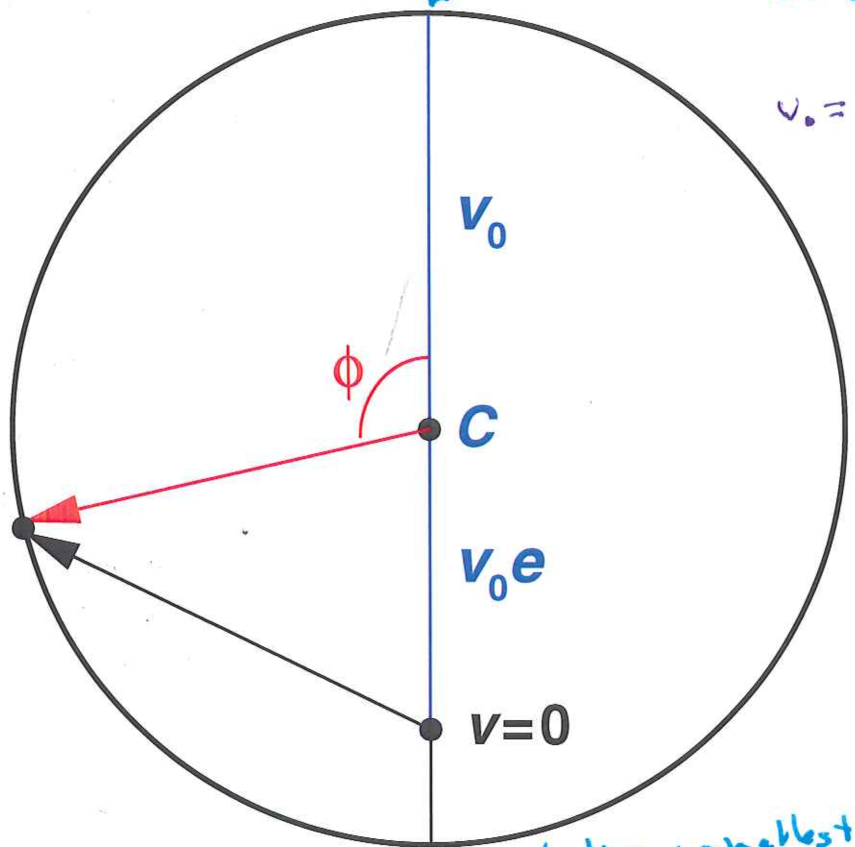
Using graphical approach: $\frac{d\vec{v}}{dt}$ points in direction $\phi + 180°$ ie in $-\hat{r}$ direction (good - that is direction of force)

$$\left|\frac{d\vec{v}}{dt}\right| = v_0 \dot{\phi} = v_0 \frac{L}{\mu r^2} = \sqrt{\frac{\alpha}{\mu a(1-e^2)}} \frac{\sqrt{\alpha \mu a(1-e^2)}}{\mu r^2} = \frac{\alpha}{\mu r^2} \checkmark$$

$\phi = 0$

perihelion: largest velocity is in +y direction

$$v_0 = \sqrt{\frac{\alpha}{\mu a(1-e^2)}}$$



aphelion: smallest velocity in −y direction

$\phi = 180°$

from focus

$$r = \frac{a(1-e^2)}{1+e\cos\phi}$$

peri $= r_{min} = a(1-e)$

apo $= r_{max} = a(1+e)$

from center

eccentric anomaly

$$\vec{r} = (a\cos u,\ b\sin u)$$

$$b = a\sqrt{1-e^2}$$

$$\tan\frac{\phi}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2}$$

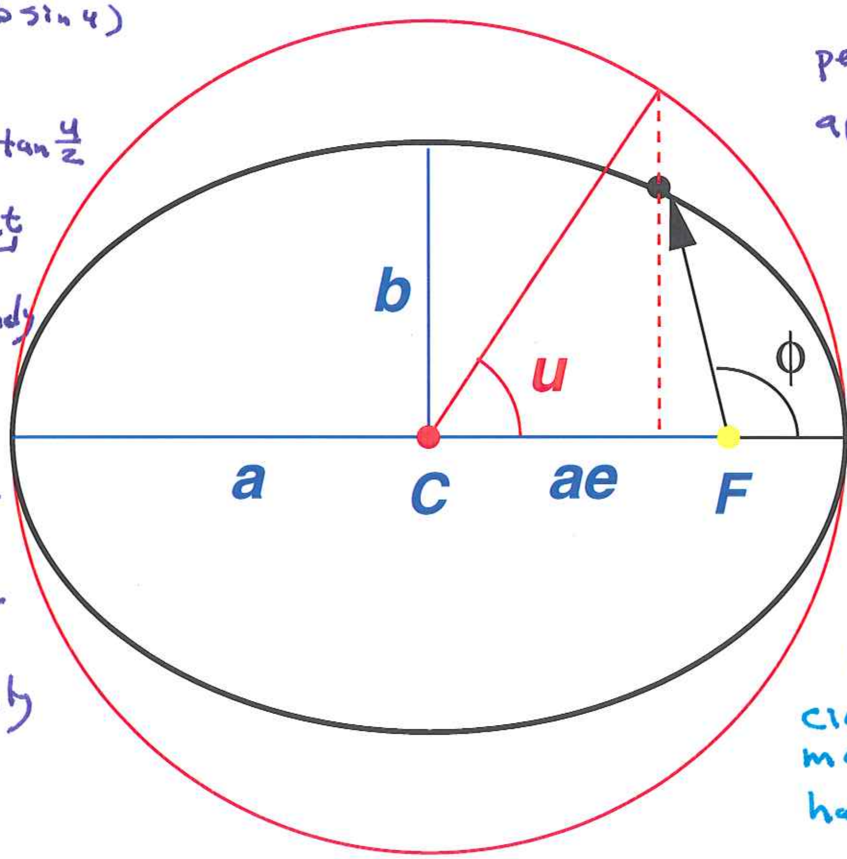
$$u - e\sin u = \underbrace{\omega t}_{\text{mean anomaly}}$$

ϕ = true anomaly

a = semi major axis

b = semi minor axis

e = eccentricity



$$\left.\begin{aligned} \omega &= \sqrt{\frac{\alpha}{\mu a^3}} \\ E &= \frac{-\alpha}{2a} \end{aligned}\right\}$$ do not depend on eccentricity

$$L = \sqrt{\alpha\mu a(1-e^2)}$$

circular orbits ($e=0$) have max L; line orbits ($e=1$) have min L