The tetrahedron is the simplest of the platonic solids: four equilateral triangles combined into a pyramid structure. It can also be thought of as a cube's top diagonal and a twisted bottom diagonal. There are 24 symmetry operations that leave a tetrahedron invariant (i.e., result in a shuffling of the vertices within in the same locations). Matrices are designed to perform such transformations. We examine below the symmetry operations that just involve rotations.

Find the spreadsheet `tetrahedron` from the class web site, `handouts` folder. The upper lhs contains cells to hold the values for the three Euler Angles: ϕ, θ, ψ; columns `F`–`H` contain the 3×3 rotation matrix defined by those Euler Angles. (Note use `=pi()` in the spreadsheet not some approximation of π.) The (x, y, z) location of the four vertices of the tetrahedron (labeled: A, B, C, D) occur below the matrix. Matrix multiplication results in the transformed point (x', y', z'). (Check out the spreadsheet formulas or decode the self-documentation.) A symmetry operation on the tetrahedron's vertices should just shuffle those points (i.e., each (x', y', z') outout should be one of the A,B,C,D inputs). Enter each of the below Euler Angle triplets, find which vertex (A,B,C,D) corresponds to the (x', y', z') output and record the results in the below table. The Trace (Tr: sum of the diagonal elements) of a rotation matrix equals $1 + 2\cos\alpha$ where α is the rotation angle of the matrix. Modify the spreadsheet to automatically calculate the Trace and calculate the corresponding angle α. Record these results also in the below table. Try to identify the rotation axis that would do the transformation.

Find below the Eular Angles defining 11 rotation symmetry operations. Fill in the table showing where each vertex (A, B, C, D) is mapped by the operation, the Trace and rotation angle. I've filled in the first row as an example.

ϕ	θ	ψ	A	B	C	D	Tr	α	axis
π	0	0	B	A	D	C	-1	$180°$	z
0	π	0							
$-\pi/2$	π	$\pi/2$							
$\pi/2$	$\pi/2$	0							
π	$\pi/2$	$\pi/2$							
$-\pi/2$	$\pi/2$	π							
0	$\pi/2$	$-\pi/2$							
$-\pi/2$	$\pi/2$	0							
π	$\pi/2$	$-\pi/2$							
$\pi/2$	$\pi/2$	π							
0	$\pi/2$	$\pi/2$							

These transformations may make more sense to you if you think about rotating the *object* in the opposite order. You usually think of a tetrahedron sitting on its triangular base. Try: $\phi = \pi/3, \theta = \arctan(\sqrt{2}), \psi = -\pi/4$. Which vertex is on top? Which bottom vertex is on the y' axis?