Consider a forcing function $f(t)$ that is a single-humped sinusoidal that starts at $t = 0$ and ends (returns to zero) at $t = T$ and has a total impulse (area) of 1:

$$f(t) = \begin{cases} 0 & t < 0 \\ \dfrac{\pi}{2T}\,\sin(\pi t/T) & 0 < t < T \\ 0 & t > T \end{cases} \tag{1}$$

If T is short compared to the free oscillation period (i.e., $T \ll 2\pi/\omega_1$) the result should be similar to a delta function impulse, i.e., a sudden change in velocity $\Delta v = 1$. If T is long compared to the free oscillation period you might expect the oscillator to 'ride out' the force. Following the example from class (which is in `greens.m`) start with variables not given any numerical value:

```
w1=Sqrt[w0^2-b^2]
begin=Pi/(2 T) Integrate[Sin[Pi tp/T] Exp[-b (t-tp)]Sin[w1(t-tp)],{tp,0,t}]/w1
end=Pi/(2 T) Integrate[Sin[Pi tp/T]  Exp[-b (t-tp)]Sin[w1(t-tp)],{tp,0,T}]/w1
x[t_]=If[t<T,Evaluate[begin],Evaluate[end]]
```

Q: The online example had terms `begin`, `start`, and `end`. Why are there only two terms in this problem; i.e., explain why the above Mathematica code matches the general formula we proved for a Green's function solution:

$$x(t) = \int_{-\infty}^{t} f(t')\,G(t,t')\,dt' \tag{2}$$

$$= \int_{-\infty}^{t} f(t')\,\frac{1}{\omega_1}\,e^{-\beta(t-t')}\,\sin(\omega_1(t-t'))\,dt' \tag{3}$$

For our usual example ($\omega_0 = 1; \beta = .01$) plot solutions $x(t)$ for the cases $T = 20\pi$, 10π, 5π, 2.5π, π, $\pi/10$. (It would be nice to put these in a `GraphicsGrid` to save paper.) For the the case $T = \pi/10$ also plot $x'(t)$ and confirm that—immediately post-impulse—the velocity is 1 as expected.